GREAT BOOKS OF THE WESTERN WORLD

ROBERT MAYNARD HUTCHINS *EDITOR IN CHIEF*

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NEWTON

HUYGENS

MATHEMATICAL PRINCIPLES OF NATURAL PHILOSOPHY

OPTICS

BY SIR ISAAC NEWTON

TREATISE ON LIGHT

BY CHRISTIAAN HUYGENS

WILLIAM BENTON *Publisher*

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GENERAL CONTENTS

MATHEMATICAL PRINCIPLES OF NATURAL PHILOSOPHY

BIOGRAPHICAL NOTE

SIR ISAAC NEWTON 164?-1727

NEWTON was born at Woolsthorpe Lincoln shire o Chr tmas D y 164.. His father a small farmer died a few mo ths before his birth, and when in 164 his mother married the rector of North Witham Newton was left with his maternal grandmothe at Woolsthorpe Aft having acqu red the rudiments of educat on at small schools close by Newton was sent t the age of twelve t the grammar school at Grantham where he lived in the

taste and ptitude fo mechanical co triv nces he made windmill. w ter-clocks kites, a d sun-dials and h is said t have in ented a f ur wheel carriage which was to be moved by the ri l

Afte th death of h second husband in 16.6 Newto mothe returned t Woolsth rpe and rem ed her eldes son from hool so that h might prepare himself to manage the farm. But t was soon evide t that his in-

boys wh perf rmed menial services in return fo their expenses. Although there is n record f his formal progress as stud t Newton is kn n t ha read widely in matl mat cs d mechanics. His first reading at Cambridge was in th pt cal w ks f Keple H turned to Eu li l because he was both red by his in bility to comprehend certain diagrams n a book astrology he l ad bo ght t a fair finding ts proposit ns self-e de t h p t it aside as a trifling book, until his teache Isaac Barrow ind ced him to take up the book again It ppears to have been tl e study f Descartes *Geom try* which inspired him to d original math mat cal work. I a mall com monplace book k pt by Newt n as n under graduate there re se eral rt les o a ular secti ns and the squaring f curves sev ral calculations bout musical notes, geometrical

problems from Vieta and Van Schooten an notati ns out of Wallis *Arithmetic of Inf nities* together with observat ons on refract on on

taking the Bachelor's degree in 1665 that Newton discovered the b omial theorem and made the first notes on his disco ery of the

shire where he co ducted *experiments in optics* and chemistry and continued his mathematical speculat ons From this forced retire-

same time his w rk on optics led to his explanation of the composit n of white light Of the w k he accomplished in these years Newton late remarked All this was in the two years f 1665 and 1666 fo in those years I was in the prime of my age for in e t on and minded M th matics and Philosophy more than at any t me since

On the re-opening of Trinity College in 1667 Newton was elected a f llow and two years later a l ttl bef re his twenty-seventh birth day he was appointed Lu asian professo of math matics succeeding his friend and teacher Dr Barrow Newton had already built a reflecting telescope in 1668 the second telescope of his making he presented to the Royal

to in l e Hooke Lucas Linus a d others N to who alw ys found contro rsy distasteful blamed my own imprudence for parti g with so substantial a blessi g as my qui t to run afte a shadow His papers on

optics the most important of wh h w –
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uu uot that Newton began to think of making known his work on gravity Hooke Halley and Sir Christopher Wren had independently come to some notion of the law of gravity but were not having any success in explaining the orbits of the planets In that year Halley consulted Newton on th e
and was as
solved it N
rems and so u pɩoɒɩcms which proved to be the nucleus of his major work In some seventeen or eighteen months during 1685 and 1686 he wrote in Latin the *Mathematical Principles of Natural Philosophy* Newton thought for some time of suppressing the third book and it was only Halley s insistence that preserved it Halley also took upon himself the cost of publishing the work in 1687 after the Royal Society proved unable to meet its cost The book caused great excitement throughout Europe and in 1689 Huygens at that time the most famous scientist came to England to make the pers n l

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allegiance and supremacy at the university Newton was elected parliamentary member for Cambr lge
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1696 he solved overnight a probl m off
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I u t to ıcel the pulse of the English analysts He was much occupied to his own distress with two mathematical controversies one regarding the astronomical observations of the astronomer royal and the other with Leibnitz regarding the invention of calculus He also worked on revisions for a second edition of the *Principles* which appeared in 1713

Newton s scientific work brought him great fame He was a popular visitor at the Court and was knighted in 1705 Many honors came to him from the continent he was in correspondence with all the leading men of science and visitors became so frequent as to prove a serious discomfort Despite his fame Newton maintained his modesty Shortly before his death he remarked I do not know what I may appear to the world but to m lf I seem
the se
then f
shell than ordinary whilst the great o f
t

I
c u u had begun to study the proph ecies In that year he wrote in the form of a letter to Locke an *Historical Account of Two Notabl Corr*

c ypse and other works of exege is

After 1725 Newton s health was much impaired and his duties at the Mint were discharged by a deputy In February 172? he presided for the last time at the Royal Society of which he had been president since 1703 and died on March 20 1727 in his eighty fifth year He was buried in Westminster Abbey after lying in state in the Jerusalem Chamber

CONTENTS

PREFACE TO THE FIRST EDITION

SINCE the ancients (as we are told by Pappus) esteemed the science of mechanics of greatest importance in the investigation of natural things and ... substantial forms and occult qualities, have endeavored

work with perfect accuracy he would be the most perfect mechanic of all for the description of right lines and circles upon which geometry is founded belongs to mechanics Geometry does not teach us to draw these lines, but re-

that geometry is commonly referred to their magnitude and mechanics to the motion In this sense rational mechanics will be the science of motions resulting from any forces whatsoever and of the forces required to produce any motions accurately proposed and demonstrated This part of mechanics as far as it extended to the five powers which relate to manual arts was cultivated by the ancients who considered gravity (it not being a manual power) no otherwise than in moving weights by those powers But I consider philoso-

therefore I offer this work as the mathematical principles of philosophy for the whole burden of philosophy seems to consist in this—from the phenomena of motions to investigate the forces of nature and then from these forces to demonstrate the other phenomena and to this end the general propositions in the first and second book are directed

the several planets Then from these forces by othe

... principles for I am induced by many reasons to suspect that they may all depend upon certain forces by which the particles of bodies by some causes hitherto unknown are either mutually impelled towards one another and cohere in regular figures or are repelled and recede from one another These forces being unknown philosophers have hitherto attempted the search of Nature in vain but I hope the principles here laid down will afford some light either to this or some truer method of philosophy

In the publication of this work the mo

Edmund Halley not only assisted

... pressed me to communicate the same to the Royal Society who afterwards by their kind encouragement and entreaties engaged me to think of publishing them But after I had begun to consider the inequalities of the lunar motions and had entered upon some other things relating to the laws and measures of gravity and other forces and the figures that would be described by bodies attracted according to given laws and the motion of seve

of bodies in resisting medium

the orbits of the comets anc

made a search into those ma ... could put forth the whole together What relates to the lunar motions (being imperfect) I have put all together in the corollaries of Prop 66 to avoid being obliged to propose and distinctly demonstrate the several things there contained in a method more prolix than the subject deserved and interrupt the series of the other propositions Some things found out after the rest I chose to insert in places less suitable rather than change the number of the propositions and the citations I heartily beg that what I have here done may be read with forbearance and that my labors in a subject so difficult may be examined not so much with the view to censure as to remedy their defects

IS NEWTON

Cambridge Trinity College *May* 8 1686

PREFACE TO THE SECOND EDITION

... and confirmed by new experiments In the third book the lunar theory and the precession of the equinoxes were more fully deduced from their principles and the theory of the comets was

confirmed by more examples of the calculation of their orbits done also with greater accuracy

IS NEWTON

London March 28 1713

PREFACE TO THE THIRD EDITION

h care by Henry Pemberton M D ... things in the second book on

... there are added new ... of the diameters of ... d on the comet which appeared in the year 1680 made in Germany ... month of November by Mr Kirk which have lately come to my hands. By the help of these it becomes apparent how nearly parabolic orbits represent the motions of comets The ... d termined somewhat more accurately than before by

by Mr Bradley Professor of Astronomy at Oxford

IS NEWTON

London Jan 12 1725-6

the several planets Then from these forces by other propositions which are also mathematical I deduce the *motions of the planets the comets the moon* and the sea I wish we could derive the rest of the phenomena of Nature by the same kind of reasoning from mechanical principles for I am induced by many reasons to suspect that they may all depend upon certain forces by which the particles of bodies by some causes hitherto unknown are either mutually impelled towards one another and cohere in regular figures or are repelled and recede from one another These forces being unknown philosophers have hitherto attempted the search of Nature in vain but I hope the principles here laid down will afford some light either to this or some truer method of philosophy

In the publication of this work the *most acute and universally learned* Mr Edmund Halley not only assisted me in correcting the errors of the press and

same to the Royal Society who afterwards by their kind encouragement and entreaties engaged me to think of publishing them But after I had begun to consider the inequalities of the lunar motions and had entered upon some other things relating to the laws and measures of gravity and other forces and the figures that would be described by bodies attracted according to given laws and the motion of several bodies moving among themselves the motion of bodies in resisting mediums the forces densities and motions of mediums the orbits of the comets and such like I deferred that publication till I had made a search into those matters and could put forth the whole together What relates to the lunar motions (being imperfect) I have put all together in the corollaries of Prop 66 to avoid being obliged to propose and distinctly demonstrate the several things there contained in a method more prolix than the subject deserved and interrupt the series of the other propositions Some things found out after the rest I chose to insert in places less suitable rather than change the number of the propositions and the citations I heartily beg that what I have here done may be read with forbearance and that my labors in a subject so difficult may be examined not so much with the view to censure as to remedy their defects

IS NEWTON

Cambridge *Trinity College May 8 1686*

PREFACE TO THE SECOND EDITION

IN this second edition of the *Principia* there are many emendations and some additions In the second section of the first book the determination of forces by which bodies may be made to revolve in given orbits is illustrated and enlarged In the seventh section of the second book the theory of the resistances of fluids was more accurately investigated and confirmed by new experiments In the third book the lunar theory and the precession of the equinoxes were more fully deduced from their principles and the theory of the comets was

DEFINITIONS

DEFINITION I

Th[illegible] f matter is the measure of the same, arising from its densi[illegible] a[illegible]d b[illegible]

[illegible] in [illegible] of a trip[illegible] space, sextuple in qua[illegible] ue-snow, and fine dust or powders, that are condensed by [illegible] con-faction, and of all bodies that are by any causes whatever differently condensed. I have no regard in this place to a medium, if any such there is, that freely pervades the interstices between the parts of bodies. It is this quantity that I mean hereafter everywhere under the name of body or mass. And the same is known by the weight of each body, for it is proportional to the weight, as I have found by experiments on pendulums, very accurately made, which shall be shown hereafter.

DEFINITION II

The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjointly.

The motion of the whole is the sum of the motions of all the parts; and therefore in a body double in quantity, with equal velocity, the motion is double; with twice the velocity, it is quadruple.

DEFINITION III

The vis insita, or innate force of matter, is a power of resisting, by which every body, as much as in it lies, continues in its present state, whether it be of rest, or of moving uniformly forwards in a right line.

This force is always proportional to the body whose force it is and differs nothing from the inactivity of the mass, but in our manner of conceiving it. A body, from the inert nature of matter, is not without difficulty put out of its state of rest or motion. Upon which account, this vis insita may, by a most significant name, be called inertia (vis inertiæ) or force of inactivity. But a body only exerts this force when another force, impressed upon it, endeavors to change its condition; and the exercise of this force may be considered as both resistance and impulse; it is resistance so far as the body, for maintaining its present state, opposes the force impressed; it is impulse so far as the body, by not easily giving way to the impressed force of another, endeavors to change the state of that other. Resistance is usually ascribed to bodies at rest, and impulse to those in motion; but motion and rest, as commonly conceived, are only relatively distinguished; nor are those bodies always truly at rest, which commonly are taken to be so.

— ... nor could the moon without

... e tu ... r
... ven velocity
... ns of a given

may be made to deviate from ...
force

The quantity of any centripetal force may be considered as of three kinds absolute accelerative and motive

DEFINITION VI

The absolute quantity of a centripetal force is the measure of the same proportional to the efficacy of the cause that propagates it from the centre through the spaces round about

Thus the magnetic force is greater in one loadstone and less in another according to their sizes and strength of intensity

DEFINITION VII

The accelerative quantity of a centripetal force is the measure of the same proportional to the velocity which it generates in a given time

Th ... f ... f the same loadstone is greater at a less distance and less at ... ng
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re
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DEFINITION VIII

sort of quantity is the centripetency or propension of the whole body ... the centre or as I may say its weight and it is always known by the quantity of an equal and contrary force just sufficient to hinder the descent of the body

These quantities of forces we may for the sake of brevity call by the names of motive accelerative and absolute forces and for the sake of distinction consider them with respect to the bodies that tend to the centre to the places of those bodies and to the centre of force towards which they tend that is to say I refer the motive force to the body as an endeavor and propensity of the whole towards a centre arising from the propensities of the several parts taken

DEFINITION IV

An impressed force is an action exerted upon a body in order to change its state either of rest or of uniform motion in a right line

This force consists in the action only and remains no longer in the body when the action is over For a body maintains every new state it acquires by its inertia only But impressed forces are of different origins as from percussion from pressure from centripetal force

DEFINITION V

A centripetal force is that by which bodies are drawn or impelled or any way tend towards a point as to a centre

Of this sort is gravity by which bodies tend to the centre of the earth magnetism by which iron tends to the loadstone and that force whatever it is by which the planets are continually drawn aside from the re t l ne

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force as it
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I u o u y orce which restrains them to and detains them in their orbits which I therefore call centripetal would fly off in right lines with an uniform motion A projectile if it was not for the force of gravity would not deviate towards the earth but would go off from it in a right line and that with an uniform motion if the resistance of the air was taken away It is by its gravity that it is drawn aside continually from its rectilinear course and made to deviate towards the earth more or less according to the force of its gravity and the velocity of its motion The less its gravity is or the quantity of its matter or the greater the velocity with which it is projected the less will it deviate from a rectilinear course and the farther it will go If a leaden ball projected from the top of

given
line to
resista u taken away with a double or decuple velocity would fly twice or ten times as far And by increasing the velocity we may at pleasure increase the distance to which it might be projected and l m n h
ture of the line which it might describe
of 10 30 or 90 degrees or even might go
falls or lastly so that it might never fall to the earth but go forwards into the celestial spaces and proceed in its motion *in infinitum* And after the same manner that a projectile by the force of gravity may be made to revolve in an orbit and go round the whole earth the moon also either by the force of gravity if it is endued with gravity or by any other force that impels it towards the earth may be continually drawn aside towards the earth out of the rectilinear way which by its innate force it would pursue and would be made

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partly from the relative motion of the ship on the earth; and if the body moves
also relatively in the ship, its true motion will arise partly from the true mo-
m the relative motions as

carried towards the west, with a velocity expressed by 10 of those parts; A
sailor walks in the ship towards the east, with 1 part of the said velocity; then
the sailor will be moved truly in immovable space towards the east, with a
velocity of 10 001 parts, and relatively on the earth towards the west, with a
velocity of 9 of those parts.

Absolute time, in astronomy, is distinguished from relative, by the equation
or correction of the apparent time. For the natural days are truly unequal

of the magnetic force or the earth in the centre of the gravit ing force) or
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causes and

Wherefore the accelerative force will stand in the same rel t on

e same quantity of matter For the sum of the
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ould always be as the product of the body by the ac
celerative gravity So in those regions where the accelerative gravity is di
minished into one-half the weight of a body two or three times less will be four
or six times less

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mot

ous I anywhere take upon me to define the kind or
the manner of any action the causes or the physical reason thereof or that I
attribute forces in a true and physical sense to certain centres (which are only
mathematical points) when at any time I happen to speak of centres as at-
tracting or as endued with attractive powers

SCHOLIUM

Hitherto I have laid down the definitions of such words as are less known
and explained the sense in which I would have them to be understood in the
following discourse I do not define time space place and motion as being
well known to all Only I must observe that the common people conceive those
quantities under no other notions but from the relation they bear to sensible
objects And thence arise certain prejudices for the removing of which it will
be convenient to distinguish them into absolute and relative true and

I

ive apparent and common time is some sensible and
external (whether accurate or unequable) measure of duration by the means of
motion which is commonly used instead of true time such as an hour a day
a month a year

of the absolute spaces which our senses determine by its posi
tion to bodies and which is commonly taken for immovable space such is the
dimension of a subterraneous an aerial or celestial space determined by its
position in respect of the earth Absolute and relative space are the same in
figure and magnitude but they do not remain always numerically the same

— ... n motion are no other than parts of

and so on, until we come to ... example of the sailor Wherefore entire and absolute motions can be no ... wise determined than by immovable places and for that reason I did before refer those absolute motions to immovable places, but relative ones to movable places. Now no other places are immovable but those that from infinity to infinity do all retain the same given position one to another and upon this account must ever remain unmoved and do thereby constitute immovable space

The causes by which true and relative motions are distinguished one from the other are the forces impressed upon bodies to generate motion. True motion is neither generated nor altered but by some force impressed upon the body moved but relative motion may be generated or altered without any force impressed upon the body For it is sufficient only to impress some force on other bodies with which the former is compared that by their giving way that relation may be changed in which the relative rest or motion of this other body did consist. Again true motion suffers always some change from any force impressed upon the moving body but relative motion does not necessarily undergo any change by such forces. For if the same forces are likewise impressed on those other bodies, with which the comparison is made that the relative position may be preserved then that condition will be preserved in which the relative motion consists. And therefore any relative motion may be changed when the true motion remains unaltered and the relative may be preserved when the true suffers some change. Thus, true motion by no means consists in such relations.

The effects which distinguish absolute from relative motion are the forces of receding from the axis of circular motion. For there are no such forces in a circular motion purely relative but in a true and absolute circular motion they are greater or less according to the quantity of the motion. If a vessel hung by a long cord, is so often turned about that the cord is strongly twisted, then filled with water and held at rest together with the water thereupon, by the sudden action of another force it is whirled about the contrary way and while the cord is untwisting itself the vessel continues for some time in this motion the surface of the water will at first be plain, as before the vessel began

concave figure (as I have experienced) and the swifter the motion becomes, the higher will the water rise till at last performing its revolutions in the same times with the vessel, it becomes relatively at rest in it. This ascent of the water shows its endeavor to recede from the axis of its motion and the true and absolute circular motion of the water which is here directly contrary to the relative becomes known, and may be measured by this endeavor At first when the relative motion of the water in the vessel was greatest it produced no endeavor to recede from the axis the water showed no tendency to the circumference nor any ascent towards the sides of the vessel but remained of a plain surface and therefore its true circular motion had not yet begun. But after

necessity of this equation for determining the times of a phenomenon is evinced as well from the experiments of the pendulum clock as by eclipses of the satellites of Jupiter

As the order of the parts of time is immutable so also is the order of the parts of space Suppose those parts to be mo
will be moved (if the e
and spaces are as it w
All things are placed i ... ccession and in space as to order of situation It is from their essence or nature that they are places and that the primary places of things should be movable is absurd These are therefore the absolute places and translations out of those places are the only absolute motions

But because the parts of space cannot be seen or distinguished from one another by our senses therefore in their stead we use sensible measures of them For from the positions and distances of things from any body considered as immovable we define all places and then with respect to such places we estimate all motions considering bodies as transferred from some of those places into others And so instead of absol

otl
rea ... another And therefore as it is possible that in the remote regions of the fixed stars or perhaps far beyond them there may be some body absolutely at rest but impossible to know from the position of bodies to one another in our regions whether any of these do keep the same position to that remote body it follows that absolute rest cannot be determined from the position of bodies in our regions

It is a property of motion that the parts which retain given positions to their wholes do partake of the motions of those wholes For all the parts of revolving bodies endeavor to recede from the axis of motion and the impetus of bodies moving forwards arises from the joint impetus of all the parts Therefore if surrounding bodies are moved those that are relatively at rest within them will partake of their motion Upon which account the true and absolute motion of a body cannot be determined by the translation of it from those which only seem to rest for the external bodies ought not only to appear at rest but to be really at rest For otherwise all included bodies besides their translation from near the surrounding ones partake likewise of their true motions and though that translation were not made they would not be really at rest but only seem to be so For the surrounding bodies stand in the like relation to the surrounded as the exterior part of a whole does to the interior or as the shell does to the kernel but if the shell moves the kernel will also move as being part of the whole without any removal from near the shell

A property near akin to the preceding is this that if a place is moved whatever is placed therein moves along with it and therefore a body which is moved from a place in motion partakes also of the motion of its place Upon

know the determination of their motions. And thus we might find both the quantity and the determination of this circular motion, even in an immense vacuum, where there was nothing external or sensible with which the globes could be compared. *But now, if in that space some remote bodies were placed* that kept always a given position one to another, as the fixed stars do in our regions, we could not indeed determine from the relative translation of the globes among those bodies, whether the motion did belong to the globes or to the bodies. *But if we observed the cord, and found that its tension was that* very tension which the motions of the globes required, we might conclude the motion to be in the globes, and the bodies to be at rest; and then, lastly, from the translation of the globes among the bodies, we should find the determination of their motions. But how we are to obtain the *true motions from their* causes, effects, and apparent differences, and the converse, shall be explained more at large in the following treatise. For to this end it was that I composed it.

wards when the relative motion of the wate h l l towards the sides of the vessel proved its e this endeavor showed the real circular mot ing till it had acq e l in the ve of the w ... not can true circular motion be defined by such translation There is only one l

... y may perhaps partake of that one only true motion And therefore in their system the ...

t ... bodies truly at rest) and being carried together with their heavens partake of their motions and as parts of revolving wholes endeavor to recede from the axis of their motions

Wherefore relative quantities are not the quantities themselves whose names they bear but those sensible measures of them (e th whi if th time ... sible) measures are properly to be understood and the expression will be unusual and purely mathematical if the measured quantities themselves are meant On this account those violate the accuracy of language which ought to be kept precise who interpret these words for the measured quantities Nor do those less defile the purity of mathematical and philosophical truths who confound real quantities with their relations and sensible measures

It is indeed a matter of great difficulty to discover and effectually to distinguish the true motions of particular bodies from the apparent because the parts of that immovable space in which those motions are performed do by no means come under the observation of our senses Yet the thing is not altogether desperate for we have some arguments to guide us partly from the apparent motions which are the differences of the true motions partly from the forces which are the causes and effects of the true motions For instance if two globes kept at a given distance one from the other by means of a cord th t con them were r l d b tension of the circular ... forces should be impressed at once on the alternate faces of the globes to augment or diminish their circular motions from the increase or decrease of the tension of the cord we might infer the increment or decrement of their motions and thence would be found on what f h ... might those being ... and consequently the opposite ones that precede we should likewise

f th motions And thus we might find both the

g those bodies whether the m tension was that
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nd the determi
otions from their
hall be explained
at I composed it

AXIOMS, OR LAWS OF MOTION

LAW I

Every body continues in its state of rest or of uniform motion in a right line unless it is compelled to change that state by forces impressed upon it

Projectiles continue in their motions so far as they are not retarded by the resistance of the air or ...
whose parts by th
tions does not ce ... wise than as it is retarded by the air The greater bodies of the planets and comets meeting with less resistance in freer spaces preserve their motions both progressive and circular for a much longer time

LAW II

The change of motion is proportional to the motive force impressed and is made in the direction of the right line in which that force is impressed

If any force generates a motion a double force will generate double the motion a triple force triple the motion whether that force be impressed altogether and at once or gradually and successively And this motion (being always directed the same way with the generating force) if the body moved before is added to or subtracted from the former motion according as they directly conspire with or are directly contrary to each other or obliquely joined when they are oblique so as to produce a new motion compounded from the determination of both

LAW III

To every act ...
two bodie

Whate ... is as much drawn or pressed by that other If you press a stone with your finger the finger is also pressed by the stone If a horse draws a stone tied to a rope the horse (if I may so say) will be equally drawn back towards the stone for the distended rope by the same endeavor to relax or unbend itself will draw the horse as much towards the stone as it does the stone towards the horse and will obstruct the progress of the one as much as it advances that of the other If a body impinge upon another and by its force change the motion of the other that body also ...
cause of th ...
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are e ... anges of the velocities made towards contrary parts are inversely proportional to the bodies This law takes place also in attractions as will be proved in the next Scholium

COROLLARY I

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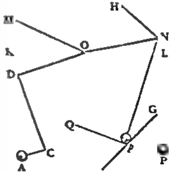
acting together it will in the same time be carried in the diagonal from A to D For since the force N acts in the direction of the line AC parallel to BD this force (by the second Law) will not at all alter the velocity generated by the other force M by which the body is carried towards the line BD The body therefore will arrive at the line BD in the same time whether the force N be impressed or not and therefore at the end of that time it will be found somewhere in the line BD By the same argument at the end of the same time it will be found somewhere in the line CD Therefore it will be found in the point D where both lines meet But it will move in a right line from A to D by Law I.

COROLLARY II

And hence is explained the composition of any one direct force AD out of any two oblique forces AC and CD and on the contrary the resolution of any one direct force AD into two oblique forces AC and CD which composition and resolution are

If of the cords be fixed to the plane of the wheel or not the weights will have the same effect whether they are suspended from the points K and L or from D and L Let the whole force of the weight A be represented by the line AD and let it be resolved into the forces AC and CD of which the force AC drawing the radius OD directly from the centre will have no effect to move the wheel but the other force DC drawing the radius DO perpendicularly will have the same effect as if it drew perpendicularly the radius OL equal to OD that is it will have the same effect as the weight P if



$$P \quad A = DC \quad DA$$

but because the triangles ADC and DOK are similar

$$DC \quad DA = OK \quad OD = OK \quad OL.$$

Therefore

P A = radius OK r d OT

As th d

in e

the s ratio its force to move the wheel will be so much greater

If the weight $p = P$ is partly suspended by the cord Np partly sustained by the oblique plane pG draw pH NH the former perpendicular to the horizon the latter to the plane pG and if the force of the weight p tending downwards is represented by the line pH it may be resolved into the forces pN HN If there was any plane pQ perpendicular to the cord pN cutting the other plane pG in a line parallel to the horizon and the weight p was supported only by those planes pQ pG it would press those planes perpendicularly with the forces pN HN to wit the plane pQ with the force pN and the plane pG with the force HN And therefore if the plane pQ was taken away so that the weight might stretch the cord because the cord now sustaining the weight supplied the place of the plane that was removed it would be strained by the same force pN which pressed upon the plane before Therefore the

tension of pN tension of PN = line pN line pH

Therefore if p is to A in a ratio which is the product of the inverse ratio of the least distances of their cords pN and AM from the centre of the wheel and of the ratio pH to pN then the weights p and A will have the same effect towards moving the wheel and will therefore sustain each other as anyone may find by experiment

But the weight p pressing upon those two oblique pl

as a we

the fo

with p pG is to the force with which the same whether by its own gravity or by the blow of a mallet is impelled in the direction of the line pH towards both the planes as

pN pH

and to the force with which it presses the other plane pG as

pN NH

And thus the force of the screw may be deduced from a like resolution of forces it being no other than a wedge impelled with the force of a lever Therefore the use of this Corollary spreads far and wide and by that diffusive extent the truth thereof is further confirmed For on what has been said depends the whole doctrine of mechanics variously demonstrated by different authors For from hence are easily deduced the forces of machines which are compounded of wheels pullies levers cords and weights ascending directly or obliquely and other mechanical powers as also the force of the tendons to move the bones of animals

COROLLARY III

The quantity of motion which is obtained by taking the sum of the motions directed towards the same parts and the difference of those that are directed to contrary parts suffers no change from the action of bodies among them elves

For action and its opposite reaction are equal by Law III and therefore by Law II they produce in the motions equal changes towards opposite parts

Therefore if the motions are directed towards the same parts whatever is added to the motion of the preceding body will be subtracted from the motion of that which follows so that the sum will be the same as before If the bodies ... there will be an equal deduction from the mo-... rence of the motions directed towards op-

Thus if a ... nes greater than the spherical body B and has a velocity = 2 and B follows in the same direction with a velocity = 10 then th ... B ... 10 ... part and the sum will ... if A acquire 3 4 or 5 ... r reflection A will pro-... the sum remaining al-ways of 16 parts as before If the body ... 11 or 12 parts of motion and therefore after meeting proceed with 15 16 17 or 18 parts the body B losing so many parts as A has got will either proceed with 1 part having lost 9 or stop and remain at rest as having lost its whole progressive motion of ... its whole motion ... 2 parts because a ... of the conspiring motion

$$15+1 \quad \text{or} \quad 16+0$$

and the differences of the contrary motions

$$17-1 \quad \text{and} \quad 18-2$$

will always be equal to 16 part as they were before the meeting and reflection ...

motion of A before reflection (6) motion of A after ...
= velocity of A before (2) velocity of A after (x)

that is

$$6 \quad 18 = 2 \quad x \quad x = 6$$

...

retained the same after reflection as before and to the perpendicular motions we are to assign equal changes towards the contrary parts in such manner that the sum of the conspiring and the difference of the contrary motions may remain the same as before From such kind of reflections sometimes arise also the circular motions of bodies about their own centres But these are cases which I do not consider in what follows and it would be too tedious to demonstrate every particular case that relates to this subject

COROLLARY IV

The common cen
or rest by the

an uniform motion in right lines and their distance be divided in a given ratio the dividing point will be either at rest or proceed uniformly in a right line This is demonstrated hereafter in Lem 23 and Corollary when the points are moved in the same plane and by a like way of arguing it may be demonstrated when the points are not moved in the same plane Therefore if any number of bodies move uniformly in right lines the common centre of gravity of any two of them is either at rest or proceeds uniformly in a right line because the line which connects the centre of the two bodies so moving is d
manner the common cen
at rest or moving uniforn
between the common cer

neither any mutual action among themselves nor any foreign force impressed upon them from without and which consequently move uniformly in right lines the common centre of gravity of them all is either at rest or mo

Moreover in a system of tw
tances between their centres an on centre of gravity of both are reciprocally as the bodies the relative motions of those bodies whether of approaching to or of receding from that centre will be equal among themselves Therefore since the changes which happen to motions are equal and directed to contrary parts the common centre of those bodies by their mutual action between themselves is neither accelerated nor retarded nor suffers any change as to its state of motion or rest But in a system of several bodies because the common centre of gravity of any two acting upon each other suffers no change in its state by that action and much less the common centre of gravity of the others with which that action does not intervene but the
those two cent

retain their state of motion or rest the common centre of all does also retain its state it is manifest that the common centre of all never suffers any change in the state of its motion or rest from the actions of any two bodies between themselves But in such a system all the actions of the bodies among themselves either happen between two bodies or are composed of actions interchanged between some two bodies and therefore they do never produce any alteration in the common centre of all as to its state of motion or rest Wherefore since that centre when the bodies do not act one upon another either is at rest or moves uniformly forwards in some right line it will notwithstanding the mutual actions of the bodies among

themselves always continue in its state either of rest or of proceeding uniformly in a right line unless it is forced out of this state by the action of some power impressed from without upon the whole system And therefore the same ... f many bodies as in one single body ... notion or of rest For the ... a whole system of bodies ... ntre of gravity

COROLLARY V

... same among themselves ... line without

... arts and the sums of those that tend towards contrary p ... suppo...tion) in both cases the same and it is from those sums and differences that the col ... impulses do arise with which the bodies impinge one upon another ... n both cases ... s in the one ... selves in the ... ip where all ... or is carried uniformly forwards in a right line

COROLLARY VI

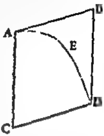
If bodies moved in any manner among themselves are urged in the direction of ... among

SCHOLIUM

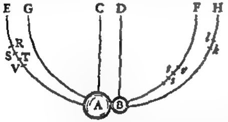
Hitherto I have laid down such principles as have been received by mathematicians and are confirmed by abundance of experiments By the first two Laws and the first two Corollaries Galileo discovered that the descent of bodies varied as the square of the time (*in duplicata ratione temporis*) and that the ...

fore generates equal velocities and in the whole time impresses a whole force and generates a whole velocity proportional to the time And the spaces described in proportional times are as the product of the velocities and the times that is as the squares of the times And when a body is thrown upwards its uniform gravity impresses forces and reduces velocities proportional to the times and the times of ascending to the greatest heights are as the velocities to be taken away and those heights are as the product of the velocities and the times or as the squares of the velocities And if a body be projected in any

direction the motion arising from its projection is compounded with the mo-
tion ... motion of projection
with its motion of
AC complete the
... ABCD and the body by that compounded motion will at the end of the time be found in the place D and the curved line AED which that body describes will be a parabola to which the right line AB will be a tangent at A and whose ordinate BD will be as the square of the line AB On the same Laws and Corollaries depend those things which have been demonstrated concerning the times of the vibration of pendulums and are confirmed by the daily experiments of pendulum clocks By the same together with Law



3 Sir Christopher Wren Dr Wallis and Mr Huygens the greatest geometers of our times did severally determine the rules of the impact and reflection of hard bodies and about the ...
Royal So...
indeed w...
Wren an... ...uygens But Sir Christopher Wren confirmed the truth of the thing before the Royal Society by the experiments on pendulums which M Mariotte soon after thought fit to explain in a treatise entirely upon that subject But to bring this experiment to an accurate agreement with the theory we are to have due regard as well to the resistance of the air as to the elastic force of the concurring bodies Let the spherical bodies A B be sus-



pended by the parallel and equal strings AC BD from ...
t...
s... ... GBH bisected respectively by the radii CA DB Bring the body A to any point R of the arc EAF and (withdrawing the body B) let it go from thence and after one oscillation suppose it to return to the point V then RV will be the retardation arising from the resistance of the air Of this RV let ST be a fourth part situated in the middle namely so that

$$RS = TV$$

and

$$RS \quad ST = 3 \quad 2$$

... he de...

...he point S the velocity thereof in the ... error will be the same as if it had descended *in vacuo* from the point T Upon which account this velocity may be represented by the chord of the arc TA For it is a proposition well known to geometers that the velocity of a pendulous body in the lowest point is as the chord of the arc which it has described in its descent After reflection suppose the body A comes to the place *s* and the body B to the place *k* Withdraw the body B and find the place *v* from which if the body A being let go should after one oscillation return to the place *r* *st* may be a fourth part of *rv* so placed in the middle thereof as to leave *rs* equal to *tv* and let the chord of the arc *t*A represent the velocity which the body A

had in the place A immediately after reflection. For *l* will be the true and correct place to which the body A should have ascended, if the resistance of the air had been taken off. In the same way we are to correct the place *k* to which the body B ascends, by finding the place *l* to which it should have ascended *in vacuo*. And thus everything may be subjected to experiment in the same manner as if we were really placed *in vacuo*. These things being done, we are to take the product (if I may so say) of the body A, by the chord of the arc TA (which represents its velocity), that we may have its motion in the place A immediately before reflection; and then by the chord of the arc *t*A, that we may have its motion in the place A immediately after reflection. And so we are to take the product of the body B by the chord of the arc B*l*, that we may have the motion ... And in like manner, when two bodies ... we are to find the motion of each ... as ... we may compare the motions between themselves, and collect the effect of the reflection. Thus trying the thing with pendulums of 10 feet, in unequal as well as equal bodies, and making the bodies to concur after a descent through large spaces, as of 8, 12, or 16 feet, I found always, without an error of 3 inches, that when the bodies concurred together directly, equal changes towards the contrary parts were produced in their ...

with those ... parts. If the bodies concurred with contrary ... B receded with 8 parts of motion, and B with 6, then if A receded with 2 ... For from the ...

from the motion of the body B ... the bodies were made ... went on with 14 parts, 9 parts being transferred from A to B. And so in other cases. By the meeting and collision of bodies, the quantity of motion, obtained from the sum of the motions directed towards the same way, or from the difference of those that were directed towards contrary ways, was never changed. For the error of an inch or two in measures may be easily ascribed to the difficulty of executing everything with accuracy. It was not easy to let go the two pendulums so exactly together that the bodies should impinge one upon the other in ...

the irregularity of the texture proceeding from other causes.

But to prevent an objection that may perhaps be alleged against the rule, for the proof of which this experiment was made, as if this rule did suppose that the bodies were either absolutely hard, or at least perfectly elastic (whereas no such bodies are to be found in Nature), I must add, that the experiments we have been describing, by no means depending upon that quality of hardness, do succeed as well in soft as in hard bodies. For if the rule is to be tried in bodies not perfectly hard, we are only to diminish the reflection in such a certain

proportion as the quantity of the elastic force requires By the theory of Wren and Huygens bodies absolutely hard return one from another with the same velocity with which they meet But this may be affirmed with more certainty of bodies perfectly elastic In bodies imperfectly elastic the velocity of the return is to be diminished together with the elastic force because that force (except when the parts of bodies are bruised by their imp
such extensio
perceive) cer
other with a given ratio to that relative velocity with which they met This I tried in balls of wool made up tightly and strongly compressed For first by letting go the pendulous bod
reflection
to this fc
impact
accordingly
velocity wh y with which they met as about 5 to 9 Balls of steel returned with almost the same velocity those of cork with a velocity something less but in balls of glass the proportion was as about 15 to 16 And thus the third Law so far as it regards percussions and reflections is proved by a theory exactly agreeing with experience

In attractions I briefly demonstrate the thing after this manner Suppose an obstacle is interposed to hinder the meeting of any two bodies A B attracting one the other then if either body as A is more attracted towards the other body B than that other body B is towards the first body A the obstacle will be more strongly urged by the pressure of the body A than by the pressure of the body B and therefore will not remain in equilibrium but the stronger pressure will prevail and will make the system of the two bodies together with the obstacle to move directly towards the parts on which B lies and in free spaces to go forwards *in infinitum* with a motion continually accelerated which is absurd and contrary to the first Law For by the first Law the system ought to continue in its state of rest or of moving uniformly forwards in a right line and therefore the bodies must equally press the obstacle and be equally attracted one by the other I made the experiment on the loadstone and iron If these placed apart in proper vessels are made to float by one another in standing water neither of them will propel the other but
they

So
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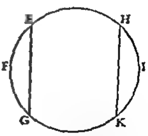
one to



by another plane HK parallel to the former EG the greater part EGI is cut into two parts EGKH and HKI whereof HKI is equal to the part EFG first cut off it is evident that the middle part EGKH will have no propension by its proper weight towards either side but will hang as it were and rest in an equilibrium between both But the one extreme part HKI will with its whole weight bear upon and press the middle part towards the other extreme part EGF and therefore the force with which EGI the sum of the parts HKI and EGKH tends towards the

third part EGF is equal to the weight of the part HKI that is to the weight of the third part EGF And therefore the weights of the two parts EGI and EGF one towards the other are equal as I was to prove And indeed if those weights were not equal the whole earth floating in the nonresisting ether would give way to the greater weight and retiring from it would be carried off in *fin tum*

And as those bodies are equipollent in the impact and reflection, whose velocities are inversely as their innate forces, so in the use of mechanic instruments those agents are equipollent and mutually sustain each the contrary pressure of the other whose velocities estimated according to the determination of the forces, are inversely as the forces.

So those weights are of equal force to move the arms of a balance which during the play of the balance are inversely as their velocities upwards and downward that is if the ascent or descent is direct those weights are of equal

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the contrary forces that promote and impede the motion of the wheel if they are inversely as the velocities of the parts of the wheel on which they are impressed will mutually sustain each other

towards the pressed body

The forces by which the wedge presses or drives the two parts of the wood it cleaves are to the force of the mallet upon the wedge as the progress of the

given of all machines

weight with a given power or with a given force to overcome any other given resistance For if machines are so contrived that the velocities of the agent and

or from the cohesion of continuous bodies that are to be separated or from the weights of bodies to be raised the excess of the force remaining after all those

resistances are overcome ... thereto as well in the part ... to treat of mechanics is not my pr... business I was aiming only to show by those examples the great extent and certainty of the third Law of Motion For if we estimate the action of the agent from the product of its force ... likewise the reaction of the ... several parts and the for... weight and acceleration o... sorts of machines will be f... another And so far as the action is propagated by the intervening instruments and at last impressed upon the resisting body the ultimate action will be always contrary to the reaction

BOOK ONE

THE MOTION OF BODIES

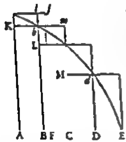
SECTION I

THE METHOD OF FIRST AND LAST RATIOS OF QUANTITIES, BY THE HELP OF WHICH WE DEMONSTRATE THE PROPOSITIONS THAT FOLLOW

LEMMA 1

ultimate difference. Therefore they cannot approach nearer to equality than by that difference D; which is contrary to the supposition.

LEMMA 2



If in any figure AacE, *terminated by the right lines* Aa, AE, *and the curve* acE, *there be inscribed any number of parallelograms* Ab, Bc, Cd, &c., *comprehended under equal bases* AB, BC, CD, &c., *and the sides* Bb, Cc, Dd, &c., *parallel to one side* Aa *of the figure; and the parallelograms* aKbl, bLcm, cMdn, &c., *are completed; then if the breadth* [illegible] *the circumscribed figure* AalbmcndoE, *and curvilinear figure* AabcdE, *will have to one another are ratios of equality.*

supposed diminished in infinitum, becomes less than any given space. And therefore (by Lem. 1) the figures inscribed and circumscribed become ultimately equal one to the other; and much more will the intermediate curvilinear figure be ultimately equal to either. Q.E.D.

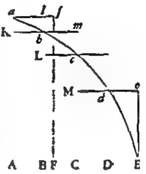
LEMMA 3

The same ultimate ratios are also ratios of equality, when the breadths AB, BC, DC, &c., *of the parallelograms are unequal, and are all diminished* in infinitum.

For suppose AF equal to the greatest breadth, and complete the parallelogram FAaf. This parallelogram will be greater than the difference of the in-

scribed and circumscribed figures but because its breadth AF is diminished *in infinitum* it will become less than any given rectangle Q.E.D.

COR. I. Hence the ultimate sum of those evanescent parallelograms will in all parts coincide with the curvilinear figure.



COR. II. Much more will the rectilinear figure comprehended under the chords of the evanescent arcs *ab, bc, cd, &c.* ultimately coincide with the curvilinear figure.

COR. III. And also the circumscribed rectilinear figure comprehended under the tangents of the same arcs.

COR. IV. And therefore these ultimate figures (as to their perimeters *acE*) are not rectilinear, but curvilinear limits of rectilinear figures.

LEMMA 4

If in two figures AacE, PprT, *there are inscribed (as before) two series of parallelograms, an equal number in each series, and their breadths being diminished* in infinitum, *if the ultimate ratios of the parallelograms in one figure to those in the other, each to each respectively, are the same, I say that those two figures* AacE, PprT, *are to each other in that same ratio.*

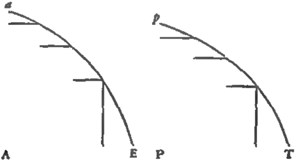


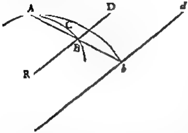
figure to the former sum, and the latter figure to the latter sum, are both in the ratio of equality. Q.E.D.

COR. Hence if two quantities of any kind are divided in any manner into an equal number of parts, and those parts, when their number is augmented and their magnitude diminished *in infinitum*, have a given ratio to each other, the first to the first, the second to the second, and so on in order, all of them taken together will be to each other in that same given ratio. For if, in the figures of this Lemma, the parallelograms are taken to each other in the ratio of the parts, the sum of the parts will always be as the sum of the parallelograms; and therefore supposing the number of the parallelograms and parts to be augmented and their magnitudes diminished *in infinitum*, those sums will be in the ultimate ratio of the parallelogram in the one figure to the correspondent parallelogram in the other; that is (by the supposition), in the ultimate ratio of any part of the one quantity to the correspondent part of the other.

LEMMA 5

All homologous sides of similar figures whether curvilinear or rectilinear are proportional and the areas are as the squares of the homologous sides

LEMMA 6



CB ... n posit on ts sub-

contained between the chord and the tangent will be diminished in infinitum *and ultimately will vanish*

For if that angle does not vanish the arc ACB will contain with the tangent AD an angle equal to a rectilinear angle and therefore the curvature at the point A will not be continued which is against the supposition

LEMMA 7

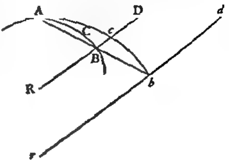
The same things being supposed I say that the ultimate ratio of the arc chord and tangent any one to any other is the ratio of equality

mate ratio of all the abscissas AD AE BF BG and of the chord and arc AB any one to any other will be *the ratio of equality*

COR III And therefore in all our reasoning about ultimate ratios we may freely use any one of those lines for any other

LEMMA 8

For while the point B approaches towards the point A consider always AB AD AR as produced to the remote points *b* *d* and *r* and *rbd* as drawn parallel to RD and let the arc A*cb* be always simi lar to the arc ACB Then supposing the points A and B to coincide the angle *b*A*d* will vanish and therefore the three tri angles *r*A*b* *r*A*cb* *r*A*d* (which are always fin to) -



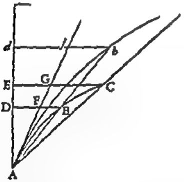
n ultimately become both simi lar and equal among themselves Q E D

COR And hence in all reasonings about ultimate ratios we may use any one of those triangles for any other

LEMMA 9

If a right line AE *and a curved line* ABC *both given by position cut each other in a given angle* A *and to that right line in another given angle* BD CE *are ordi natcly applied meeting the curve in* B C *and the points* B *and* C *together approach towards and meet in the point* A *I say that the areas of the triangles* ABD ACE *will ultimately be to each other as the squares of homologous sides*

For while the points B C approach to ... ays AD ... or to ... parallel to the ordinates DB and EC ... and ... ABC ... right line A*g* so as to touch both curves in A and cut the ordinates DB EC *db* *ec* in F G *f* *g* Then supposing the length A*e* to remain the same let the points B and C meet in the point A and the angle *c*A*g* vanishing the ... up



cate ratio of the sides A*d* A*e* But the areas ABD ACE are always propor tional to these areas and so the sides AD AE are to these sides And there fore the areas ABD ACE are ultimately to each other as the squares of the sides AD AE Q E D

LEMMA 10

The spaces which a body describes by any finite force urging it whether that force is determined and immutable or is continually augmented or continually dimin ished are in the very beginning of the motion to each other as the squares of the times

Let the times be represented by the lines AD AE and the velocities gen erated in those times by the ordinates DB EC The spaces described with

these velocities will be as the areas ABD ACE described by those ordinates that is at the very beginning of the motion (by Lem 9) in the duplicate ratio of the times AD AE Q E D

COR. I And hence one may easily infer that the errors of bodies describing similar parts of imular figures in proportional times the errors being generated ... similarly applied to the bodies and measured by the dis- ... t which without ... proportional ... enerated ... l forces imilarly applied to the bodies at similar parts ... are as the product of the forces and the quares of the times

COR III The same thing is to be understood of any paces whatsoever described by bodies urged with different forces all which in the very beginning of the motion are as the product of the forces and the squares of the times

COR IV And therefore the forces are directly as the paces described in the very beginning of the motion and inversely as the squares of the times

COR V And the squares of the times are directly as the spaces described and inversely as the forces

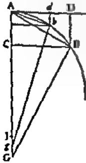
SCHOLIUM

If in comparin with each other indeterminate qu ntities of different sorts any one is said to be directly or inversely as any other the meaning is that the former is augmented or diminished in the same ratio as the latter or as its reciprocal And if any one is said to be as any other two or more directly or in ... first a gmented or diminished in the ratio ...

in the same ratio as ... at A and $\frac{BC}{D}$ are to each other in a gi en ratio

LEMMA 11

The eva escent subtense of the an le of contact in all curves which at the poi t of contact have a fi ite curvatu e is ultimately as the square of the subtense of the con terminous arc



CASE I Let AB be that arc AD its tangent BD the subtense of the angle of contact perpendicular on the tan ...

of the circles passing through the points A, B G and through A b g)

$$AB^2 = AG \cdot BD \text{ and}$$
$$Ab^2 = Ag \cdot bd$$

But because GJ may be assumed of less length than any assignable the ratio of AG to Ag may be such as to differ from unity by less than any assignable difference and therefore the ratio of AB^2 to Ab may be such as to differ from the ratio of BD to bd by less than any assignable difference Therefore by Lem 1, ultimately,

$$AB \quad Ab^2 = BD \quad bd$$

Q E D

CASE 2 Now let BD be inclined to AD in any given angle and the ultimate ratio of BD to bd will always be the same as before and therefore the same with the ratio of AB^2 to Ab^2 Q E D

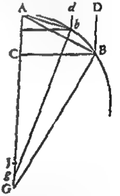
CASE 3 And if we suppose the angle D not to be given but that the right line BD converges to a given point or is determined by any other condition whatever nevertheless the angles D d being determined by the same law will always draw nearer to equality and approach nearer to each other than

ual and

Q E D

Co Therefore since the tangents AD Ad the arcs AB Ab and their sines BC bc become ultimately equal to the chords AB Ab their squares will ultimately become as the subtenses BD bd

COR II Their squares are also ultimately as the versed sines of the arcs bisecting the chords and converging to a given point For those versed sines



ocity

COR IV The ultimate proportion

$$\triangle ADB \quad \triangle Adb = AD^3 \quad Ad^3 = DB^{3/2} \quad db^{3/2}$$

is derived from

$$\triangle ADB \quad \triangle Adb = AD \quad DB \quad Ad \quad db$$

and from the ultimate proportion

$$AD^2 \quad Ad = DB \quad db$$

So also is obtained ultimately

$\triangle ABC \quad \triangle Abc = BC^3 \quad bc^3$

COR nd as the squares of the lines AL will be (by the nature of the p s ADB Adb and the segments AB Ab will be one third of the same triangles And thence those areas and those segments will be as the cubes of the tangents AD Ad and also of the chords and arcs AB Ab

SCHOLIUM

But we have all along supposed the angle of contact to be neither infinitely greater nor infinitely less than the angles of contact made by circles and their tangents that is that the curvature at the point A is neither infinitely small nor infinitely great and that the interval AJ is of a finite magnitude I or DB may be taken as AD^3 in which case no circle can be drawn through the point A between the tangent AD and the curve AB and therefore the angle of contact will be infinitely less than those of circles And by a like reasoning if DB be made successfully as AD^4 AD^5 AD^6 AD^7 &c we shall have a series of angles of contact proceeding *in infinitum* wherein every succeeding term is in

h preceding And if DB be made successively as AD^2

h 6 te series of

an les of contact may be interp

wherein every succeeding angle shall be infinitely greater or infinitely less than the preceding As if between the terms AD and AD there were interposed the series AD AD AD AD AD AD AD AD AD &c And again between any two angles of this series a new series of intermediate an les may be interposed differing from one another by infinite intervals Nor is Nature confined to any bounds

Those things which have been demonstrated of curved lines and the sur

mprehend may be easily applied to the curved surfaces

trated we may use them

sums and ratios of determinate parts but always the limits of sums and ratios and that the force of such demonstrations always depends on the method laid down in the foregoing Lemmas

nt that with

understood the ratio of the quantities not before they vanish nor afterwards but with which they vanish In like manner the first ratio of nascent quantities is that with which they begin to be And the first or last sum is that with which they begin and cease to be (or to be augmented or diminished) There is a limit hich the velocity at the end of the motion may attain but not exceed This is the ultimate velocity And there is the like limit in all quantities and propor tions that begin and cease to be And since such limits are certain and definite to determine the same is a problem strictly geometrical But whatever is geo-

metrical we may use in determining and demonstrating any other thing that is also geometrical

It may also be objected that if the ultimate ratios of evanescent quantities are given their ultimate magnitudes will be also given and so all quantities will consist of indivisibles which is contrary to what Euclid has demonstrated concerning incommensurables in the tenth book of his *Elements* But this objection is founded on a false supposition For those ultimate ratios with which quantities vanish are not truly the ratios of ultimate quantities but limits towards which the ratios of quantities decreasing without limit do always converge and to which they approach nearer than by any given difference but never go beyond nor in effect attain to till the quantities are diminished *in infinitum* This thing will appear more evident in quantities infinitely great If two quantities whose difference is given be augmented *in infinitum* the ultimate ratio of t
it does not fro
selves whose
sake of being more easily understood I should happen to mention quantities as least or evanescent or ultimate you are not to suppose that quantities of any determinate magnitude are meant but such as are conceived to be always diminished without end

SECTION II

THE DETERMINATION OF CENTRIPETAL FORCES

PROPOSITION 1 THEOREM 1

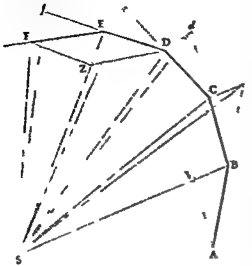
The areas which revolving bodies describe by radii drawn to an immovable centre of force do lie in the same immovable planes and are proportional to the times in which they are described

For suppose the time to be divided into equal parts and in the first part of that time let the body by its innate force describe the right line AB In the second part of that time the same would (by Law I) if not hindered proceed directly to c along the line Bc equal to AB so that by the radii AS BS cS drawn to the centre the equal areas ASB BSc would be described But when
at once with a
Bc compels it
cC parallel to
BS meeting BC in C and at the end of the second part of the time the body (by Cor I of the Laws) will be found in C in the same plane with the triangle ASB Join SC and because SB and Cc are parallel the triangle SBC will be equal to the triangle SBc and therefore also to the triangle SAB By the like

by com
the times
be aug

mented, and their breadth diminished in infinitum and (by Cor. iv Lem 3) their ultimate perimeter ADF will be a curved line and therefore the centripetal force by which the body is continually drawn back from the tangent of this curve will act continually and any described areas SADS SAFS which are always proportional to the times of description will in this case also be proportional to those times

Q.E.D



Cor. i. The velocity of a body attracted towards an immovable centre in spaces void of resistance is inversely as the perpendicular let fall from that centre on the right line that touches the orbit For the velocities in those places A, B C D E, are as the bases AB BC CD DE, EF of equal triangles and these bases are inversely as the perpendiculars let fall upon them.

Cor. ii If the chords AB BC of two arcs successively described in equal [illegible]

in spaces void of resistance are completed into the parallelogram ABCV DEFZ, the forces in B and E are one to the other in the ultimate ratio of the [illegible]

[illegible]

Cor iii

Cor. v And therefore those forces are to the force of gravity as the said versed sines to the versed sines perpendicular to the horizon of those parabolic arcs which projectiles describe in the same time

Cor. vi And the same things do all hold good (by Cor. v of the Laws) when the planes in which the bodies are moved together with the centres of force which are placed in those planes are not at rest but move uniformly forwards in right lines

metrical we may use in determining and demonstrating any other thing that is also geometrical

It may also be objected that if the ultimate ratios of evanescent quantities are given their ultimate magnitudes will be also given and so all quantities will consist of indivisibles which is contrary to what Euclid has demonstrated concerning incommensurables in the tenth book of his *Elements* But this objection is founded on a false supposition For those ultimate ratios with which quantities vanish are not truly the ratios of ultimate quantities but limits towards which the ratios of quantities decreasing without limit do always converge and to which they approach nearer than by any given difference but never go beyond nor in effect attain to till the quantities are diminished *in infinitum* This thing will appear more evident in quantities infinitely great If two quantities whose difference is given be augmented *in infinitum* the ultimate ratio of these quantities will be given namely the ratio of equality but it does not from thence follow that the ultimate or greatest quantities themselves whose ratio that is will be given Therefore if in what follows for the sake of being more easily understood I should happen to mention quantities as least or evanescent or ultimate you are not to suppose that quantities of any determinate magnitude are meant but such as are conceived to be always diminished without end

SECTION II

THE DETERMINATION OF CENTRIPETAL FORCES

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For suppose the time to be divided into equal parts and in the first part of that time let the body by its innate force describe the right line AB In the second part of that time the same would (by Law I) if not hindered proceed directly to c along the line Bc equal to AB so that by the radii AS BS cS drawn to the centre the equal areas ASB BSc would be described But when the body is arrived at B suppose that a centripetal force acts at once with a great impulse and turning aside the body from the right line Bc compels it

BC Draw cC parallel to

he time the body

with the triangle

ASB Join SC and because SB and Cc are parallel the triangle SBC will be equal to the triangle SBc and therefore also to the triangle SAB By the like argument if the centripetal force acts successively in C D E &c and makes

CD DE

l be equal

refore in

by com

position any sums SADS SAFS of those areas are to each other as the times in which they are described Now let the number of those triangles be aug

PROPOSITION 3 THEOREM 3

Every body that by a radius drawn to the centre of another body howsoever moved about that centre proportional to the times is urged by a force that other body and of all the ac lled

dy and (by Cor VI of the Laws) equal L will ut the a equal left to body ht line and the first y the force remaining ortional to the times s directed to the other Q E D

And therefore (by body T as its centre body L by a radius drawn to the other body T h which Cor II e Cor) e whole body T remaining force by which the first body as its centre

COR II And if these areas are proportional to the times nearly the remain h th r body T nearly h bod

s y ion of or it is nd the nother r body taken is moved by any motion what oever provided that centr pe which remains after subtracting that whole force acting upon that other body T

SCHOLIUM

l cates that there is a centre to which it is hy may we scription of areas as an indication of a centre about which all circular motion is performed in free spaces?

PROPOSITION 4 THEOREM 4

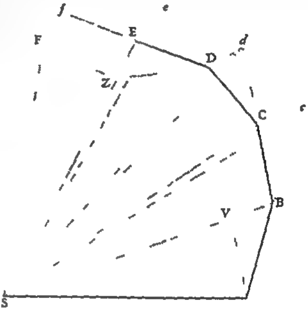
Th centripetal forces of bod es wh ch by equ ble motions describe different circles tend t the centres of the same ci cles and are to each other as the squares of the arcs described n equal t mes di ided respectively by the radii of the circles

PROPOSITION I THEOREM 2

Every body that moves in any curved line described in a plane and by a radius drawn to a point either immovable or moving forwards with an uniform rectilinear motion describes about that point areas proportional to the times is urged by a centripetal force directed to that point

CASE 1 For every body that moves in a curved line is (by Law 1) turned aside from its rectilinear course by the action of some force that impels it And that force by which the body is turned off from its rectilinear course and is made to describe in equal times the equal least triangles SAB SBC SCD &c about the immovable point S (by Prop 40 Book 1 *Elements* of Euclid and Law II) acts in the place B according to the d

BS and in the place C according to the direction of a line parallel to dD that is in the direction of the line CS &c and therefore acts always in the direction of lines tending to the immovable point S Q E D



CASE 2 And (by Cor v of the Laws) it is indifferent whether the surface in which a body d

the fi

Co resisting spaces or m to the time the forces are not direct but devi to th descripti

COR I areas is accelerat the forces deviate from the point in which the radii meet towards the part to which the motion tends

SCHOLIUM

A body may be urged by a centripetal force compounded of several forces in which case the meaning of the Proposition is that the force which results out of all tends to the point S But if any force acts continually in the direction of lines perpendicular to the described surface this force will make the body to deviate from the plane of its motion but will neither augment nor diminish the area of the described surface and is therefore to be neglected in the composition of forces

such propositions Mr Huygens in his excellent book *De horologio oscillatorio* has compared the force of gravity with the centrifugal forces of revolving

k

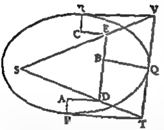
demon trated after this manner ed of any number of sides And if ie ides of the polygon is reflected s the force with which at every

f — h rcle at the

velocity and therefore the um of he num length of the t length hed d in a h th

PROPOSITION 5 PROBLEM

r locity with which a body describes a given



locities of the body in the points which the perpendiculars were raised that so that PA may be to QB as the velocity in Q to the velocity in P and QB to RC as the velocity in R to the velocity in Q Through the ends A B C of the perpendiculars draw AD DBE EC at right angles meeting in D and E and the right lines TD VE produced will meet in S the centre required

For the perpendiculars let fall from the centre S on the tangents PT QT are inversely as the velocities of the bodies in the points P and Q (by Cor I Prop 1) and therefore by construction directly as the perpendiculars AP BQ that is as the perpendiculars let fall from the point D on the tangent Whence it is easy to infer that the points S D T are in one right line And by the like argument the points S E V are also in one right line and therefore the centre S is in the point where the right lines TD VE meet Q.E.D

PROPOSITION 6 THEOREM 5

In a space void of resistance if a body revolves in any orbit about an immovable centre and in the least time describes any arc just then nascent and the versed sine of that arc is supposed to be drawn bisecting the chord and produced passing through the centre of force the centripetal force in the middle of the arc will be directly as the versed sine and inversely as the square of the time

For the versed sine in a given time is as the force (by Cor IV Prop 1) and augmenting the time in any ratio because the arc will be augmented in the

These forces tend to the centres of the circles (by Prop 2 and Cor II Prop 1) and are to one another as the versed sines of the least arcs described in equal times (by Cor IV Prop 1) that is as the squares of the same arcs divided by the diameters of the circles (by Lem 7) and therefore since those arcs are as arcs described in any equal times and the diameters are as the radii the forces will be as the squares of any arcs described in the same time divided by the radii of the circles Q E D

COR I Therefore since those arcs are as the velocities of the bodies the centripetal forces are as the squares of the velocities divided by the radii

COR II And since the periodic times are as the radii divided by the velocities the centripetal forces are as the radii divided by the square of the periodic times

COR III Whence if the periodic times are equal and the velocities therefore as the radii the centripetal forces will be also as the radii and conversely

COR IV If the periodic times and the velocities are both as the square roots of the radii the centripetal forces will be equal among themselves and conversely

COR V If the periodic times are as the radii and therefore the velocities equal the centripetal forces will be inversely as the radii and conversely

COR VI If the periodic times are as the $^{3}/$th powers of the radii and therefore the velocities inversely as the square roots of the radii the centripetal forces will be inversely as the squares of the radii and conversely

COR VII And universally if the periodic time is as any power R of the radius R and therefore the velocity inversely as the power $R^{\ \ 1}$ of the radius the centripetal force will be inversely as the power $R^{\ \ \ 1}$ of the radius and conversely

COR VIII The same things hold concerning the times the velocitie and the forces by which bodies describe the similar parts of any similar figures that

equable motion and using the distances of the bodies from the centres instead of the radii

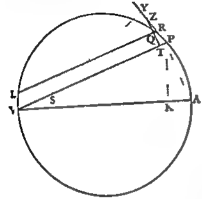
COR IX From the same demonstration it likewise follows that the arc which a body uniformly revolving in a circle with a given centripetal force describes in any time is a mean proportional between the diameter of the circle and the space which the same body falling by the same given force would describe in the same given time

SCHOLIUM

The case of the sixth Corollary obtains in the celestial bodies (as Sir Christopher Wren Dr Hooke and Dr Halley have everally observed) and therefore in what follows I intend to treat more at large of those things which relate to centripetal force decreasing as the squares of the distances from the centres

Moreover by means of the preceding Proposition and its Corollaries we may discover the proportion of a centripetal force to any other known force such as that of gravity For if a body by means of its gravity revolves in a circle concentric to the earth this gravity is the centripetal force of that body But from the descent of heavy bodies the time of one entire revolution as well as the arc described in any given time is given (by Cor IX of this Prop) And by

meeting the circle in L and the tangent PZ in R And because of the similar triangles ZQR ZTP VPA we shall have



$$RP^2 \; QT^2 = AV^2 \; PV$$

Since $RP^2 = RL \; QR \; QT^2 = \dfrac{RL \; QR \; PV}{AV}$

Multiply those equals by $\dfrac{SP^2}{QR}$ and the points P and Q coinciding for RL write PV then we shall have

$$\frac{SP \; PV^3}{AV} = \frac{SP^2 \; QT^2}{QR}$$

And therefore (by Cor I and V Prop 6) the centripetal force is inversely as $\dfrac{SP^2 \; PV^3}{AV}$ that is (because AV^2 is given) inversely as the product of SP and PV^3

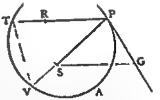
Q E I

The same otherwise

On the tangent PR produced let fall the perpendicular SY and (because of the similar triangles SYP VPA) we shall have AV to PV as SP to SY and therefore $\dfrac{SP \; PV}{AV} = SY$ and $\dfrac{SP^2 \; PV^3}{AV^2} = SY^2 \; PV$ And therefore (by Cor III and V Prop 6) the centripetal force is inversely as $\dfrac{SP^2 \; PV^3}{AV}$ that is (because AV is ... inversely as $SP^2 \; PV^3$

Q E I

about any other centre of force R as RP ... to the cube of the right line SG which from the first centre of force S is drawn parallel to the distance PR of the body from the second centre of force R meeting the tangent PG of the orbit in G For by the construction of this Proposition the former force is to the latter as $RP^2 \; PT^3$ to $SP^2 \; PV^3$ that is as $SP \; RP^2$ to $\dfrac{SP^3 \; PV^3}{PT^3}$ or (because of the similar triangles PSG TPV) to SG^3

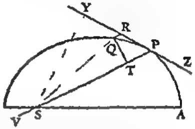


meeting the tangent PG of the orbit in G For the force in this orbit at any point P is the same as in a circle of the same curvature

same ratio the versed sine will be augmented in the square of that rat... Cor II and III Lem 11) and therefo... time ... as t... ...e time Q E D

A... ...ing may also be easily demonstrated by Cor IV Lem 10

COR I If a body P revolving about the centre S describes a curved line APQ which a right line ZPR touches in any point P and from any other point Q of the curve QR is drawn parallel to the distan... SP ...



...y as the solid $\frac{\ldots QT}{QR}$ if the solid be taken of that magnitude which it ultimately acquires when the points P and Q coincide For QR is equal to the versed sine of double the arc QP whose middle is P and double the triangle SQP or SP QT is proportional to the time in which that double arc is described and therefore may be used to represent the time

COR II By a like reasoning the centripetal force is inversely as the solid $\frac{SY^2\ QP^2}{QR}$ if SY is a perpendicular from the centre of force on PR the tangent of the orbit For the rectangles SY QP and SP QT are equal

COR III If the orbit is either a circle or t... ...cally that is contains with a circle the least ... the same curvature and the same radius of c... be a chord of this circle drawn from the body ...rough the centre of force the centripetal force will be inversely as the solid SY^2 PV For PV is $\frac{QP^2}{QR}$

COR IV The same things being supposed the centripetal force is as the square of the velocity directly and the chord inversely For the velocity is reciprocally as the perpendicular SY by Cor I Prop 1

COR V Hence if any curvilinear figure APQ ... also ... tripe... back... ...ated in the perimeter of that figure ...describe the same by a continual revolution That is we are to find by computation either the solid $\frac{SP\ QT^2}{QR}$ or the solid SY^2 PV inversely proportional to this force Examples of this we shall give in the following Problems

PROPOSITION 7 PROBLEM 2

If a body revolves in the circumference of a circle it is ... the law of ...

Let VQPA be t... ...which as to a centre the forc... ...next ... ...ced ... the ... ...which produced may meet the tangent PR in Z and lastly through the point Q draw LR parallel to SP

The same otherwise

The perpendicular SY let fall upon the tangent and the chord PV of the circle concentrically cutting the spiral are in given ratios to the height SP and therefore SP^3 is as SY^2 PV that is (by Cor III and V Prop 6) inversely as the centripetal force

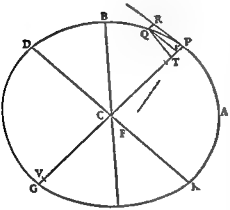
LEMMA 12

All parallelograms circumscribed about any conjugate diameters of a given ellipse or hyperbola are equal among themselves

This is demonstrated by the writers on the conic sections

PROPOSITION 10 PROBLEM V

... find the law of the centripetal force



QvPR be completed then (by the properties of the conic sections) Pv vG : $Qv^2 = PC^2$: CD^2 and because of the similar triangles QvT PCF Qv^2 : $QT^2 = PC^2$: PF^2 and by eliminating Qv^2 vG : $\frac{QT^2}{Pv} = PC^2$: $\frac{CD^2\ PF^2}{PC^2}$ Since QR = Pv and (by Lem 12) BC CA = CD PF and when the points P and Q coincide 2PC = vG we shall have multiplying the extremes and means together $\frac{QT^2\ PC^2}{QR} = \frac{2BC^2\ CA^2}{PC}$ Therefore (by Cor V Prop 6) the centripetal force is inversely as $\frac{2BC^2\ CA^2}{PC}$ that is (because $2BC^2\ CA^2$ is given) inversely as $\frac{1}{PC}$ that is directly as the distance PC Q E I

The same otherwise

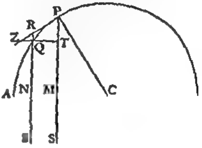
ratio of DC^2 to PC^2 will become the ratio of PV to PG or PV to 2PC and therefore PV will be equal to $\frac{2DC^2}{PC}$ And therefore the force by which the body P revolves in the ellipse will be inversely as $\frac{2DC^2}{PC}$ PF^2 (by Cor III Prop 6) that is (because $2DC^2\ PF^2$ is given) directly as PC Q E I

PROPOSITION 8 PROBLE 3

ro its drawn

for parallels

From C the centre of the semicircle let the semidiameter CA be drawn cutting the parallels at right angles in M and N and join CP Because of the similar triangles CPM PZT and RZQ we shall have CP PM^2=PR^2 QT^2 From the nature of the circle PR^2=QR(RN+QN)= QR 2PM when the points P and Q coincide Therefore CP^2 PM^2=QR 2PM QT^2 and $\frac{QT^2}{QR}=\frac{2PM^3}{CP^2}$ and $\frac{QT^2\ SP^2}{QR}=\frac{2PM^3\ SP^2}{CP^2}$ And therefore (by Cor 1 and 5 Prop 6) the centripetal force is inversely as $\frac{2PM^3\ SP^2}{CP^2}$ that is



(neglecting the given ratio $\frac{2SP^2}{CP^2}$) inversely as PM^3 Q E I

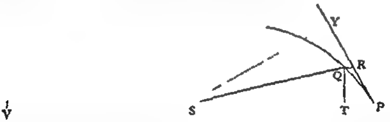
And the same thing is likewise easily inferred from the preceding Proposition

SCHOLIUM

And by a like reasoning a body will be moved in an ellipse or even in an hyperbola or parabola by a centripetal force which is inversely as the cube of the ordinate directed to an infinitely remote centre of force

PROPOSITION 9 PROBLEM 4

If a body revolves in a spiral PQS cutting all the radii SP SQ &c in a given angle it is proposed to find the law of the centripetal force tending to the centre of that spiral



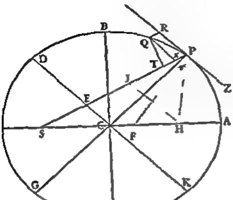
Suppose the indefinitely small angle PSQ to be given because then all the angles are given, the figure SPRQT will be given in kind Therefore the

QT QT^2

QT Therefore the ratio $\frac{QT^2}{QR}$ remains the same as before that is as SP And $\frac{QT^2\ SP^2}{QR}$ is as SP^3 and therefore (by Cor 1 and 5 Prop 6) the centripetal force is inversely as the cube of the distance SP Q E I

L QR L $Pv = QR$ $Pv = PE$ $PC = AC$ PC

also L Pv Gv $Pv = L$ Gv and Gv Pv $Qv^2 = PC^2$ CD



By Cor II Lem 7 when the points P and Q coincide $Qv^2 = Qx^2$ and Qx or Qv^2 $QT^2 = EP^2$ $PF^2 =$ CA PF^2 and (by Lem 12) = CD CB^2 Multiplying together corresponding terms of the four proportions and implifying we shall have L QR $QT^2 = AC$ L PC^2 CD^2 PC Gv CD $CB^2 =$ 2PC Gv since AC $L = 2BC^2$ But the points Q and P coinciding 2PC and Gv are equal And therefore the quantities L QR and QT^2 proportional to the e will be also equal Let those equals be multiplied by $\frac{SP^2}{QR}$ and L SP^2 will become equal to $\frac{SP^2\ QT^2}{QR}$ And therefore (by Cor I and V Prop 6) the centripetal force is inversely as L SP^2 that is inversely as the square of the distance SP

Q E I

The same otherwise

P may he body t PR of the ellipse and the force by which the same body may ... about any ther point S of the ellipse if CE and PS intersect in E will be as $\frac{PE}{SP}$ (by Cor III Prop) that is if the point S is the focus of the ellipse and therefore PE be given as SP^2 reciprocally

Q E I

With the same brevity with which we reduced the fifth Problem to the parabola and hyperbola we might do the like here but because of the dignity of the Problem and its use in what follows I shall confirm the other cases by particular demonstrations

PROPOSITION 12 PROBLEM 7

Suppose a body to move in an hyperbola it is required to find the law of the centripetal force tending to the focus of that figure

Let CA CB be the semiaxes of the hyperbola PG KD other conjugate

PR and the equal angles IPR, HPZ) of PS PH the difference of which is

COR I And therefore the force is as the distance of the body from the centre of the ellipse and *vice versa* if the force is as the distance the body will move in an ellipse whose centre coincides with the centre of force or perhaps in a circle into which the ellipse may degenerate

COR II And the periodic times of
soever about the same centre will be
will be equal (by Cor III and VIII ellipses that have their greater axis common they are to each other as the whole areas of the ellipses directly and the parts of the areas described in the same time inversely
is as the lesser axes directly and the
vertices inversely that is as those less
same point of the common axes invers efore (because of the equality of the direct and inverse ratios) in the ratio of equality I I

SCHOLIUM

If the ellipse by having its centre removed to
erates into
tending to
theorem A
of the cuttin
move in the
into a cent

ordinates augmented or
in the ratio of the distances from the centre

SECTION III

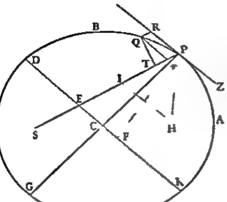
THE MOTION OF BODIES IN ECCENTRIC CONIC SECTIONS

PROPOSITION 11 PROBLEM

the focus of the ellipse Draw SP cutting the diameter DK of the ellipse in E and the ordinate Qv in x and complete the
is evident that EP
other focus H of
will also be equal that is (because of the parallels HI PR and the equal angles IPR HPZ) of PS PH which taken together are equal to the whole axis 2AC Draw QT perpendicular to SP and putting L for the principal latus rectum of the ellipse (or for $\frac{2BC^2}{AC}$) we shall have

L QR L Pv=QR Pv=PE PC=AC PC

also L Pv Gv Pv=L Gv and Gv Pv Qv =PC² CD²



By Cor II Lem 7 when the points P and Q coincide Qv²=Qx² and Qx² or Qv² QT²=EP² PF²=CA² PF² and (by Lem 12)=CD CB Multiplying together corresponding terms of the four proportions and simplifying we shall have L QR QT²=AC L PC CD PC Gv CD CB²=2PC Gv since AC L=2BC² But the points Q and P coinciding 2PC and Gv are equal And therefore the quantities L QR and QT² proportional to these will be also equal Let those equals be multiplied by $\frac{SP^2}{QR}$ and L SP² will become equal to $\frac{SP^2\ QT^2}{QR}$ And therefore (by Cor I and v Prop 6) the centripetal force is inversely as L SP that is inversely as the square of the distance SP

Q E I

The same otherwise

L h had P m r

other point S of the ellipse if CE and PS intersect in E will be as $\frac{PE}{SP}$ (by Cor III Prop 6) that is if the point S is the focus of the ellipse and therefore PE be given as SP² reciprocally

Q E I

With the same brevity with which we reduced the fifth Problem to the parabola and hyperbola we might do the like here but because of the dignity of the Problem and its use in what follows I shall confirm the other cases by particular demonstrations

PROPOSITION 12 PROBLEM 7

Suppose a body to move in an hyperbola it is required to find the law of the centripetal force tending to the focus of that figure

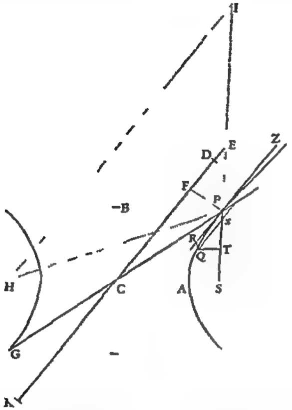
Let CA CB be the semiaxes of the hyperbola PG KD other conjugate diameters PF a perpendicular to the diameter KD and Qv an ordinate to the diameter GP Draw SP cutting the diameter DK in E and the ordinate Qv in x and complete the parallelogram QRPx It is evident that EP is equal to the semitransverse axis AC for drawing HI from the other focus H of the hyperbola parallel to EC because CS CH are equal ES EI will be also equal so that EP is the half difference of PS PI that is (because of the parallels IH PR, and the equal angles IPR, HPZ) of PS PH the difference of which is

equal to the whole axis 2AC Draw QT perpendicular to SP and putting L for the principal latus rectum of the hyperbola (that is for $\frac{2BC^2}{AC}$) we shall have

$$L \; QR \quad L \; Pt = QR \quad Pv = Px \quad Pv = PE \quad PC = AC \quad PC$$

also $L \; Pv \quad Gv \; Pv = L \quad Gt$ and $Gv \; Pv \quad Qt^2 = PC \quad CD^2$ By Cor II Lem 7 when P and Q coincide $Qx = Qt^2$ and

$$Qx \text{ or } Qv^2 \quad QT = EP^2 \quad PF^2 = CA^2 \quad PF^2, \text{ by Lem 12 } = CD \quad CB^2$$



Multiplying together corresponding terms of the four proportions and simplifying

$$L \; QR \quad QT^2 = AC \; L \; PC^2 \; CD \quad PC \; Gv \; CD \quad CB^2 = 2PC \quad Gt$$

since $AC \; L = 2BC^2$ But the points P and Q coinciding 2PC and Gt are equal And therefore the quantities L QR and QT² proportional to them will also be equal Let those equals be drawn into $\frac{SP}{QR}$ and we shall have L SP² equal to $\frac{SP^2 \; QT^2}{QR}$ And therefore (by Cor I and v Prop 6) the centripetal force is inversely as L SP² that is inversely as the square of the distance SP Q E I

The same otherwise

Find out the force tending from the centre C of the hyperbola This will be proportional to the distance CP But from thence (by Cor III Prop 7) the force tending to the focus E will be as $\frac{PE^3}{SP^2}$ that is because PE is given reciprocally as SP² Q E I

And the same way may it be demonstrated that the body having its centripetal changed into a centrifugal force will move in the conjugate hyperbola

LEMMA 13

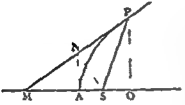
The la..is rectum of a parabola belonging to any vertex is four times the distance of that vertex from the focus of the figure

Th.. is demonstrated by the writers on the conic section.

LEMMA 14

The perpendicular let fall from the focus of a parabola on its tangent is a mean proportional between the di..ances of the focus from the point of contact and from

of



pendicular from the focu on the tange.. join AN and because of the equal lines MS and SP MN and NP MA and AO the right lines AN OP will be parallel and thence the triangle SAN will be right angled at A, and similar to the equal triangles SNM SNP therefore PS i to SN as SN is to SA. Q.E.D

COR. I PS^2 is to SN^2 as PS is to SA.

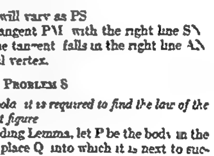
COR. II. And because SA is given, SN^2 will vary as PS

COR. III. And the intersection of any tangent PM with the right line SN drawn from the focu.. perpendicular on the tangent falls in the right line AN that touches the parabola in the principal vertex.

PROPOSITION 13 PROBLEM 8

If a body moves in the perimeter of a parabola it is required to find the law of the centripetal force tending to the focus of that figure

Retaining the con..truction of the preceding Lemma, let P be the body in the perimeter of the parabola and from the place Q into which it is next to succeed draw QR parallel and QT perpendicular to SP as also Q parallel to the tangent ..nd meeting the diameter PG in and the di..tance SP in x Now because of the similar triangles Pxx SPM and of the equal



the rectangle under the latu.. rectum and the segment Px of the diameter that is (by Lem 13) to the rectangle 4PS P or 4PS QP and the points P and Q coinciding (by Cor II, Lem. 7) Q = Q And theref re Qx^2 in this case becomes equal to the rectangle 4PS QR But (because of the similar triangles QxT SPN)

Qx^2 QT^2 = PS SN^2 = PS SA (by Cor I Lem. 14)
= 4PS QR 4SA QR.

Therefore (by Prop [illegible] Book V *Elements of* Euclid) $QT^2 = 4SA \cdot QR$ Multiply these equals by $\frac{SP^2}{QR}$ and $\frac{SP^2 \cdot QT}{QR}$ will become equal to $SP^2 \cdot 4SA$ and therefore (by Cor I and V Prop 6) the centripetal force is inversely as $SP^2 \cdot 4SA$ that is because 4SA is given inversely as the square of the distance SP Q E I

COR I From the three last Propositions it follows that if any body P goes from the place P with any velocity in the direction of any right line PR and at the same time is urged by the action of a centripetal force that is inversely proportional to the square of the distance of the places from the centre the body will move in one of the conic sections having its focus in the centre of force and conversely For the focus the point of contact and the position of the tangent being given a conic section may be described which at that point shall have a given curvature But the curvature is given from the centripetal force and velocity of the body being given and two orbits touching one the other cannot be described by the same centripetal force and the same velocity

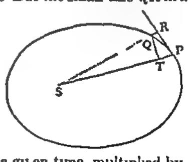
COR II If the velocity with which the body goes from its place P is such that in any infinitely small moment of time the small line PR may be thereby described and the centripetal force such as in the same time to move the same body through the space QR the body will move in one of the conic sections whose principal latus rectum is the quantity $\frac{QT^2}{QR}$ in its ultimate state when the small lines PR QR are diminished *in infinitum* In these Corollaries I consider the circle as an ellipse and I except the case where the body descends to the centre in a right line

PROPOSITION 14 THEOREM 6

If several bodies revolve about one common centre and the centripetal force is inversely as the square of the distance of places from the centre I say that the principal latera recta of their orbits are as the squares of the areas which the bodies by radii drawn to the centre describe in the same time

For (by Cor II Prop 13) the latus rectum L is equal to the quantity $\frac{QT^2}{QR}$ in

le But the small line QR in a



therefore $\frac{QT^2}{QR}$ is as $QT^2 \cdot SP$ that is the latus rectum L is as the square of the area $QT \cdot SP$ Q E D

COR Hence the whole area of the ellipse and the rectangle under the axes which is proportional to it is as the product of the square root of the latus rectum and the periodic time For the whole area is as the area $QT \cdot SP$ described in a given time multiplied by the periodic time

PROPOSITION 15 THEOREM 7

The same things being supposed I say that the periodic times in ellipses are as the $^3/_2$*th power* (in ratione sesquiplicata) *of their greater axes*

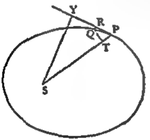
For the lesser axis is a mean proportional between the greater axis and the latus rectum and therefore the product of the axes is equal to the product of

of the greater axis But

PROPOSITION 16 THEOREM 8

drawn to the bodies that on those tangents from the ... inversely as the perpen... al latera recta

...gent PR, and the velocity of the body P varies inversely as the square root of the quantity $\frac{SY^2}{L}$ For that velocity is as the infinitely small arc PQ described in a given moment of time that is (by Lem 7) as the tangent PR that is (because of the proportion PR QT = SP SY) as $\frac{SP\ QT}{SY}$ or inversely as SY and directly as SP QT but SP QT is as the area described in the given time that is (by Prop 14) as the square root of the latus rectum Q E D



COR. I The principal latera recta vary as the squares of the perpendiculars and the squares of the velocities.

distance from the focus is to the velocity in a circle at the same distance from the centre as the square root of the principal latus rectum is to the square root of double that distance

root of the inverse ratio of the distances

COR V In the same figure or even in different figures whose principal latera recta are equal the velocity of a body is inversely as the perpendicular let fall from the focus on the tangent

COR. VI In a parabola the velocity is inversely as the square root of the

parabola is as the square root of the distance In the hyperbola the perpendicular is less variable in the ellipse more

COR VII In a parabola the velocity of a body at any distance from the for s is to the velocity of a body revolving n a
centre as the square root of
le

OU VIII The velocity of a body revolving in any conic section is to the velocity of a body revolving in a circle at the distance of half the principal latus rectum of the section as that distance to the perpendicular let fall from the focus on the tangent of the section This appears from Cor v

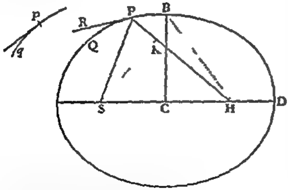
COR IX Wherefore since (b Co

c distances therefore likewise
velocity of a body revolving in a conic
body
that c section to
the p all from the common focus upon the tangent of the section

PROPOSITION 17 PROBLEM 9

Supposing the centripetal force to be inversely
distances of
it is require
given place

Let the c
p revolve i velocity of this body in the place *p* is known Then from the place P suppose the body P to be let go with a given velocity in the direction of the line PR but by virtue of a centripetal force to be immediately turned aside from that right line into the conic section PQ This the right line PR will therefore touch in P Suppo e likewise that the right line *pr* touches the orbit *pq* in *p* and if from S you suppose perpendiculars



let fall on those tangents the principal latus rectum of the conic section (by Cor I Prop 16) will be to the principal latus re t m of

given by

position Let fall SK perpendicular on PH and erect the conjugate semiaxis BC this done we shall have

$$SP^2-2PH \cdot PK+PH^2=SH^2=4CH^2=4(BH^2-BC^2)=$$
$$(SP+PH)^2-L(SP+PH)=SP^2+2PS \cdot PH+PH^2-L(SP+PH)$$

Add on both sides

$$2PK \cdot PH-SP^2-PH^2+L(SP+PH)$$

and we shall have

$$L(SP+PH)=2PS \cdot PH+2PK \cdot PH \text{ or}$$
$$(SP+PH) : PH=2(SP+KP) : L$$

... velocity of the ... will be on ... figure will ... +PH is ... us rectum L ... herefore the ... e PK and is ... ater velocity ... o the tangent passing between the foci the figure ... its principal ... SP and PH and thence is given For if ...

... iven place P with ... iven by position Q E F ... ertex D the latus ... by taking DH to ... rectum and 4DS

For the proportion

$$SP+PH : PH=2SP+2KP : L$$

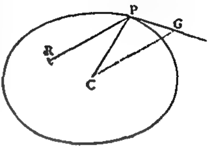
becomes in the case of this Corollary

$$DS+DH : DH=4DS : L$$
$$\text{and } DS : DH=4DS-L : L$$

COR. III Whence if the velocity of a body in the principal vertex D is given the orbit may be readily found namely by taking its latus rectum to twice the distance DS in the squared ratio of this given velocity to the velocity of a body revolving in a circle at the distance DS (by Cor III Prop 16) and then taking DH to DS as the latus rectum to the difference between the latus rectum and 4DS

will undergo in the intermediate places from the analogy that appears in the progress of the series

SCHOLIUM



If a body P by means of a centripetal *force tending to any given point R move in* the perimeter of any given conic section whose centre is C and the law of the centripetal force is required draw CG parallel to the radius RP and meeting the tangent PG of the orbit in G and the force required (by Cor I and Schol, Prop 10 and Cor III Prop 7) will be as $\frac{CG^3}{RP^2}$

SECTION IV

THE FINDING OF ELLIPTIC PARABOLIC AND HYPERBOLIC ORBITS FROM THE FOCUS GIVEN

LEMMA 15

If from the two foci S H *of any ellipse or hyperbola we draw to any third point* V *the right lines* SV HV, *whereof one* HV *is equal to the principal axis of the figure that is to the axis in which the foci are situated the other* SV *is bisected in* T *by the perpendicular* TR *let fall upon it that perpendicular* TR *will somewhere touch the conic section and vice versa if it does touch it* HV *will be equal to the principal axis of the figure*

For let the perpendicular

R and join SR Because TS

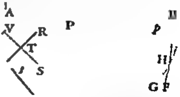
well as the angles *TRS TRV*

the conic section and the perpendicular TR will touch the same and the contrary Q E D

PROPOSITION 18 PROBLEM 10

From a focus and the principal axes given to describe elliptic and hyperbolic curves which shall pass through given points and touch right lines given by position

Let S be the common focus of the figures AB the length of the principal axis of any conic P a point through which the conic should pass and TR a right line which it should touch About the centre P with the radius AB−SP if the orbit is an ellipse or AB+SP if the orbit is an hyperbola describe the circle HG On the tangent TR let fall the perpendicular ST and produce the same to V so that TV may be equal to ST and about V as a centre with the interval AB describe the circle FH In this manner whether two points P *p* are given or two tangents TR *tr* or a point P and a tangent TR we are to describe two circles Let *H* be their

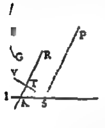


——— — describe the
—SP
rough
in the hyperbola, is equal to ......
the point P and (by the preceding Lemma) will touch the right line ... And by the same argument it will either pass through the two points P ... or touch the two right lines TR tr Q.E.F

PROPOSITION 19 PROBLEM 11

... scribe a parabola which shall pass through given points

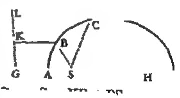
R a tangent of the curve to be described
About P as a centre with the radius S describe the circle FG From the focus let fall ST perpendicular on the tangent and produce the same to V so as TV may be equal to ST After the same manner another circle *fg* is to be described if another point *p* is given or another point *v* is to be found if another tangent *tr* is given then draw the right line IF which shall touch the two circles FG *fg* if two points P *p* are given or pass through the two points V *v* if two tangents TR *tr* are given or touch the circle FG and pass through the point V if the point P and the tangent TR are given On FI let fall the perpendicular SI and bisect the same in K and with the axis SK and principal vertex K describe a parabola I say the thing is done For this parabola (because SK *is equal to* IK and SP *to* FP) will pass through the point P and (by Cor III Lem 14) because ST is equal to TV and STR a right angle it will touch the right line TR. Q.E.F



PROPOSITION 20 PROBLEM 12

About a given focus to describe any given conic which shall pass through given points and touch right lines given by position

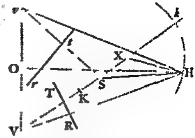
CASE 1 About the focus S it is required to describe a conic ABC passing through two points B C Because the conic is given in kind, the ratio of the principal axis to the distance of the foci will be given. In that ratio take KB to BS and LC to CS About the centres B C with the intervals BK, CL describe two circles and on the right line KL that touches the same in K and L, let fall the perpendicular SG which ...



we shall have
Ga—GA aS—AS=GA AS or Aa SH=GA AS and therefore GA and AS are in the ratio which the principal axis of the figure to be described has to the distance of its foci and therefore the described figure is of the same kind with the figure which was to be described And since KB to BS and LC to CS are in the same ratio this figure will pass through the points B C as is manifest from the conic sections

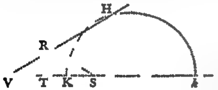
CASE 2 About the focus S it is required to describe a conic which shall some-

where touch two right lines TR tr From the focus on those tangents let fall the perpendiculars ST St which produce to V v so that TV tv may be equal to TS tS Bisect Vv in O and erect the indefinite perpendicular OH, and cut the right line VS infinitely produced in K and k so that VK be to KS and Vk to kS as the principal axis of the conic to be described is to the distance of its foci On the diameter Kk describe a circle cutting OH in H and with the foci S H, and principal axis equal to VH describe a conic I say the thing is done For bisecting Kk in X and joining HX HS HV Hv because VK is to KS as Vk to kS and by composition as



VK+Vk to KS+kS and by subtraction as Vk−VK to kS−KS that is as 2VX to 2KX and 2KX to 2SX and therefore as VX to HX and HX to SX the triangles VXH HXS will be similar therefore VH will be to SH as VX to XH and therefore as VK to KS Wherefore VH the principal axis of the described conic has the same ratio to SH the distance of the foci as the principal axis of the conic which was to be described has to the distance of its foci and is therefore of the same kind And seeing VH vH are equal to the principal axis and VS vS are perpendicularly bisected by the right lines TR tr it is evident (by Lem 15) that those right lines touch the described conic Q E F

CASE 3 About the focus S it is required to describe a conic which shall touch a right line TR in a given point R On the right line TR let fall the perpendicular ST which produce to V so that TV may be equal to ST join VR and cut the right line VS indefinitely produced in K and k so that VK may be to SK and Vk to Sk as the principal axis of the ellipse to be described to the distance of its foci and on the diameter Kk describing a circle cut the right line VR produced in H then with the foci S H and principal axis equal to VH describe a conic I say the thing is done For VH SH = VK SK and therefore as the principal axis of the conic which was



what we have dem

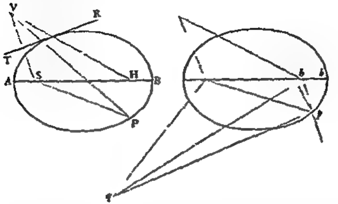
is of the same kind

line TR by which

the angle VRS is bisected touches the conic in the point R is certain from the properties of the conic sections Q E F

CASE 4 About the focus S it is required to describe a conic APB that shall touch a right line TR and pass through any given point P without the tangent and shall be similar to the figure *apb* described with the principal axis *ab* and foci *s* *h* On the tangent TR let fall the perpendicular ST which produce to V so that TV may be equal to ST and making the angles *hsq* *shq* equal to the angles VSP SVP about *q* as a centre and with a radius which shall be to *ab* as SP to VS describe a circle cutting the figure *apb* in *p* Join *sp* and draw SH such that it may be to *sh* as SP is to *sp* and may make the angle I SH equal to the angle *psh* and the angle VSH equal to the angle *psq* Then with the foci S H and principal axis AB equal to the distance VH describe a conic section I say the thing is done for if *sv* is drawn so that it shall be to *sp* as *sh* is to *sq*

and shall make the angle *rep* equal to the angle *hsq* and the angle *rsh* equal to the angle *peq* the triangles *rsh* *epq* will be similar and therefore *rh* will be to *pq* as *sh* is to *sq* that is (because of the similar triangles VSP *hsq*) as VS is to SP or as *ab* to *pq* Wherefore *rh* and *ab* are equal But because of the similar

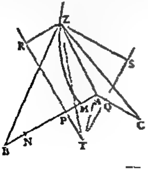


[illegible] VH is to SH as *rh* to *sh* that is the axis of the conic section [illegible] the axis *ab* to the distance of the [illegible] similar to the figure *aph* But because the triangle PSH is [illegible] angle *psh* this figure passes through the point P and because VH is equal to its axis, and VS is perpendicularly bisected by the right line TP the said figure touches the right line TR. Q.E.F.

LEMMA 16

From three given points to draw to a fourth point that is not given three right lines whose differences either shall be given or are zero

CASE 1 Let the given points be A B C and Z the fourth point which we are to find because of the given difference of the lines AZ BZ the locus of the point Z will be an hyperbola whose foci are A and B and whose principal axis is the given difference Let that axis be MN Taking PM to MA as MN to AB erect PR per [illegible]



[illegible] bola, whose foci are A C and whose principal axis is the difference between AZ and CZ and QS a perpendicular on AC may be drawn to which (QS) if from any point Z of this hyperbola a perpendicular ZS is let fall (this ZS) shall be to AZ as the difference between AZ and CZ is to AC [illegible] the ratio [illegible] meet in [illegible] and and [illegible] given in [illegible] TZ and

because the ratios of AZ and TZ to ZS are given their ratio to each other is given also and thence will be given likewise the triangle ATZ whose vertex L the point Z

CASE 2 If two of ...
right line TZ so ...
above Q E I

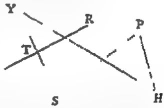
CASE 3 If all the three are equal the point Z will be placed in the centre of a circle that passes through the points A B C Q E I

This problematic Lemma is likewise solved in the *Book of Tactions* of Apollonius [of Perga] restored by Vieta

PROPOSITION 21 PROBLEM 13

About a given focus to describe a conic that shall pass through given points and touch right lines given by position

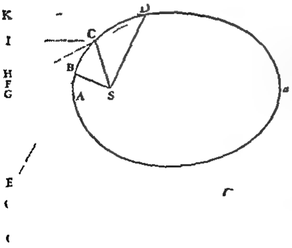
Let the focus S the point P and the tangent TR be given and suppose that the other focus H is to be found On the tangent let fall the perpendicular ST which produce to Y so that TY may be equal to ST and YH will be equal to the principal axis Join SP HP and SP will be the difference between HP and the principal axis After this manner if more tangents TR are given or more points P we shall always determine as many lines YH or PH drawn from the said points Y or P to the focus H which either shall be equal to the axes or differ from the axes by given lengths SP and therefore which shall either be equal among themselves or shall have given differences from whence (by the preceding Lemma) that other focus H is given But having the foci and the length of the axis (which is either YH or if the conic be an ellipse PH+SP or PH−SP if it be an hyperbola) the conic is given Q E I



SCHOLIUM

When ...
under
never

...ase when three points are given is more readily solved thus Let B C D be the given points Join BC CD and produce them to E F so as EB may be to EC as SB to SC and FC to FD as SC to SD On EF drawn and produced let fall the perpendiculars SG BH and in GS produced indefinitely take GA to AS and Ga to aS as HB is to BS then A will be the vertex and Aa the prin...



other side of the line GF

to SA. And, by the like ar ... ratio. Wherefore the point B C D lie in a conic section ... the focus S in such manner that all the right lines drawn from the focus S to the several point of the section and the perpendiculars let fall from the same poin on the right lin GF are in that given ratio

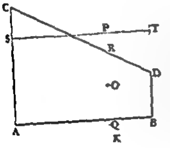
That excellent geometer M de la Hire has solved this Problem much after the same way in his Conics Prop 25 Book viii

SECTION V

How the orbits are to be found when neither focus is given

Lemma 17

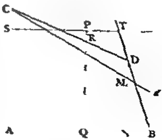
... P ... in a conic section to the four produced sides AB CD ... lines ... of those on the other two opposite sides AC BD ...



Case 1 Let us suppose first that the lines drawn to one pair of opposite sides are parallel to either of the other sides as PQ and PR to the side AC and PS and PT to the side AB And further that one pair of the opposite sides, as AC and BD are parallel between themselves then the right line which bisects those parallel sides will be one of the diameters of the conic section, and will likewise bisect RQ Let O be the point in which RQ is bisected and PO will ... diameter Produce PO

tangle AQ QB in a given ratio But Qk and PR are equal as being the differences of the equal lines Ok OP and OQ OR whence the rectangles PQ Qk and PQ PR are equal and therefore the rectangle PQ PR is to the rectangle AQ QB that is to the rectangle PS PT in a given ratio.

Q.E.D



Case 2 Let us next suppose that the opposite sides AC and BD of the trapezium are not parallel Draw Bd parallel to AC and meeting as well the right line ST in t as the conic section in d Join Cd cutting PQ in and draw DM parallel to PQ cutting Cd in M and AB in N Then (because of

because the ratios of AZ and TZ to ZS are given their ratio to each other is given also and thence will be given likewise the triangle ATZ whose vertex is the point Z Q E I

CASE 2 If two of the three lines for example AZ and BZ are equal draw the right line TZ so as to bisect the right line AB then find the triangle ATZ as above Q E I

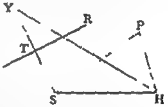
CASE 3 If all the three are equal the point Z will be placed in the centre of a circle that passes through the points A B C Q E I

This problematic Lemma is likewise solved in the *Book of Tactions* of Apollonius [of Perga] restored by Vieta

PROPOSITION 21 PROBLEM 13

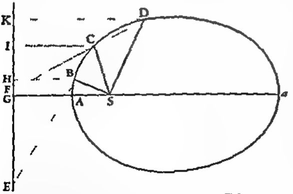
About a given focus to describe a conic that shall pass through given points and touch right lines given by position

Let the focus S the point P and the tangent TR be given and suppose that the other focus H is to be found On the tangent let fall the perpendicular ST which produce to Y so that TY may be equal to ST and YH will be equal to the principal axis Join SP HP and SP will be the difference between HP and the principal axis After this manner if more tangents TR are given or more points P we shall always determine as many lines YH or PH drawn from the said points Y or P to the focus H which either shall be equal to the axes or differ from the axes by given lengths SP and therefore which shall either be equal among themselves or shall have given differences from whence (by the preceding Lemma) that other focus H is given But having the foci and the length of the axis (which is either YH or if the conic be an ellipse PH+SP or PH−SP if it be an hyperbola) the conic is given Q E I



SCHOLIUM

When the conic is an hyperbola I do not include its conjugate hyperbola under the name of this conic For a body going on with a continued motion can never pass out of one hyperbola into its conjugate hyperbola



The case when three points are given is more readily solved thus Let B C D be the given points Join BC CD and produce them to E F so as FB may be to EC as SB to SC and FC to FD as SC to SD On EF drawn and produced let fall the perpendiculars SG BH and in GS produced indefinitely take GA to AS and Ga to aS as HB is to BS then A will be the vertex and Aa the principal axis of the conic which according as GA is greater than equal to or less than AS will be either an ellipse a parabola or an hyperbola the point a in the first case falling on the same side of the line GF as the point A in the second going

on the other side of the line GF

to SA And by the like argum
ratio Wherefore the points B C D lie in a conic section described a the focus S in such manner that all the right lines drawn from the focus S to the several points of the section and the perpendiculars let fall from the same points on the right line GF are in that given ratio

That excellent geometer M de la Hire has solved this Problem much after the same way in his Conics Prop 25 Book VIII

SECTION V

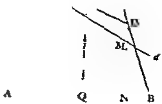
HOW THE ORBITS ARE TO BE FOUND WHEN NEITHER FOCUS IS GIVEN

LEMMA 17

If from any point P *of a given conic section to the four produced sides* AB CD AC DB *of any trapezium* ABDC *inscribed in that section as many right lines* PQ PR PS PT *are drawn in given angles each line to each side the rectangle* PQ PR *of those on the opposite sides* AB CD *will be to the rectangle* PS PT *of those on the other two opposite sides* AC BD *in a given ratio*

CASE 1 Let us suppose first that the lines drawn to one pair of opposite sides are parallel to either of the other sides as PQ and PR to AB And

rectangles PQ QR and PQ PR are equal and therefore the rectangle PQ PR is to the rectangle AQ QB that is to the rectangle PS PT in a given ratio Q E D



CASE 2 Let us next suppose that the opposite sides AC and BD of the trapezium are not parallel Draw Bd parallel to AC and meeting as well the right line ST in t as the conic section in d Join Cd cutting PQ in r and draw DM parallel to PQ cutting Cd in M and AB in N Then (because of

because the ratios of AZ and TZ to ZS are given their ratio to each other is given also and thence will be given likewise the triangle ATZ whose vertex is the point Z Q E I

CASE 2 If two of the three lines for example AZ and BZ are equal draw the right line TZ so as to bisect the right line AB then find the triangle ATZ as above Q E I

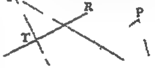
CASE 3 If all the three are equal the point Z will be placed in the centre of a circle that passes through the points A B C Q E I

This problematic Lemma is likewise solved in the *Book of Tactions* of Apollonius [of Perga] restored by Vieta

PROPOSITION 21 PROBLEM 13

About a given focus to describe a conic that shall pass through given points and touch right lines given by position

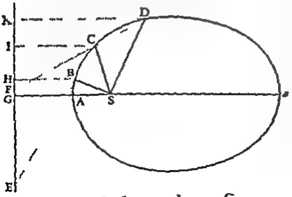
Let the focus S the point P and the tangent TR be given and suppose that the other focus H is to be found On the ... perpendicular ST which produce to Y so that ... H will be equal to the principal axis Join SP ... difference between HP and the principal axis After this manner if more tangents TR are given or more points P we shall always determine as many lines YH or PH drawn from the said points Y or P to the focus H which ... the ... to ...



... that other focus H ... foci and the length of the axis (which is either YH or if the conic be an ellipse PH+SP or PH−SP if it be an hyperbola) the conic is given Q E I

SCHOLIUM

When the conic is an hyperbola I do not include its conjugate hyperbola under the name of this conic For a body going on with a continued motion can never pass out of one hyperbola into its conjugate hyperbola



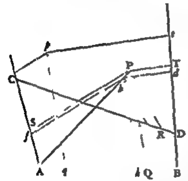
The case when three points are given is more readily solved thus Let B C D be the given points Join BC CD and produce them to E F so as EB may be to EC as SB to SC and FC to FD as SC to SD On EF drawn and produced let fall the perpendiculars SG BH and in GS produced indefinitely take GA to AS and Ga to aS as HB is to BS then A will be the vertex and A... of the conic ...

touches the lines AB CD in A and C and the contrary For the position of the ... right lines AB CD AC remaining the same let the line BD approach to ... PT come likewise to coincide ... he right lines ... and D can no

longer cut but only touch ...

SCHOLIUM

In this Lemma the name of conic section is to be understood in a large sense ... ell the rectilinear section through the vertex of the cone ... to be in a



which is that right line upon which the point p falls and the other is a right line that joins the other two of the four points If the two opposite angles of the trapezium taken together are equal to two right angles and if the four lines PQ PR, PS PT are drawn to the sides thereof at right angles or any other equal angles and the rectangle PQ PR under two of the lines drawn PQ and PR, is equal to the rectangle PS PT under the other two PS and PT the conic section will become a circle And the same thing will happen if the four lines are drawn in any angles and the rectangle PQ PR ... one pair of the lines drawn, is to the rectangle PS PT under

quadrilateral figure whose two opposite sides cross one another like diagonals And one or two of the four points A, B C D may be supposed to be removed

llum

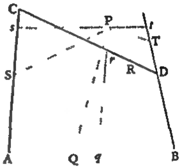
LEMMA 19

To find a point P *from which if four right lines* PQ PR PS PT *are drawn to as many other right lines* AB CD AC BD *given by position each to each at given angles the rectangle* PQ PR *under any two of the lines drawn shall be to the rectangle* PS PT *under the other two in a given ratio*

Suppose the lines AB CD to which the two right lines PQ PR, containing one of the rectangles, are drawn to meet two other lines given by position in the points A B C D From one of those as A draw any right line AH in which you would find the point P Let this cut the opposite lines BD CD in H and I and because all the angles of the figure are given, the ratio of PQ to PA

the similar triangles BT*t* DBN) B*t* or PQ T*t* = DN NB And so R*r* AQ or PS = DM AN Wherefore by multiplying the antecedents by the antecedents and the consequents by the consequents as the rectangle PQ R*r* is to the rectan le PS T*t* so will the rectangle DN DM be to the rectangle NA NB and (by Case 1) so is the rectangle PQ P*r* to the rectangle PS P*t* and by division so is the rectangle PQ PR to the rectangle PS PT Q E D

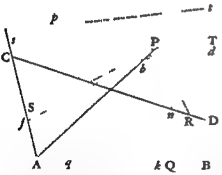
CASE 3 Let us suppose lastly the four lines PQ PR PS PT not to be parallel to the sides AC AB but any way inclined to them In their place draw P*q* P*r* parallel to AC and P*s* P*t* parallel to AB and because the angles of the triangles PQ*q* PR*r* PS*s* PT*t* are given the ratios of PQ to P*q* PR to P*r* PS to P*s* PT to P*t* will be also given and therefore the compounded ratios PQ PR to P*q* P*r* and PS PT to P*s* P*t* are given But from what we have demonstrated before the ratio of P*q* P*r* to P*s* P*t* is given and therefore also the ratio of PQ PR to PS PT Q E D



LEMMA 18

The same things supposed if the rectangle PQ PR *of the lines drawn to the two opposite sides of the trapezium is to the rectangle* PS PT *of those drawn to the other two sides in a given ratio the point* P *from whence those lines are drawn will be placed in a conic section described about the trapezium*

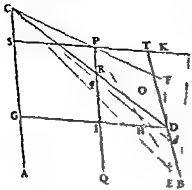
Conceive a conic section to be described passing through the points A B C D and any one of the infinite number of points P as for example *p* I say the point P will be always placed in this section If you deny the thing join AP cutting this conic section somewhere else if possible than in P as in *b* Therefore if from those points *p* and *b* in the given angles to the sides of the trapezium we draw the right lines *pq pr ps pt* and *bk bn bf bd* we shall have as *bk bn* to *bf bd* so (by Lem 17) *pq pr* to *ps pt* and so (by supposition) PQ PR to PS PT And because of the similar trapezia *bkAf* PQAS as *bk* to *bf* so PQ to PS Wherefore by dividing the terms of the preceding proportion by the corresponding terms of this we shall have *bn* to *bd* as PR to PT And therefore the equiangular trapezia D*nbd* DRPT are similar and consequently their diagonals D*b* DP do coincide Wherefore *b* falls in the intersection of the right lines AP DP and consequently coincides with the point P And therefore the point P wherever it is taken falls within the assigned conic section Q E D



COR Hence if three right lines PQ PR PS are drawn from a common point P to as many other right lines given in position AB CD AC each to each in as many angles respectively given and the rectangle PQ PR under any two of the lines drawn be to the square of the third PS in a given ratio the point P from which the right lines are drawn will be placed in a conic section that

other in a given ra io the locus of the point D will be a conic section passing through the four poi s A, B C P

CASE 1 Join BP CP and from the point D draw the two right lines DG DE, of which the first DG shall be parallel to AB and meet PB PQ CA, in H I G and the other DE hall be parallel to AC and meet PC PS AB in F K E and (by Lem 17) the rectangle DE DF will be



PP ı. to PS as D G DG ...

pounding those ratios, the rectangle PQ PR will be to the rectangle PS PT as the rectangle DE DF is to the rectangle DG DH and consequently in a given ratio But PQ and PS are given and therefore the ratio of PR to PT is given.

Q E.D

the

the

point

C P

as its locus.

Q E D

COR. I. Hence if we draw BC cutting PQ in r and in PT take Pt to Pr in the same ratio which PT has to PR then Bt will touch the conic section in the point B For suppose the point D to coalesce with the point B so that the chord BD vanishing BT shall become a tangent and CD and BT will coincide with CB and Bt

BD CD

conic section.

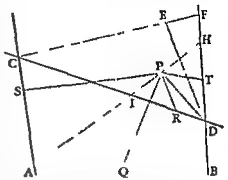
COR. III One conic section canno cut another conic section in more than

LEMMA 21

passing throu h th poi ts B C And conversely f the right l es BD CD do by their point of meet ng M describe a conic section passing through the given points B C A, and the angle DBM is always equal to the given an le ABC as well as the

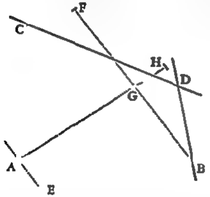
and PA to PS and therefore of PQ to PS will be also given This ratio taken as a divisor of the given ratio of PQ PR to PS PT gives the ratio of PR to PT, and multiplying the given ratios of PI to PR and PT to PH the ratio of PI to PH, *and therefore the point P, will be given*

Q E I



COR I Hence also a tangent may be drawn to any point D of the locus of all the points P For the chord PD where the points P and D meet that is where AH is drawn through the point D becomes a tangent In which case the ultimate ratio of the evanescent lines IP and PH will be found as above Therefore draw CF parallel to AD meeting BD in F and cut it in E in the same ultimate ratio then DE will be the tangent because CF and the evanescent IH are parallel and similarly cut in E and P

COR II Hence also the locus of all the points P may be determined Through any of the points A B C D as A draw AE touching the locus and through any other point B parallel to the tangent draw BF meeting the locus in F and find the point F by this Lemma Bisect BF in G and drawing the indefinite line AG this will be the position of the diameter to which BG and FG are ordinates Let this AG meet the locus in H and AH will be its diameter or latus transversum to which the latus rectum will be as BG^2 to AG GH If AG nowhere meets the locus the line AH being infinite the locus will be a parabola and its latus rectum corresponding to the diameter AG



will be $\frac{BG^2}{AG}$ But if it does meet it anywhere the locus will be an hyperbola when the points A and H are placed on the same side of the point G and an ellipse if the point G falls between the points A and H unless perhaps the angle AGB is a right angle and at the same time BG^2 equal to the rectangle GA GH in which case the locus will be a circle

And so we have given in this Corollary a solution of that famous Problem of the ancients concerning four lines begun by Euclid and carried on by Apollonius and this not an analytical calculus but a geometrical composition such as the ancients required

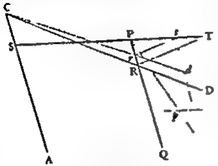
LEMMA 20

If the two opposite angular points A *and* P *of any parallelogram* ASPQ *touch any conic section in the points* A *and* P *and the sides* AQ AS *of one of those angles indefinitely produced meet the same conic section in* B *and* C *and from the points of meeting* B *and* C *to any fifth point* D *of the conic section two right lines* BD CD *are drawn meeting the two other sides* PS PQ *of the parallelogram indefinitely produced in* T *and* R *the parts* PR *and* PT *cut off from the sides will always be one to the other in a given ratio And conversely if those parts cut off are one to the*

PROPOSITION 22 PROBLEM 14

To describe a conic that shall pass through five given points

Let the five given points be A B C P D From any one of them as A to any other two as B C which may be called the poles draw the right lines AB AC and parallel to those the lines TPS PRQ through the fourth point P Then from the two poles B C draw through the fifth point D two indefinite lines BDT CRD meeting with the last drawn lines TPS PRQ (the former with the former and the latter with the latter) in T and R. And then draw the right line *tr* parallel to TR cutting off from the right lines PT



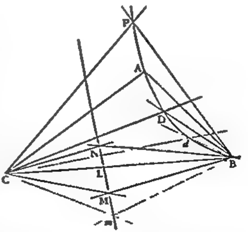
tremities *t* *r* and the poles B

d that point *d* will be placed in the conic required For (by Lem 20) the point *d* is placed in a conic section passing through the four points A B C P and the lines Rr Tt vanishing the point *d* comes to coincide with the point D Wherefore the conic section passes through the five points A B C P D

Q E D

The same otherwise

point D then to the point P and mark the points M N in which the other legs BL CL intersect each other in both cases Draw the indefinite right line MN and let those movable angles revolve about their poles B C in such manner that the intersection which is now supposed to be *m* of the legs BL CL or BM CM may always fall in that indefinite right line MN and the intersection which is now supposed to be *d* of the legs BA CA or BD CD will describe the conic required PAD*d*B For (by Lem



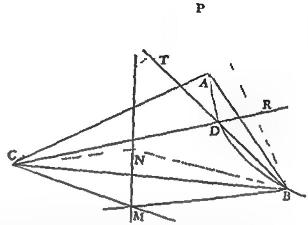
1) the point *d* will be placed in a conic section passing through the points B C and when the point *m* comes to coincide with the points L, M N the point *d* will (by construction) come to coincide with the points A, D P Wherefore a conic section will be described that shall pass through the five points A B C P D

Q E F

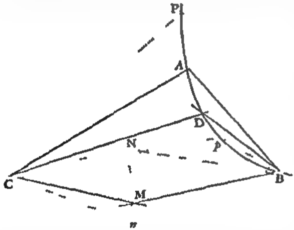
COR. 1 Hence a right line may be readily drawn which shall be a tangent to

angle DCM *always equal to the given angle* ACB *the point* M *will lie in a right line given by position as its locus*

For in the right line MN let a point N be given and when the movable point M falls on the immovable point N, let the movable point D fall on an immovable point P Join CN BN CP BP and from the point P draw the right lines PT PR meeting BD CD in T and R and making the angle BPT equal to the given angle BNM and the angle CPR equal to the given angle CNM Wherefore since (by supposition) the angles MBD NBP are equal as also the angles MCD NCP take away the angles NBD and NCD that are common and there will remain the angles NBM and PBT NCM and PCR equal and therefore the triangles NBM PBT are similar as also the triangles NCM PCR Wherefore PT is to NM as PB to NB and PR to NM as PC to NC But the points B C N P are immovable wherefore PT and PR have a given ratio to NM and consequently a given ratio between themselves and therefore (by Lem 20) the point D wherein the movable right lines BT and CR continually concur will be placed in a conic section passing through the points B C P Q E D



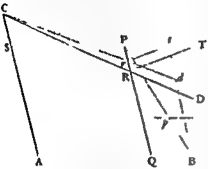
And conversely if the movable point D lies in a conic section passing through the given points B C A and the angle DBM is always equal to the given angle ABC and the angle DCM always equal to the given angle ACB and when the point D falls successively on any two immovable points p P of the conic section the movable point M falls successively on two immovable points n N Through these points n N draw the right line nN this line nN will be the continual locus of that movable point M For if possible let the point M be placed in any curved line Therefore the point D will be placed in a conic section passing through the five points B C A p P when the point M is continually placed in a curved line But from what was demonstrated before the point D will be also placed in a conic section passing through the same five points B C A p P when the point M is continually placed in a right line Wherefore the two conic sections will both pass through the same five points against Cor III Lem 20 It is therefore absurd to suppose that the point M is placed in a curved line Q E D



PROPOSITION 22 PROBLEM 14

To describe a conic that shall pass through five given points

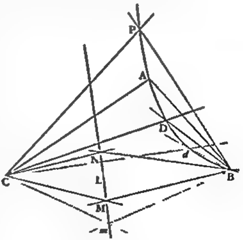
Let the five given points be A B C P D From any one of them as A to any other two as B C which may be called the poles draw the right lines AB AC and parallel to those the lines TPS PRQ through the fourth point P Then from the two poles B C draw through the fifth point D two indefinite lines BDT CRD meeting with the last drawn lines TPS PRQ (the former with the former and the latter with the latter) in T and R And then draw the right line *tr* parallel to TR cutting off from the right lines PT PR any segments P*t* P*r* proportional to PT PR and if through their extremities *t r* and the poles B C the right lines B*t* C*r* are drawn meeting in *d* that point *d* will be placed in the conic required For (by Lem 20) that



Q E D

The same otherwise

Of the given points join any three as A B C and about two of them B C as poles making the angles ABC ACB of a given magnitude to revolve apply the legs BA CA first to the point D then to the point P and mark the points M N in which the other legs BL CL intersect each other in both cases Draw the indefinite right line MN and let those movable angles revolve about their poles B C in such manner that the intersection which is now supposed to be *m* of the legs BL, CL or BM CM may always fall in that indefinite right line MN and the intersection which is now supposed to be *d* of the legs BA, CA, or BD CD will describe the conic required, PAD*d*B For (by Lem 21) the point *d* will be placed in a conic section passing through the points B C and when the point *m* comes to coincide with the points L M N the point *d* will (by construction) come to coincide with the points A D P Wherefore a conic section will be described that shall pass through the five points A, P C P D



Q E F

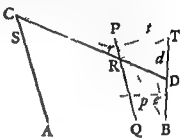
COR. 1 Hence a right line may be readily drawn which shall be a tangent to

the conic in any given point B Let the point *d* come to coincide with the point B and the right line B*d* will become the tangent required

COR II Hence also may be found the centres diameters and latera recta of the conics as in Cor II Lem 19

SCHOLIUM

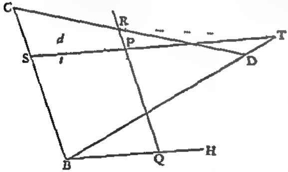
The former of these constructions will become something more simple by joining B P and in that line produced if need be taking B*p* to BP as PR is to PT and through *p* draw the indefinite right line *pe* parallel to SPT and in that line *pe* taking always *pe* equal to P*r* and draw the right lines B*e* C*r* to meet in *d* For since P*r* to P*t* PR to PT *p*B to PB *pe* to P*t* are all in the same ratio *pe* and P*r* will be always equal After this manner the points of the conic are most readily found unless you would rather describe the curve mechanically as in the second construction



PROPOSITION 23 PROBLEM 15

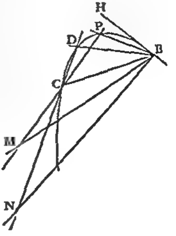
To describe a conic that shall pass through four given points and touch a given right line

CASE 1 Suppose that HB is the given tangent B the point of contact and C D P the three other given points Join BC and draw PS parallel to BH and PQ parallel to BC complete the parallelogram BSPQ Draw BD cutting SP in T and CD cutting PQ in R Lastly draw any line *tr* parallel to TR cutting off from PQ PS the segments P*r* P*t* proportional to PR PT respectively and draw C*r* B*t* their point of intersection *d* will (by Lem 20) always fall on the conic to be described



The same otherwise

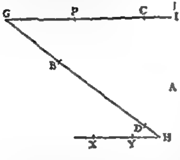
Let the angle CBH of a given magnitude revolve about the pole B as also the rectilinear radius DC both ways produced about the pole C Mark the points M N on which the leg BC of the angle cuts that radius when BH the other leg thereof meets the same radius in the points P and D Then drawing the indefinite line MN let that radius CP or CD and the leg BC of the angle continually meet in this line and the point of meeting of the other leg BH with the radius will delineate the conic required



For if in the constructions of the preceding Problem the point A comes to a coincidence with the point B the lines CA and CB will coincide and the line AB in its last situation will become the tan

gent BH and therefore the constructions there set down will become the same with the constructions here described Wherefore the intersection of the leg BH with the radius will describe a conic section passing through the points C D P and touching the line BH in the point B Q E F

CASE 2 Suppose the four points B C D P given being situated without the tangent HI Join each two by the lines BD CP meeting in G and cutting the tangent in H and I Cut the tangent in A in such manner that HA may be to IA as the product of the mean propor ... CP and the mean

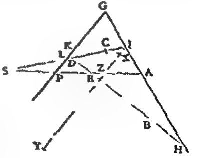


tween ... f contact For if HX is parallel to the right line PI cuts the conic in any points X and Y the point A (by the properties of the conic section) will come to be so placed that HA^2 will become to AI^2 in a ratio that is compounded out of the ratio of the rectangle HX HY to the rectangle BH HD or of the rectangle CG GP to the rectangle DG GB and the ratio of the rectangle BH HD to the rectangle PI IC But after the point of contact A is found the conic will be described as in the first Case Q E F But the point A may be taken either between or without the points H and I upon which account a two-fold conic may be described

PROPOSITION 24 PROBLEM 16

To describe a conic that shall pass through three given points and touch two given right lines

Suppose HI KL to be the given tangents and B C D the given points Through any two of those points as B D draw the indefinite right line BD meeting the tangents in the points H K Then likewise through any other two of these points as C D draw the indefinite ... th



and IS to LS as the mean proportional between CI and ID is to the mean proportional between CL and LD But you may cut at pleasure either within or between the points K and H I and L or without them Then draw RS cutting the tangents in A and P and A and P will be the points of contact For if A and P are supposed to be

conic sections) be to LP as the rectangle CI ID is to the rectangle CL LD that is (by the construction) as SI is to SL^2 and therefore IZ LP = SI SL Wherefore the points S P Z are in one right line Moreover since the tangents meet in G the rectangle XI IY or IZ^2 will (by the properties of the

IA = GP GA

the points

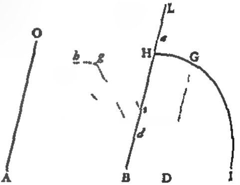
I and A are in one right line And the same argument will prove that the points R, P and A are in one right line Wherefore the points of contact A and P lie in the right line RS But after these points are found the conic may be described as in the first Case of the preceding Problem Q E F

In this Proposition and Case 2 of the foregoing the constructions are the same whether the right line XY cuts the conic in X and Y or not neither do they depend upon that section But the constructions being demonstrated where that right line does cut the conic the constructions where it does not are also known, and therefore for brevity s sake I omit any further demonstration of them

LEMMA 22

To transform figures into other figures of the same kind

Suppose that any figure HGI is to be transformed Draw at pleasure two parallel lines AO BL cutting any given third line AB in A and B and from any point G of the figure draw out any right line GD parallel to OA till it meets the right line AB Then from any given point O in the line OA draw to the point D the right line OD meeting BL in d



having such ratio to *Od* as DG has to OD and *g* will be the point in the new figure *hgi* corresponding to the point G And in like manner the several points of the first figure will give as many correspondent points of the new figure If we therefore conceive the point G to be carried along by a continual motion through all the points of the first figure the point *g* will be likewise carried along by a continual motion through all the points of the new figure and describe the same For distinction s sake let us call DG the first ordinate *dg* the new ordinate AD the first abscissa *ad* the new abscissa O the pole OD the abscinding radius OA the first ordinate radius and O*a* (by which the parallelogram OAB*a* is completed) the new ordinate radius

order and so on in curved lines of higher orders The two lines in which the points G *g* are placed will be always of the same analytical order For as *ad* OA = *Od* OD = *dg* DG = AB AD and therefore AD is equal to $\frac{OA\ \ AB}{ad}$

and DG equal to $\frac{OA \cdot dg}{ad}$ Now if the point G is placed in a right line and therefore in any equation by which the relation between the abscissa AD and the ordinate GD is expressed those indetermined lines AD and DG rise no higher than to one dimension by writing this equation $\frac{OA \cdot AB}{ad}$ in place of AD and $\frac{OA \cdot dg}{ad}$ in place of DG a new equation will be produced in which the new abscissa ad and new ordinate dg rise only to one dimension and which therefore ... ht line But if AD and DG (or either of them) had risen to two ... risen to two ... nsions. The ... he first will ... nes in which

I say further that if any right line touch ... first figure the same right line transferred the same way with the curve into the new figure will touch that curved line in the new figure and conversely For if any two points of the curve in the first figure are supposed to approach one the other till they come to coincide the same points transferred will approach one the other till they come to coincide in the new figure and therefore the right lines with hich those points are joined will become together tangents of the curves in both figures. I might have given demonstrations of these assertions in a more geometrical form but I study to be brief

Wherefore if one rectilinear figure is to be transformed into another we need only transfer the intersections of the right lines of which the first figure con... ... to draw right lines in the new ... we must transfer the ... which the curved line is defined This Lemma is of use in the olution of the more difficult Problems for thereby we may transform the proposed figures if they are intricate into

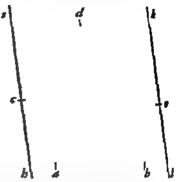
... transformed into a right line and a circle

PROPOSITION 25 PROBLEM 17

To describe a conic that shall pass through two given points and touch three given right lines

Through the intersection of any two of the tangents one with the other and

the intersection of the third tangent with the right line which passes through the two given points draw an indefinite right line and taking this line for the first ordinate radius transform the figure by the preceding Lemma into a new figure In this figure those two tangents will become parallel to each other and the third tangent will be parallel to the right line that passes through the two given points Suppose hi kl to be those two parallel tangents ik the third tangent and hl a right line parallel thereto passing through those points a b through which the conic section ought to pass in this new figure and right be to d and



ke to kd as the sum of the right lines hi and kl is to the sum of the three lines the first whereof is the right line ik and the other two are the square roots of the rectangles ahb and alb and c d e will be the points of contact For by the properties of the conic sections

$$hc \quad ah\ hb = ic^2 \quad id^2 = ke^2 \quad kd^2 = el^2 \quad al\ lb$$

Therefore

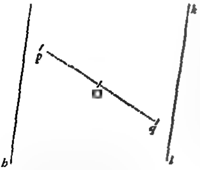
$$hc \quad \surd(ah\ hb) = ic \quad id = ke \quad kd = el \quad \surd(al\ lb)$$
$$= hc + ic + ke + el \quad \surd(ah\ hb) + id + kd + \surd al\ lb$$
$$= hi + kl \quad \surd(ah\ hb) + ik + \surd(al\ lb)$$

Wherefore from that given ratio we have the points of contact c d e in the new figure By the inverted operations of the last Lemma let those points be transferred into the first figure and the conic will be there described by Prob 14 Q E F But according as the points a b fall between the points h l or without them the points c d e must be taken either between the points h i k l or without them If one of the points a b falls between the points h i and the other without the points h l the Problem is impossible

PROPOSITION 26 PROBLEM 18

To describe a conic that shall pass through a given point and touch four given right lines

From the common intersections of any two of the tangents to the common intersection of the other two draw an indefinite right line and taking this line for the first ordinate radius transform the figure (by Lem 22) into a new figure and the two pairs of tangents each of which before concurred in the first ordinate radius will now become parallel Let hi and kl ik and hl be those pairs of parallels completing the parallelogram $hikl$ And let p be the point in this new figure corresponding to the given point in the first figure Through O the centre of the figure draw pq and Oq being equal to Op q will be the other point through which the conic section must pass in this new figure Let this point be transferred by the inverse operation of Lem 22 into the first figure and there we shall have the two points through which

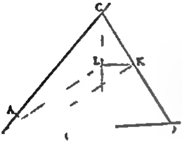


the conic is to be described But through those points that conic may be described by Prop 17

LEMMA 23

AC BD *terminating in given points* A B *are in a* *nde ermined points* *H will be placed in*

BG to AE as BD d by constructe at 1 to EF as atio l be s CL CD



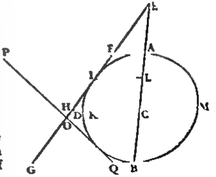
and because that i a given rat o, e tri angle EFL will be given in kind and therefore the point L will be placed in the given ri ht line EL. Join LK and the trian les CLK CFD will be similar and because

the given

COR. Because the figure EFLC is given in kind the three right lines EF EB and EC that is GD HK and EC will have given ratios to each other

LEMMA 24

Let AF GB be the two parallels touching the conic section ADB in A and B EF the third right line touching the conic section in I and meeting the two former tangents in F and G and let CD be the semidiameter of the figure parallel to those tangents I say that AF CD BG are continually proportional For if the conjugate diameters AB DM meet the tangent FG in E and H and cut one the other in C and the parallelogram IKCL be completed from the nature of the conic sections,



$$EC : CA = CA : CL$$

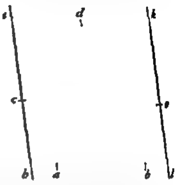
thence $$EC - CA : CA - CL = EC : CA$$

or $$EA : AL = EC : CA$$

thence $$EA : EA + AL = EC : EC + CA$$

or $$EA : EL = EC : EB$$

the intersection of the third tangent with the right line which passes through the two given points draw an indefinite right line and taking this line for the first ordinate radius transform the figure by the preceding Lemma into a new figure In this figure those two tangents will become parallel to each other and the third tangent will be parallel to the right line that passes through the two given points Suppose hi kl to be those two parallel tangents ik the third tangent and hl a right line parallel thereto passing through those points a b through which the conic section ought to pass in this new figure and completing the parallelogram $hikl$ let the right lines hi ik kl be so cut in c d e that hc may be to the square root of the rectangle ahb ic to id and ke to kd as the sum of the right lines hi and kl is to the sum of the three lines the first whereof is the right line ik and the other two are the square roots of the rectangles ahb and alb and c d e will be the points of contact For by the properties of the conic sections



$$hc^2\quad ah\ hb = ic^2\quad id^2 = ke^2\quad kd^2 = el^2\quad al\ lb$$

Therefore

$$hc\quad \sqrt{(ah\ hb)} = ic\quad id = ke\quad kd = el\quad \sqrt{(al\ lb)}$$
$$= hc + ic + ke + el\quad \sqrt{(ah\ hb)} + id + kd + \sqrt{al\ lb}$$
$$= hi + kl\quad \sqrt{(ah\ hb)} + ik + \sqrt{(al\ lb)}$$

Wherefore from that given ratio we have the points of contact c d e in the new figure By the inverted operations of the last Lemma let those points be transferred into the first figure and the conic will be there described by Prob 14 Q E F But according as the points a b fall between the points h l or without them the points c d e must be taken either between the points h i k l or without them If one of the points a b falls between the points h i and the other without the points h l the Problem is impossible

PROPOSITION 26 PROBLEM 18

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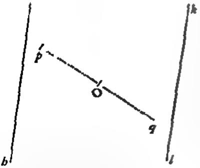


the conic is to be described But through those points that conic may be described by Prop 17

LEMMA 23

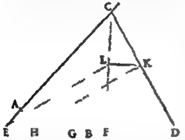
~ ~ points A B are in a

nnts

ed in

a given fig e

For let the right lines AC BD meet in E and in BL ... ; BD is to AC and let FD be always equal to the given line EG and by construc- ... be to CD that is to EF as

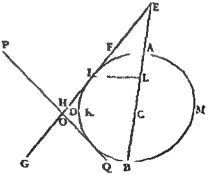


given in kind Let CF be cut in L so as CL may be to CF in the ratio of CK to CD and because that i a given ratio the triangle EFL will be given in kind and therefore the point L will be placed in the given right line EL Join LK and the triangles CLK CFD will be similar and because FD is a given line and LK is to FD in a given ratio LK will be also given To this let EH be taken equal and ELKH will be always a parallelogram And therefore the point K is always placed in the given side HK of that parallelogram Q E D

COR. Because the figure EFLC is given in kind the three right lines EF EL and EC that is GD HK and EC will have given ratios to each other

LEMMA 24

~ ~



~ ~ ~

IKCL be completed from the nature of the conic sections

EC CA=CA CL

thence EC−CA CA−CL=EC CA

or EA AL=EC CA

thence EA EA+AL=EC EC+CA

or EA EL=EC EB

Therefore because of the similitude of the triangles EAF ELI, ECH EBG

AF LI=CH BG

Likewise from the nature of the conic sections

LI or CK CD=CD CH

Taking the products of corresponding terms in the last two proportions and simplifying

AF CD=CD BG Q E D

COR I Hence if two tangents FG PQ meet two parallel tangents AF BG in F and G P and Q and cut one the other in O then by the Lemma applied to EG and PQ

AF CD=CD BG

BQ CD=CD AP

Therefore AF AP=BQ BG

and AP−AF AP=BG−BQ BG

or PF AP=GQ BG

and AP BG=PF GQ=FO GO=AF BQ

COR II Whence also the two right lines PG FQ drawn through the points P and G F and Q will meet in the right line ACB passing through the centre of the figure and the points of contact A B

LEMMA 25

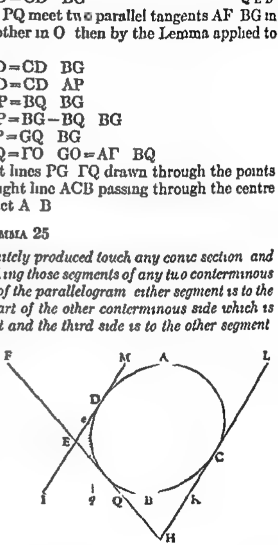
If four sides of a parallelogram indefinitely produced touch any conic section and are cut by a fifth tangent I say that taking those segments of any two conterminous sides that terminate in opposite angles of the parallelogram either segment is to the side from which it is cut off as that part of the other conterminous side which is intercepted between the point of contact and the third side is to the other segment

Let the four sides ML IK KL MI of the parallelogram MLIK touch the conic section in A B C D and let the fifth tangent FQ cut those sides in F Q H and E and taking the segments ME KQ of the sides MI KI or the segments KH MF of the sides KL ML I say that

ME MI=BK KQ

and KH KL=AM MF

For by Cor I of the preceding Lemma



ME EI=AM or BK BQ

and by addition

ME MI=BK KQ Q E D

Also KH HI =BK or AM AF

and by subtraction

KH KL=AM MF Q E D

COR I Hence if a parallelogram IKLM described about a given conic section is given the rectangle KQ ME as also the rectangle KH MF equal thereto will be given For by reason of the similar triangles KQH MFE those rectangles are equal

COR II And if a sixth tangent *eq* is drawn meeting the tangents KI MI in *q*

and e the rectangle KQ ME will be equal to the rectangle Kq Me and

KQ Me=Kq ME

and by subtraction

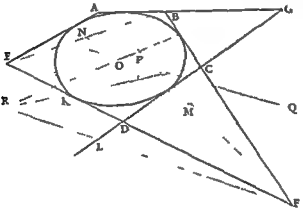
KQ Me=Qq Ee

centre of the conic section For ... will pass throu h the middle of all the lines Eq eQ MK (by Lem 23) and the middle point of the right line MK is the centre of the section

PROPOSITION 27 PROBLEM 19

To describe a conic that may touch five right lines given in position

... ABC BCF GCD FDE EA to be the tangents given in position ... ABFE ... MN



... through the centre of the conic

bisecting lines Suppose it to be O Parallel to any tangent ... distance that the centre O may be placed in the middle between the parallels this KL will touch the conic to be described Let this cut any other two tan gent GCD FDF in I and K Through the points C and K, F and L where

and then the conic may be described by Prob 14 Q E F

SCHOLIUM

Under the preceding Propositions are comprehended those Problems where in either the centres or asymptotes of the conics are given For when points and tangents and the centre are given as many other points and as many other

Therefore because of the similitude of the triangles EAF, ELI ECH EBG

AF LI=CH BG

Likewise from the nature of the conic sections

LI or CK CD=CD CH

Taking the products of corresponding terms in the last two proportions and simplifying

AF CD=CD BG Q E D

COR I Hence if two tangents FG PQ meet two parallel tangents AF BG in F and G P and Q and cut one the other in O then by the Lemma applied to EG and PQ

AF CD=CD BG

BQ CD=CD AP

Therefore AF AP=BQ BG

and AP−AF AP=BG−BQ BG

or PF AP=GQ BG

and AP BG=PF GQ=FO GO=AF BQ

COR II Whence also the two right lines PG FQ drawn through the points P and G F and Q will meet in the right line ACB passing through the centre of the figure and the points of contact A B

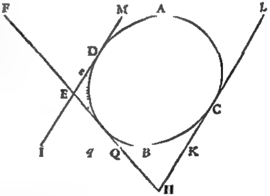
LEMMA 25

If four sides of a parallelogram indefinitely produced touch any conic section and are cut by a fifth tangent I say that taking those segments of any two conterminous sides that terminate in opposite angles of the parallelogram either segment is to the side from which it is cut off as that part of the other conterminous side which is intercepted between the point of contact and the third side is to the other segment

Let the four sides ML IK KL MI of the parallelogram MLIK touch the conic section in A B C D and let the fifth tangent FQ cut those sides in F Q H and E and taking the segments ME KQ of the sides MI KI or the segments KH MF of the sides KL ML I say that

ME MI=BK KQ

and KH KL=AM MF



For by Cor I of the preceding Lemma

ME EI=AM or BK BQ

and by addition

ME MI=BK KQ Q E D

Also KH HI =BK or AM AF

and by subtraction

KH KL=AM MF Q E D

~

tangles are equal

COR II And if a sixth tangent *eq* is drawn meeting the tangents KI MI in *q*

LEMMA 26

lines

EMF capable of angles equal to the angles BAC ABC ACB respectively But those segments are to be described towards such sides of the lines DE DF EF that the letters DRED may turn round about in the same order with the letters BACB the letters DGFD in the same order with the letters ABCA and the letters EMFE in the same order with the letters ACBA then completing those segments into entire circles let the two former circles cut each other in G and suppose P and Q to be their centres Then joining GP PQ take

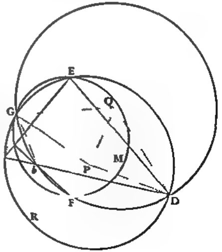
$$Ga \quad AB = GP \quad PQ$$

and about the centre G with the interval Ga describe a circle that may cut the first circle DGE in a Join aD cutting the second circle DFG in b as well as aE cutting the third circle EMF in c Complete the figure ABCdef similar and

to the angle ACB and therefore the triangle anc equiangular to the triangle ABC Wherefore the angle anc or FnD is equal to the angle ABC and consequently to the angle FbD and therefore the point n falls on the point b Moreover the angle GPQ which is half the angle GPD at the centre is equal to the angle GaD at the circumference and the angle GQP which is half the angle GQD at the centre is equal to the supplement of the angle GbD at the circumference and therefore equal to the angle Gba Upon which account the triangles GPQ Gab are similar and

$$Ga \quad ab = GP \quad PQ$$

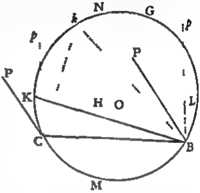
and by construction



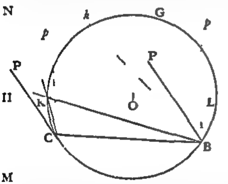
$$GP \quad PQ = Ga \quad AB$$

tangents are given at an equal distance on the other side of the centre And an asymptote is to be considered as a tangent and its infinitely remote extremity (if we may say so) is a point of contact Conceive the point of contact of any tangent removed *in infinitum* and the tangent will degenerate into an asymptote and the constructions of the preceding Problems will be changed into the constructions of those Problems wherein the asymptote is given

After the conic is described we may find its axes and foci in this manner In the construction and figure of Lem 21 let those legs BP CP of the movable angles PBN PCN by the intersection of which the conic was described be made parallel one to the other and retaining that position let them revolve about their poles B C in that figure In the meanwhile let the other legs CN BN of those angles by their intersection K or *k* describe the circle BKGC Let O be the centre of this circle and from this centre upon the ruler MN wherein those legs CN BN did concur while the conic was described let fall the perpendicular OH meeting the circle in K and L And when those other legs CK BK meet in the point K that is nearest to the ruler the first legs CP, BP will be parallel to the greater axis and perpendicular on the lesser and the contrary will happen if those legs meet in the remotest point L Whence if the centre of the conic is given the axes will be given and those being given the foci will be readily found

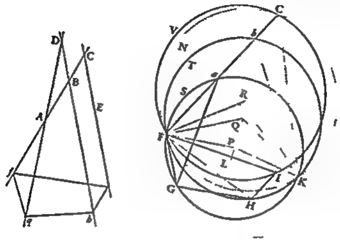


But the squares of the axes are one to the other as KH to LH and thence it is easy to describe a conic given in kind through four given points For if two of the given points are made the poles C B the third will give the movable angles PCK PBK but those being given the circle BGKC may be described Then because the conic is given in kind the ratio of OH to OK and therefore OH itself will be given About the centre O with the interval OH describe another circle and the right line that touches this circle and passes through the intersection of the legs CK BK when the first legs CP BP meet in the fourth given point will be the ruler MN by means of which the conic may be described Whence also on the other hand a trapezium given in kind (excepting a few cases that are impossible) may be inscribed in a given conic section



There are also other Lemmas by the help of which conics given in kind may be described through given points and touching given lines Of such a sort is this that if a right line is drawn through any point given in position that may cut a given conic section in two points and the distance of the intersections is bisected the point of bisection will touch another conic section of the same kind with the former and having its axes parallel to the axes of the former But I hasten to things of greater use

angle ACE But the segments are to be described towards those sides of the FG FH FI that the circular order of the letters FSGF may be the same ... FTHF may turn about in the ... in the same order as the ... es and let P be the centre



... as FGH GHI be so f ... n creased that the right

and CE will be to each other as the lines FG GH HI and will observe the same order among themselves. But the same thing may be more readily done in this manner

Wherefore ab and AB are equal and consequently the triangles abc ABC which we have now proved to be ...
angles D E F of the triangle DE...
the triangle abc the figure ABCd...
figure abcDFF and by completi... ...roblem will be solved Q E F

Cor Hence a right line may be drawn whose parts given in length may be intercepted between three right lines given in po...
DEF by the ...
DE DF ...

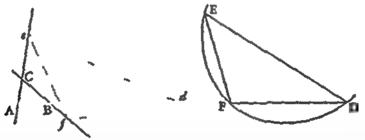
...

Problem will be solved ... preceding construction to this case the

PROPOSITION 28 PROBLEM 20

To describe a conic given both in kind and in magnitude given parts of which shall be placed between three right lines given in position

Suppose a conic is to be described that may be similar and equal to the curved line DEF and may be cut by three right lines AB AC BC given in position into parts DE and EF similar and equal to the given parts of this curved line



Draw the right lines DE EF DF and place the angles D E F of this triangle DEF so as to touch those right lines given in position (by Lem 26) Then about the triangle describe the conic similar and equal to the curve DEF Q E F

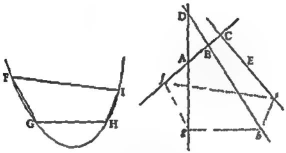
LEMMA 27

To describe a trapezium given in kind the angles whereof may respectively touch four right lines given in position that are neither all parallel among themselves nor converge to one common point

Let the four right lines ABC AD BD CE be given in position the first cutting the second in A the third in B and the fourth in C and suppose a trapezium $fghi$ is to be described that may be simila... to the ...pezium FGHI and ... ...ght line ABC and ... I may touch the ... ... BD CE respectively Join FH and upon FG FH FI describe as many segments of circles FSG FTH FVI the first of ...
may
of an

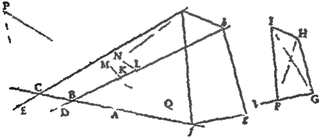
the second between the second and the third between the third Draw the right lines FG GH HI FI and (by Lem 27) describe a trapezium *fghi* that ... FGHI and whose angles *f g h i* may touch ...

to the curved line ...



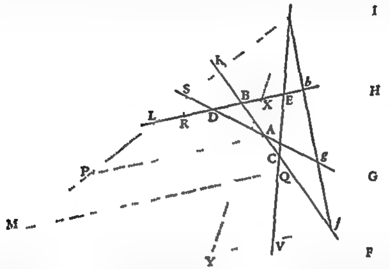
SCHOLIUM

This problem may be likewise constructed in the following manner Joining ... CF to ... and join FH IG and make the angles



Produce AB to K and BD to L ... DL to BD as GI to FG and iL to M so as LM may be to ... and meeting the right line AD in g and join gi cutting AB BD in f, h I say the thing is done

For let Mg cut the right line AB in Q and AD the right line KL in S and draw AP parallel to BD and meeting iL in P and gM to Lh (gi to hi Mi to Li GI to HI, AK to BK) and AP to BL will be in the same ratio Cut DL in R so



as DL to RL may be in that same ratio and because gS to gM AS to AP and DS to DL are proportional therefore as gS to Lh so will AS be to BL and DS ...

to FG Therefore fh is to fg as IH to FG Since therefore gi to hi likewise is as Mi to Li that is as GI to HI it is manifest that the lines FI fi are similarly cut in G and H g and h Q E F

In the construction of this Corollary after the line LK is drawn cutting CE in i we may produce iE to V so as EV may be to Ei as FH to HI and then draw Vf parallel to BD It will come to the same if about the centre i with an interval IH we describe a circle cutting BD in X and produce iX to Y so as iY may be equal to IF and then draw Yf parallel to BD

Sir Christopher Wren and Dr Wallis have long ago given other solutions of this Problem

PROPOSITION 29 PROBLEM 21

To describe a conic given in kind that may be cut by four right lines given in position into parts given in order kind and proportion

...

BD and CE given in position viz the first between the first pair of those lines

line a movable point going out from the pole moves always forwards with a velocity proportional to the square of that right line within the oval By this motion that point will describe a spiral with infinite circumgyrations Now if a portion of the area of the oval cut off by that right line could be found by a finite equation the distance of the point from the pole which is proportional to this area, might be found by the same equation and therefore all the points of the spiral might be found by a finite equation also and therefore the intersection of a right line given in position with the spiral might also be found by a

cuts a spiral in an

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intersection may be also found. Because there may be tions of two

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may amount to six, come out together by equations of six dimensions and the intersections of two curves of the third order because they may amount to nine come out together by equations of nine dimensions If this did not necessarily happen we might reduce all solid to plane Problems, and those higher than solid to solid Problems But here I speak of curves irreducible in power For if the equation by which the curve is defined may be reduced to a lower power the curve will not be one single curve but composed of two or more whose intersections may be severally found by different calculi After the same manner the two intersections of right lines with the conic sections come out

the intersecting line revolves about the pole the intersections of the spiral will mutually pass the one into the other and that which was first or nearest after one revolution will be the second after two the third and so on nor will the equation in the meantime be changed but as the magnitudes of those quantities are changed by which the position of the intersecting line is determined Therefore since those quantities after every revolution return to their first magnitudes the equation will return to its first form and consequently one and the same equation will exhibit all the intersections, and will therefore have an infinite number of roots, by which they may be all exhibited Therefore the intersection of a right line with a spiral cannot be universally found by any

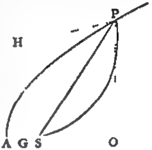
SECTION VI

HOW THE MOTIONS ARE TO BE FOUND IN GIVEN ORBITS

PROPOSITION 30 PROBLEM 22

To find at any assigned time the place of a body moving in a given parabola

Let S be the focus and A the principal vertex of the parabola and suppose 4AS M equal to the parabolic area to be cut off APS which either was described by the radius SP since the body's departure from the vertex or is to be described thereby before its arrival there Now the quantity of that area to be cut off is known from the time which is proportional to it Bisect AS in G and erect the perpendicular GH equal to 3M and a circle described about the centre H with the radius HS will cut the parabola in the place P required For letting fall PO perpendicular on the axis and drawing PH there will be



$$AG^2+GH^2(=HP^2=(AO-AG)^2+(PO-GH)^2)$$
$$=AO^2+PO^2-2AO\ AG-2GH\ PO+AG^2+GH^2$$

Whence

$$2GH\ PO(=AO^2+PO^2-2AO\ AG)=AO^2+\tfrac{3}{4}PO^2$$ For AO^2 write $AO\ \frac{PO^2}{4AS}$ then dividing all the terms by 3PO and multiplying them by 2AS we shall have

$$\tfrac{4}{3}GH\ AS(=\tfrac{1}{6}AO\ PO+\tfrac{1}{2}AS\ PO=\frac{AO+3AS}{6}\ PO=\frac{4AO-3SO}{6}\ PO=\text{to the}$$ area $APO-SPO$) = to the area APS But GH was 3M and therefore

$$\tfrac{4}{3}GH\ AS \text{ is } 4AS\ M$$

Therefore the area cut off APS is equal to the area that was to be cut off 4AS M Q E D

COR I Hence GH is to AS as the time in which the body described the arc AP to the time in which the body described the arc between the vertex A and the perpendicular erected from the focus S upon the axis

COR II And supposing a circle ASP continually to pass through the moving body P the velocity of the point H is to the velocity which the body had in the vertex A as 3 to 8 and therefore in the same ratio is the line GH to the right line which the body in the time of its moving from A to P would describe with that velocity which it had in the vertex A

COR III Hence also on the other hand the time may be found in which the body has described any assigned arc AP Join AP and on its middle point erect a perpendicular meeting the right line GH in H

LEMMA 28

There is no oval figure whose area cut off by right lines at pleasure can be universally found by means of equations of any number of finite terms and dimensions

Suppose that within the oval any point is given about which as a pole a right line is continually revolving with an uniform motion while in that right

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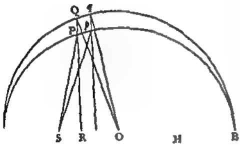


Secondly a certain length L which may be to the radius in the same ratio inversely And these being found the Problem may be solved by the following analysis. By any construction (or even by conjecture) suppose we know P the place of the body near its true place p Then letting fall on the axis of the ellipse the ordinate PR from the

we also know the angle proportional to the time that is which

t me of one

ch may

angle E

which may be to the angle $N-AOQ+D$ as the length L to the same length L diminished by the cosine of the angle AOQ when that angle is less than a right

the ordinate pr which is to its sine qr as the lesser axis of the ellipse to the greater we shall have p the correct place of the body When the angle $N-$

finite equation and hence there is no oval figure whose area cut off by right lines at pleasure can be universally exhibited by any such equation

By the same argument if the interval of the pole and point by which the spiral is described is taken proportional to that part of the perim

ed by conjugate figures running out *in infinitum*

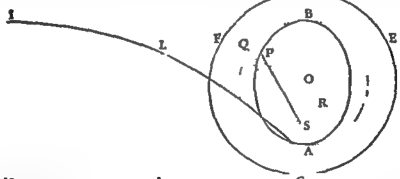
COR Hence the area of an ellipse described by a radius drawn from the focus to the moving body is not to be found from the time given by a finite equation and therefore cannot be determined by the description of curves geometrically rational Those curves I call geometrically rational all the points whereof may be determined by lengths that are definable by equations that

in the following manner

PROPOSITION 31 PROBLEM 23

To find the place of a body moving in a given ellipse at any assigned time

Suppose A to be the principal vertex S the focus and O the centre of the ellipse APB and let P be the place of the body to be found Produce OA to G so that OG OA = OA OS Erect the perpendicular GH and about the centre



O with the radius OG describe the circle GEF and on the ruler GH as a base suppose the wh GEF
meantime by
perimeter GE
whole revolution in the
id in L then LP drawn
place of the body

AFS varies as the area AQS that is as the difference between the sector OQA and the triangle OQS or as the difference of the rectangles ½OQ AQ and

$$CO : BO = BO : TO$$

thence $$CO + BO : BO = BO + TO : TO$$

and $$CO : BO = CB : BT$$

From this, $$BO - CO : BO = BT - CB : BT$$

and $$AC : AO = TC : BT = CP : BQ$$

and since $$CP = \frac{BQ \cdot AC}{AO}$$

one obtains $$\frac{CP^2 \cdot AO \cdot SP}{AC \cdot CB} \text{ equal to } \frac{BQ^2 \cdot AC \cdot SP}{AO \cdot BC}$$

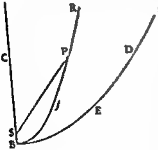
Now suppose CP the breadth of the figure RPB to be diminished *in infinitum* so that the point P may come to coincide with the point C and the point S with the point B and the line SP with the line BC and the line SY with the line BQ and the velocity of the body now descending perpendicularly in the line CB will be to the velocity of a body describing a circle about the centre B at the distance BC as the square root of the ratio of $\frac{BQ^2 \cdot AC \cdot SP}{AO \cdot BC}$ to SY^2 that is (neglecting the ratios of equality of SP to BC and BQ^2 to SY^2) as the square root of the ratio of AC to AO or ½AB. Q.E.D.

COR. I When the points B and S come to coincide TC will become to TS as AC to AO.

COR. II A body revolving in any circle at a given distance from the centre by its motion converted upwards will ascend to double its distance from the centre.

PROPOSITION 34 THEOREM 10

If the figure BED is a parabola I say that the velocity of a falling body in any place C is equal to the velocity by which a body may uniformly describe a circle about the centre B at half the interval BC.



For (by Cor. VII Prop. 16) the velocity of a body describing a parabola RPB about the centre S in any place P is equal to the velocity of a body uniformly describing a circle about the same centre S at half the interval SP. Let the breadth CP of the parabola be diminished *in infinitum* so that the parabolic arc PfB may come to coincide with the right line CB the centre S with the vertex B and the interval SP with the interval BC and the Proposition will be manifest. Q.E.D.

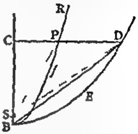
PROPOSITION 35 THEOREM 11

... figure DES described by the ...

... ing about the centre S

For suppose a body C in the smallest moment of time describes in falling the infinitely little line Cc while another body K, uniformly revolving about the centre S in the circle OKk describes the arc Kk. Erect the perpendiculars CD, cd meeting the figure DES in D, d. Join SD, Sd, SK, Sk and draw Dd meeting the axis AS in T and thereon let fall the perpendicular SY.

CSP CSD = CB*f*P CBED = SP*f*B SDEB = CP CD and since the area SP*f*B varies as the time in which the body P will move through the arc P*f*B the area SDEB will also vary as that time Let the latus rectum of the hyperbola RPB be diminished *in infinitum* the transverse axis remaining the same and the arc PB will come to coincide with the right line CB and the focus S with the vertex B and the right line SD with the right line BD And therefore the area BDEB will vary as the time in which the body C by its perpendicular descent describes the line CB Q E I

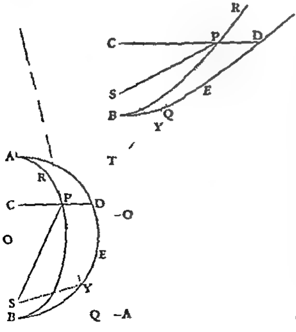


CASE 3 And by the like argument if the figure RPB is a parabola and to the same principal vertex B another parabola BED is described that may always remain given while the former parabola in whose perimeter the body P moves by having its lat … diminished … ith the line CB the parabolic seg … that body P or C will descend to … Q E I

PROPOSITION 33 THEOREM 9

The things above found being supposed I say that the velocity of a falling body in any place C *is to the velocity of a body describing a circle about the centre* B *at the distance* BC *as the square root of the ratio of* AC *the distance of the body from the remoter vertex* A *of the circle or rectangular hyperbola to* ½AB *the principal semi diameter of the figure*

Let AB the common diameter of both figures RPB DEB be bisected in O and draw the right line PT that may touch the fi… RPB … P …

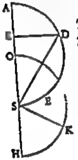


to be L From Cor IX Prop 16 it is manifest that the velocity of a body moving in the line RPB about the centre S in any place P is to the velocity of a body describing a circle about the same centre at the distance SP as the square root of the ratio of the rectangle ½L SP to SY^2 For by the properties of the conic sections AC CB is to CP^2 as 2AO to L and therefore $\frac{2CP^2 \, AO}{AC \, CB}$ is equal to L Therefore those velocities are to each other as the square root of the ratio of $\frac{CP^2 \, AO \, SP}{AC \, CB}$ to SY^2 Moreover by the properties of the conic sections

equal to the velocity with which a circle may be uniformly described at the ... And this velocity to the velocity with which a ... to the arc to ¼SC ¼CD Cc

that is, in the ratio ... and therefore equal to ½SY Dd that is the area KSK is equal to the area SDd as above Q E D

PROPOSITION 36 PROBLEM 25

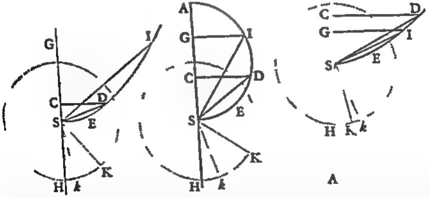


To determine the times of the descent of a body falling from a given place A

Upon the diameter AS the distance of the body from the centre at the beginning describe the semicircle ADS as likewise the semicircle OKH equal thereto about the centre S ... of the body erect the ordinate CD Join SD

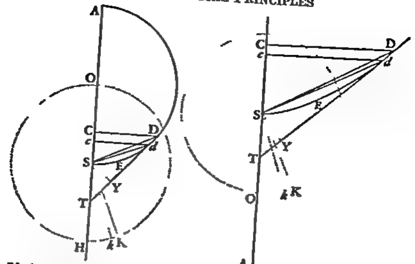
PROPOSITION 3 PROBLEM 26

To define the times of the ascent or descent of a body projected upwards or downwards from a given place



Suppose the body to go off from the given place G in the direction of the line GS with any velocity Take GA to ½AS as the square of the ratio of this velocity to the uniform velocity in a circle with which the body may revolve about the centre S at the given interval SG If that ratio is the same as of the ...

diameter SA as appears by Prop 33 Then about the centre S with a radius equal to half the latus rectum describe the circle HkK and at the place G of the ascending or descending body and at any other place C erect the perpen



CASE 1 If the figure DES is a circle or a rectangular hyperbola bisect its transverse diameter AS in O and SO will be half the latus rectum And becau e

$$TC \quad TD = Cc \quad Dd$$

and

$$TD \quad TS = CD \quad Sy$$

there follows

$$TC \quad TS = CD \quad Cc \quad SY \quad Dd$$

But (by Cor I Prop 33) $\quad TC \quad TS = AC \quad AO$

namely if in the coalescence of the points D d the ultimate ratios of the lines are taken Therefore

$$AC \quad AO \text{ or } SK = CD \quad Cc \quad SY \quad Dd$$

Further the velocity of the descending bo[illegible] n C

(by Cor [illegible]

the little l[illegible]

is in the r[illegible]

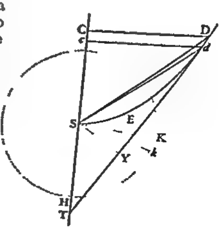
hence $\quad CD \quad Cc = AC \quad Kk$

and $\quad AC \quad SK = AC \quad Kk \quad SY \quad Dd$

and $\quad SK \quad Kk = SY \quad Dd$

and $\quad \frac{1}{2}SK \quad Kk = \frac{1}{2}SY \quad Dd$

that is the area KSk is equal to the area SDd Therefore in every moment of time two equal particles KSk and SDd of areas are generated which if their magnitude is diminished and their number increased *in infinitum* obtain the ratio of equality and consequently (by Cor Lem IV) the whole areas together generated are always equal Q E D



CASE 2 But if the figure DES is a parabola we shall find as above

$$CD \quad Cc \quad SY \quad Dd = TC \quad TS$$

that is $= 2 \quad 1$ therefore

$$\tfrac{1}{4}CD \quad Cc = \tfrac{1}{2} \, SY \quad Dd$$

But the velocity of the falling body in C is

DLF be the place of the line EMG when the body was in D and if the centripetal force be such, that a right line whose square is equal to the area ABGE, is as the velocity of the descending body the area itself will be as the square of that velocity that is if for the velocities in D and E we write V and V+I the area ABFD will be as VV and the area ABGE as VV+2VI+II and by subtraction the area DFGE as 2VI+II and therefore $\frac{DFGE}{DE}$ will be as $\frac{2VI+II}{DE}$ that is, if we take the first ratios of those quantities when just nascent the length DF is as the quantity $\frac{2VI}{DE}$ and therefore also as half that quantity $\frac{IV}{DE}$ But the time in which the body in falling describes the very small line DE, is directly as that line and inversely as the velocity V and the force will be directly as the increment I of the velocity and inversely as the time and therefore if we take the first ratios when those quantities are just nascent as $\frac{IV}{DE}$ that is as the length DF Therefore a force proportional to DF or EG will cause the body to descend with a velocity that is as the right line whose square is equal to the area ABGE Q E D

Moreover since the time in which a very small line DE of a given length may be described is inversely as the velocity and therefore also inversely as a right line whose square is equal to the area ABFD and since the line DL and by consequence the nascent area DLME will be inversely as the same right line the time will be as the area DLME and the sum of all the times will be as the sum of all the areas that is (by Cor Lem 4) the whole time in which the line AE is described will be as the whole area ATVME. Q E D

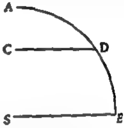
COR. I Let P be the place from whence a body ought to fall so as that when urged by any known uniform centripetal force (such as gravity is commonly ... in the place D a velocity equal to the velocity ... d in that place ... be to DF as ... the rectangle PDPQ and cut off the area ABFD equal to that rectangle ... then A will be the ... DRSE ... be ... he ... he ... he by falling by the uniform force and since those increments (by reason of the equality of the nascent times) are as the generating forces, that is as the ordinates DF DR and consequently as the nascent areas DFGE DRSE therefore the whole areas ABFD PQRD will be to each other as the halves of the whole velocities and therefore because the velocities are equal they become equal also

COR II Whence if any body be projected either upwards or downwards with a given velocity from any place D and there be given the law of centripetal force acting on it its velocity will be found in any other place as e by erecting the ordinate eg and taking that velocity to the velocity in the place D as a right line whose square is equal to the rectangle PQRD either increased by the

– I –d D Then joining SEIS SEDS same time in which the body K may describe the arc KL Q E F

PROPOSITION 38 THEOREM 12

Supposing that the centripetal force is proportional to the altitude or distance of places from the centre I say that the times and velocities of falling bodies and the spaces which they describe are respectively proportional to the arcs and the sines and versed sines of the arcs



Suppose the body to fall from any place A in the right line AS and about the centre of force S with the radius AS, describe the quadrant of a circle AE and let CD be the sine of any arc AD and the body A will in the time AD in falling describe the space AC and in the place C will acquire the velocity CD

This is demonstrated the same way from Prop 10 as Prop 32 was demonstrated from Prop 11

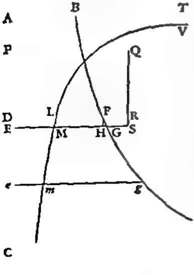
COR I Hence the times are equal in which one body falling from the place A arrives at the centre S and another body revolving describes the quadrantal arc ADE

COR II *Therefore all the times are equal in which bodies falling from what*soever places arrive at the centre For all the periodic times of revolving bodies are equal (by Cor III Prop 4)

PROPOSITION 39 PROBLEM 27

Supposing a centripetal force of any kind and granting the quadratures of curvilinear figures it is required to find the velocity of a body ascending or descending in a right line in the several places through which it passes as also the time in which it will arrive at any place and conversely

Suppose the body E to fall from any place A in the right line ADEC and from its place E imagine a perpendicular EG always erected proportional to the centripetal force in that place tending to the centre C and let BFG be a



curved line the locus of the point G And in the beginning of the motion suppose EG to coincide with the perpendicular AB and the velocity of the body in any place E will be as a right line whose square is equal to the curvilinear area ABGE Q E I

In EG take EM inversely proportional to a right line whose square is equal to the area ABGE and let VLM be a curved line wherein the point M is always placed and to which the right line AB produced is an asymptote and the time in which the body in falling describes the line AE will be as the curvilinear area ABTVME Q E I

For in the right line AE let there be taken the very small line DE of a given length and let

DLF be the place of the line EMG when the body was in D and if the centripetal force be such, that a right line whose square is equal to the area ABGE, is as the velocity of the descending body the area itself will be as the square of that velocity that is if for the velocities in D and E we write V and V+I the area ABFD will be as VV and the area ABGE as VV+2VI+II and by subtraction, the area DFGE as 2VI+II and therefore $\frac{DFGE}{DE}$ will be as $\frac{2VI+II}{DE}$ that is if we take the first ratios of those quantities when just nascent the length DF is as the quantity $\frac{2VI}{DE}$ and therefore also as half that quantity $\frac{I \times V}{DE}$. But the time in which the body in falling describes the

nascent as $\frac{I}{DE}$ that is as the length DF Therefore a force proportional to DF or EG will cause the body to descend with a velocity that is as the right line whose square is equal to the area ABGE. Q.E.D.

Moreover ...

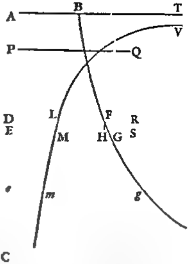
may be d...

right line ...

by consequence the nascent area DLME, will be inversely as the same right line the time will be as the area DLME and the sum of all the times will be as the sum of all the areas that is (by Cor. Lem. 4) the whole time in which the line AE is described will be as the whole area ATVME. Q.E.D.

COR. I Let P be the place from whence a body ought to fall so as that when urged by any known uniform centripetal force (such as gravity is commonly supposed to be) it may acquire in the place D a velocity equal to the velocity which the other body falling by any force whatever hath acquired in that place D In the perpendicular DF let there be taken DR, which may be to DF as that uniform force to the other force in the place D Complete the rectangle PDRQ and cut off the area ABFD equal to that rectangle Then A will be the place from whence the other body fell For completing the rectangle DRSE, since the area ABFD is to the area DFGE as VV to 2VI and therefore as ½V to I that is as half the whole velocity to the increment of the velocity of the body falling by the variable force and in like manner the area PQRD to the area DRSE as half the whole velocity to the increment of the velocity of the body falling by the uniform force and since those increments (by reason of the equality of the nascent times) are as the generating forces, that is as the ordinates DF DR and consequently as the nascent areas DFGE, DRSE therefore the whole areas ABFD PQRD will be to each other as the halves of the whole velocities and therefore because the velocities are equal, they become equal also.

COR. II. Whence if any body be projected either upwards or downwards with a given velocity from any place D and there be given the law of centripetal force acting on it its velocity will be found in any other place as e by erecting the ordinate eg and taking that velocity to the velocity in the place D as a right line whose square is equal to the rectangle PQRD either increased by the



curvilinear area DFge if the place e is below the place D or diminished by the same area DFge if it be higher is to the right line whose square is equal to the rectangle PQRD alone

COR III The time is also known by erecting the ordinate em inversely proportional to the square root of PQRD+or−DFge and taking the time in which the body has described the line De to the time in which another body has fallen with an uniform force from P and in falling arrived at D in the proportion of the curvilinear area DLme to the rectangle 2PD DL For the time in which a body falling with an uniform force hath described the line PD is to the time in which the same body hath described the line PE as the square root of the ratio of PD to PE that is (the very small line DE being just nascent) in the ratio of PD to PD+½DE or 2PD to 2PD+DE and by subtraction to the time in which the body hath described the small line DE as 2PD to DE and therefore as the rectangle 2PD DL to the area DLME and the time in which both the bodies described the very small line DE is to the time in which the body with the variable motion described the line De as the area DLME to the area DLme and therefore the first mentioned of these times is to the last as the rectangle 2PD DL to the area DLme

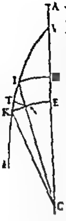
SECTION VIII

THE DETERMINATION OF ORBITS IN WHICH BODIES WILL REVOLVE BEING ACTED UPON BY ANY SORT OF CENTRIPETAL FORCE

PROPOSITION 40 THEOREM 13

If a body acted upon by any centripetal force is moved in any manner and another body ascends or descends in a right line and their velocities be equal in any one case of equal altitudes their velocities will be also equal at all equal altitudes

Let a body descend from A through D and E to the centre C and let another body move from V in the curved line VIKk From the centre C with any distances describe the concentric circles DI EK meeting the right line AC in D and E and the curve VIK in I and K Draw IC meeting KE in N and on IK let fall the perpendicular NT and let the interval DE or IN between the circumferences of the circles be very small and imagine the bodies in D and I to have equal velocities Then because the distances CD and CI are equal the centripetal forces in D and I will be also equal Let those forces be expressed by the equal short lines DE and IN and let the force IN (by Cor II of the Laws of Motion) be resolved into two others NT and IT Then the force NT acting in the direction of the line NT perpendicular to the path ITK of the body will not at all affect or change the velocity of the body in that path but only draw it aside from a rectilinear course and make it deflect continually from the tangent of the orbit and proceed in the curvilinear path ITKk That



whose force therefore will be spent in producing that effect but the other force IT acting in the direction of the course of the body will be all employed in accelerating it and in the least given time will produce an acceleration proportional to itself Therefore the accelerations of the bodies in D and I produced in equal times are as the lines DE IT (if we take the first ratios of the nascent lines DE IN II IT NT) and in unequal times as the product of those lines and the times But the times in which DE and Ik are described are by reason of the equal velocities (in D and I) as the spaces described DE and Ik and therefore the accelerations in the course of the bodies through the lines DE ar DE nd IT and DE and Ik conjointly that is, a I

of the bodies from D and I to E and k are equal the bodies in E and k are also equal and by the same reasoning they will always be found equal in any subsequent equal distances. Q E D

By the same reasoning bodies of equal velocities and equal distances from the centre will be equally retarded in their ascent to equal distances. Q E D

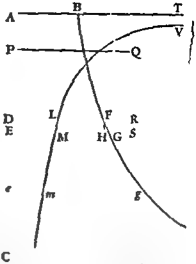
COR. I Therefore if a body either oscillates by hanging to a string or by any polished and perfectly smooth impediment is forced to move in a curved line and another body ascends or descends in a right line and their velocities be n on equal altitude their velocities will be also equal at all other

ohliged to leave its rectilinear course

COR II Suppose the quantity P to be the greatest distance from the centre to which a body can ascend whether it be oscillating or revolving in a curve lso the same projected upwards from any point of a curve with the velocity it has in that point Let the quantity A be the distance of the body from the centre in any other point of the orbit and let the centripetal force be always as the power A^{n-1} of the quantity A the index of which power $n-1$ is any number n diminished by unit Then the velocity in every altitude A will be a $(P^n - A^n)$ and therefore will be given For by Prop 39 the velocity of a body ascending and descending in a right line is in that very ratio.

PROPOSITION 41 PROBLEM 28

Let any centripetal force tend to the centre C and let it be required to find the curve VIkK Let there be given the circle VR described from the centre C



Cor. III. The time is also known by erecting the ordinate em inversely proportional to the square root of PQRD+or−DFge, and taking the time in which the body has described the line De to the time in which another body has fallen with an uniform force from P, and in falling arrived at D in the proportion of the curvilinear area DLme to the rectangle 2PD·DL. For the time in which a body falling with an uniform force hath described the line PD is to the time in which the same body hath described the line PE as the square root of the ratio of PD to PE; that is (the very small line DE being just nascent), in the ratio of PD to PD+½DE or 2PD to 2PD+DE; and, by subtraction, to the time in which the body hath described the small line DE as 2PD to DE, and therefore as the rectangle 2PD·DL to the area DLME; and the time in which both the bodies described the very small line DE is to the time in which the body with the variable motion described the line De as the area DLME to the area DLme; and therefore the first mentioned of these times is to the last as the rectangle 2PD·DL to the area DLme.

SECTION VIII

The determination of orbits in which bodies will revolve, being acted upon by any sort of centripetal force

Proposition 40. Theorem 13

If a body, acted upon by any centripetal force, is moved in any manner, and another body ascends or descends in a right line, and their velocities be equal in any one case of equal altitudes, their velocities will be also equal at all equal altitudes.

Let a body descend from A through D and E, to the centre C; and let another body move from V in the curved line VIt ... C

... force IN (by Cor. II of the ... and IT. Then the force NT ... ular to the path ITK of the body will not at all affect or change the velocity of the body in that path, but only draw it aside from a rectilinear course, and make it deflect continually from the tangent of the orbit, and proceed in the curvilinear path ITKk. That

VD a VCX are always equal therefore the generated area

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For the apsides are those points

centre falls perpendicularly upon

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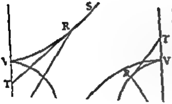
e point T and then



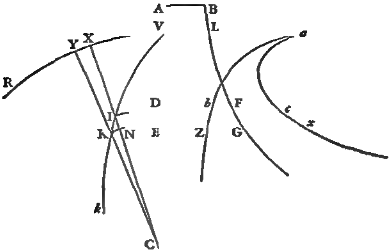
joining C n the right line CP equal to the abscissa CT making an angle VCP proportional to the sector VCR and if a centripetal force inversely proportional to the cubes of the distances of the places from the centre tend to the centre C and from the place V there sets out a body with a just velocity in the

to th right line

nd the length CP

from the foregoing Proposition by the quadrature of a certain curve the invention of which as being easy enough for brevity s sake I omit

PROPOSITION 42 PROBLEM 29



be the place from which another body is to fall so as in the place D to acquire a velocity equal to the velocity of the first body in I And things remaining as in Prop 39 the short line IK described in the least given time will be as the velocity and therefore as the right line whose square is equal to the area ABFD and the triangle ICK proportional to the time will be given and therefore KN will be inversely as the altitude IC that is (if there be given any quantity Q and the altitude IC be called A) as $\frac{Q}{A}$ This quantity $\frac{Q}{A}$ call Z and suppose the magnitude of Q to be such that in some one case

$$\sqrt{ABFD} \quad Z = IK \quad KN$$

and then in all cases

$$\sqrt{ABFD} \quad Z = IK \quad KN$$

and

$$ABFD \quad ZZ = IK^2 \quad KN^2$$

and by subtraction

$$ABFD - ZZ \quad ZZ = IN^2 \quad KN$$

and therefore

$$\sqrt{(ABFD - ZZ)} \quad Z \text{ or } \frac{Q}{A} = IN \quad KN$$

and

$$A \quad KN = \frac{Q \quad IN}{\sqrt{(ABFD - ZZ)}}$$

Since

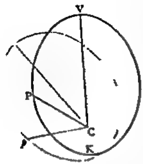
$$YX \quad XC \quad A \quad KN = CX \quad AA$$

it follows that

$$YX \quad XC = \frac{Q \quad IN \quad CX^2}{AA\sqrt{(ABFD - ZZ)}}$$

Therefore in the perpendicular DF let there be taken continually *Db* *Dc* equal to $\frac{Q}{2\sqrt{(ABFD-ZZ)}}$ $\frac{Q \quad CX}{2AA\sqrt{(ABFD-ZZ)}}$ respectively and let the curved lines *ab* *ac* the foci of the points *b* and *c* be described and from the point V let the perpendicular V*a* be erected to the line AC cutting off the curvilinear areas VD*ba* VD*ca* and let the ordinates E E*x* be erected also Then because the rectangle D*b* IN or D*b* E is equal to half the rectangle A KN or to the triangle ICK and the rectangle D*c* IN or D*cx*E is equal to half the rectangle YX XC or to the triangle XCY that is because the nascent particles D*b* E ICK of the areas VD*ba* VIC are always equal and the nascent particles D*cx*E

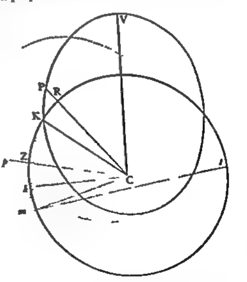
... a given ratio and ...ribed by the line ...hat a body being ...tal force may revolve with the point p in the curved line which the same point p by the method just now explained may be made to describe in a fixed plane Make the angle VCu equal to the angle PCp and the line Cu equal to CV and the figure uCp equal to the figure VCP and the body being always in the point p will move in the perimeter of the revolving figure uCp and will describe its (revolving) arc up in the same time that the other body P describes the similar and equal arc VP in the fixed figure VPK Find then by Cor v Prop 6 the centripetal force by which the body may be made to revolve in the curved line which the point p describes in a fixed plane and the Problem will be solved Q E F



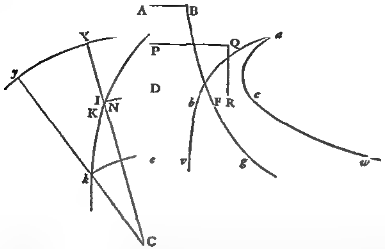
PROPOSITION 44 THEOREM 14

The difference of the forces by which two bodies may be made to move equally one in a fixed the other in the same orbit revolving varies inversely as the cube of their common altitudes

Let the parts of the fixed orbit VP PK be similar and equal to the parts of the revolving orbit up pk and let the distance of the points P and K be supposed of the utmost smallness. Let fall a perpendicular kr from the point k to the right line pC and produce it to m so that mr may be to kr as the angle VCp to the angle VCP Because the altitudes of the bodies PC and pC KC and kC are always equal it is manifest that the increments or decrements of the lines PC and pC are always equal and therefore if each of the several motions of the bodies in the places P and p be resolved into two (by Cor II of the Laws of Motion) one of which is directed towards the centre or according to the lines PC pC and the other transverse to the former hath a direction perpendicular to the lines PC and pC the motions towards the centre will be equal and the transverse motion of the body p will be to the transverse motion of the body P as the angular motion of the line pC to the angular motion of the line PC that is as the angle VCp to the angle VCP Therefore at the same time that the body P by both its motions comes to the point K, the body p having an equal motion towards the centre will be equally moved



velocity as anothe body h f ll ng th n n fo m n n t
place P may
the body is at



about the centre C with the radius Ck describe the circle *ke* meeting the right line PD in *e* and let there be erected the lines *eg* *ev* *ew* ordinately applied to the curves BΓ*g* *abv* *acw* From the given rectangle PDRQ and the given law of centripetal force by which the first body is acted on the curved line BΓ*g* is also given by the construction of Prop 27 and its Cor I Then from the given angle CIK is given the proportion of the nascent lines IK KN and thence by the construction of Prob 28 there is given the quantity Q with the curved lines *abv* *acw* and therefore at the end of any time D*bve* there is given both the altitude of the body C*e* or C*k* and the area D*cue* with the sector equal to it XC*y* the angle IC*k* and the place *k* in which the body will then be found

Q E I

We suppose in these Propositions the centripetal force to vary in its recess from the centre according to some law which anyone may imagine at pleasure but at equal distances from the centre to be everywhere the same

I have hitherto considered the motions of bodies in immovable orbits It remains now to add something concerning their motions in orbits which revolve round the centres of force

SECTION IX

THE MOTION OF BODIES IN MOVABLE ORBITS AND THE MOTION OF THE APSIDES

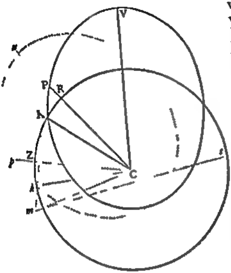
PROPOSITION 43 PROBLEM 30

It is required to make a body move in a curve that revolves about the centre of force in the same manner as another body in the same curve at rest

In the fixed orbit VPK let the body P revolve proceeding from V towards

as the velocity of the describing line C*p* to the velocity of the describing line

ratio of G to ... and



rectum ... which a body may be made to revolve in a movable ellipse will be as $\frac{FF}{AA} + \frac{RGG-RFF}{A}$ and conversely Let the force with which a body may revolve in a fixed ellipse be expressed by the quantity $\frac{FF}{AA}$ and the force in V will be $\frac{FF}{CV^2}$ But the force with which a body may revolve in a circle at the distance CV with the same velocity as a body revolving in an ellipse has in V is to the force with which a body revolving in an ellipse is acted upon in the apse V as half the latus rectum of the ellipse to the semidiameter CV of the circle and therefore is as $\frac{RFF}{CV}$ and the force which is to this as GG−FF to FF is as $\frac{RGG-RFF}{CV}$ and this force (by Cor 1 of this Prop) is the difference of the forces in V with which the body P revolves in the fixed ellipse VPK and the body p in the movable ellipse upk Then since by this Proposition that difference at any other altitude A is to itself at the altitude CV as $\frac{1}{A}$ to $\frac{1}{CV^2}$ the same difference in every altitude A will be as $\frac{RGG-RFF}{A}$ Therefore to the force $\frac{FF}{AA}$ by which the body may revolve in a fixed ellipse VPK add the excess $\frac{RGG-RFF}{A}$ and the sum will be the whole force $\frac{FF}{AA}+\frac{RGG-RFF}{A}$ by which ... h m ... llipse upk

VPK be upposed the prin

COR IV And universally if the greatest altitude CV of the body be called T and the radius of the curvature which the orbit VPK has in V that is the

from p towards C and therefore that time being expired it will be found somewhere in the line mkr which passing through the point k is perpendicular to the line pC and by its transverse motion will acquire a distance from the line pC that will be to the distance which the other body P acquires from the line PC as the transverse motion of the body p to the transverse motion of the other body P Therefore since kr is equal to the distance which the body P acquires from the line PC and mr is to kr as the angle VCp to the angle VCP that is as the transverse motion of the body p to the transverse motion of the body P it is manifest that the body p at the expiration of that time will be found in the place m These things will be so if the bodies p and P are equally moved in the directions of the lines pC and PC and are therefore urged with equal forces in those directions But if we take an angle pCn that is to the angle pCk as the angle VCp to the angle VCP and nC be equal to kC in that case the body p at the expiration of the time will really be in n and is therefore urged with a greater force than the bod P f C r
than the

t

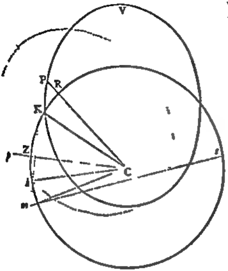
i

c
o
d

i ... the rectangle mk ms and therefore mn will be equal to $\frac{mk\ ms}{mt}$ But since the triangles pCk pCn in a given time are of a given magnitude kr and mr and their difference mk and their sum ms are inversely as the altitude pC and therefore the rectangle mk ms is inversely as the square of the altitude pC Moreover mt is directly as $\frac{1}{2}mt$ that is as the altitude pC These are the first ratios of the nascent lines and hence $\frac{mk\ ms}{mt}$ that is the nascent short line mn and the difference of the forces proportional thereto are inversely as the cube of the altitude pC Q E D

COR I Hence the difference of the forces in the places P and p or K and k is to the force with which a body may revolve with a circular motion from R to K in the same time that the body P in a fixed orbit describes the arc PK as the nascent line mn to the versed sine of the nascent arc RK that is as $\frac{mk\ ms}{mt}$ to $\frac{rk^2}{2kC}$ or as mk ms to the square of rk that is if we take given quantities F and G in the same ratio to each other as the angle VCP bears to the angle VCp as GG − FF to FF And therefore if from the centre C with any distance CP or Cp there be described a circular sector equal to the whole area VPC which the body revolving in a fixed orbit hath by a radius drawn to the centre described in any certain time the difference of the forces with which the body P revolves in a fixed orbit and the body p in a movable orbit will be to the centripetal force with which another body by a radius drawn to the centre can ... he same time as the area VPC is described and the area pCk are to each other as the

COR II If the orbit VPK be an ellipse having its focus C and its highest ... and equal to it so that pC may to the angle VCP in the given we put A, and 2R for the latus um of the ellipse the force with which a body may be made to revolve in a movable ellipse will be as $\frac{FF}{AA}+\frac{RGG-RFF}{A}$ and conversely Let the force with which a body may revolve in a fixed ellipse be expressed by the quantity $\frac{FF}{AA}$ and the force in V will be $\frac{FF}{CV^2}$. But the force with which a body may revolve in a circle at the distance CV with the same velocity as a body revolving in an ellipse has in V is to the force with which a body re... nd ... ...



fore is as $\frac{RFF}{CV}$ and the force which is to this as GG−FF to FF is as $\frac{RGG-RFF}{CV}$ and this force (by Cor I of this Prop) is the difference of the forces in V with which the body P revolves in the fixed ellipse VPK, and the body p in the movable ellipse *upk* Then since by this Proposition that difference at any other altitude A is to itself at the altitude CV as $\frac{1}{A^3}$ to $\frac{1}{CV^3}$ the same difference in every altitude A will be as $\frac{RGG-RFF}{A}$ Therefore to the force $\frac{FF}{AA}$ by which the body may revolve in a fixed ellipse VPK add the excess $\frac{RGG-RFF}{A}$ and the sum will be the whole force $\frac{FF}{AA}+\frac{RGG-RFF}{A}$ by which a body may revolve in the same time in the movable ellipse *upk*

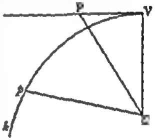
COR. III. In the same manner it will be found that if the fixed orbit VPK be an ellipse having its centre in the centre of the forces C and there be supposed a movable ellipse *upk* similar equal and concentric to it and 2R be the principal latus rectum of that ellipse and 2T the latus transversum or greater axis and the angle VCp be continually to the angle VCP as G to F the forces with which bodies may revolve in the fixed and movable ellipse in equal times will be as $\frac{FFA}{T^3}$ and $\frac{FFA}{T}+\frac{RGG-RFF}{A}$ respectively

COR. IV And universally if the greatest altitude CV of the body be called T and the radius of the curvature which the orbit VPK has in V that is the

radius of a circle equally curved be called R and the centripetal force with which a body may revolve in any fixed curve VPK at the place V be called $\frac{VFF}{TT}$ and in other places P be indefinitely styled X and the altitude CP be called A and G be taken to F in the given ratio of the angle VCp to the angle VCP the centripetal force with which the same body will perform the same motions in the same time in the same curve upk revolving with a circular motion will be as the sum of the forces $X+\frac{VRGG-VRFF}{A^3}$

COR V Therefore the m
lar motion round the c
given ratio and thence
revolv with new centr

COR VI Therefore if there be erected the line VP of an indeterminate length perpendicular to the line CV given by position and CP be drawn and Cp equal to it making the angle VCp having a given ratio to the angle VCP the force with which a body may revolve in the curved line Vpk which the point p is continually describing will be inversely as the cube of the altitude Cp For the body P by its inertia alone no other force impelling it will proceed uniformly in the right line VP Add then a force tending to the centre C inversely as the cube of the altitude CP or Cp and (by what was just demonstrated) the body will deflect from the rectilinear motion into the Vpk But this curve Vpk is the 41 in which I said bodi



PROPOSITION 45 PROBLEM 31

To find the motion of the apsides in orbits approaching very near to circles

This problem is solved arithmetically by reducing the orbit which a body revolving in a movable ellipse (as in Cor II and III of the above Prop) describes in a fixed plane to the figure of the orbit whose apsides are required and then seeking the apsides of the orbit which that body describes in a fixed plane But orbits acquire the same figure if the centripetal forces with which they are described compared between themselves are made proportional at equal altitudes Let the point V be the highest apse and write T for the greatest altitude CV A for any other altitude CP or Cp and X for the difference of the altitudes CV−CP and the force with which a body moves in an ellipse revolving about its focus C (as in Cor II) and which in Cor II was as $\frac{FF}{AA}+\frac{RGG-RFF}{A^3}$ that is as $\frac{FFA+RGG-RFF}{A^3}$ by substituting T−X for A will become as $\frac{RGG-RFF+TFF-FFX}{A^3}$ In like manner any other centripetal force is to be reduced to a fraction whose denominator is A^3 and the numerators are to be made analogous by collating together the homologous terms This will be made plainer by Examples

EXAM I Let us suppose the centripetal force to be uniform and therefore as

$\frac{A^3}{A^3}$ or writing $T-X$ for A in the numerator as $\frac{T^3-3TTX+3TXX-X^3}{A^3}$. Then collating together the correspondent terms of the numerators that is those that consist of given quantities with those of given quantities and those of quantities not given with those of quantities not given it will become

$$RGG-RFF+TFF : T^3 = -FFX : -3TTX+3TXX-X^3$$
$$= -FF : -3TT+3TX-XX.$$

Now since the orbit is supposed extremely near to a circle let it coincide with a circle and because in that case R and T become equal and X is infinitely diminished the last ratios will be

$$GG : T^2 = -FF : -3TT$$

and again $\quad GG : FF = TT : 3TT = 1 : 3$

and therefore G is to F that is the angle VCp to the angle VCP as 1 to $\sqrt{3}$. Therefore since the body in a fixed ellipse in descending from the upper to the lower apse describes an angle if I may so speak of 180° the other body in a movable ellipse and therefore in the fixed plane we are treating of will in its descent from the upper to the lower apse describe an angle VCp of $\frac{180^\circ}{\sqrt{3}}$. And this comes to pass by reason of the likeness of this orbit which a body acted [illegible] describes and of that orbit which a body

[illegible]

moving from the upper apse to the lower apse when it has once [illegible] that angle and thence returning to the upper apse when it has described that angle again and so on in infinitum.

EXAM. 2. Suppose the centripetal force to be as any power of the altitude A as for example A^{n-3} or $\frac{A^n}{A^3}$ where $n-3$ and n signify any indices of powers whatever whether integers or fractions rational or surd affirmative or negative. That numerator A^n or $(T-X)^n$ being reduced to an indeterminate series by my method of converging series will become

$$T^n - nXT^{n-1} + \frac{nn-n}{2} XXT^{n-2} \text{ \&c.}$$

And comparing these terms with the terms of the other numerator

$$RGG-RFF+TFF-FFX,$$

it becomes

$$RGG-RFF+TFF : T^n = -FF : -nT^{n-1} + \frac{nn-n}{2} XT^{n-2} \text{ \&c.}$$

And taking the last ratios where the orbits approach to circles it becomes

$$RGG : T^n = -FF : -nT^{n-1}$$
$$GG : T^{n-1} = FF : nT^{n-1}$$

and again $\quad GG : FF = T^{n-1} : nT^{n-1} = 1 : n$

and therefore G is to F that is the angle VCp to the angle VCP as 1 to $\sqrt{n}$. Therefore since the angle VCP described in the descent of the body from the upper apse to the lower apse in an ellipse is of 180° the angle VCp described

in the descent of the body from the upper apse to the lower apse in an orbit nearly circular which a body describes with a centripetal force proportional to the power A^{-3} will be equal to an angle of $\frac{180}{\sqrt{n}}$ and this angle being repeated the body will return from the lower to the upper apse and so on *in infinitum* As if the centripetal force be as the distance of the body from the centre that is as A or $\frac{A^4}{A^3}$ n will be equal to 4 and $\sqrt{n}$ equal to 2 and therefore the angle between the upper and the lower apse will be equal to $\frac{180}{2}$ or 90 Therefore the body having performed a fourth part of one revol t on [illegible] at the upper [illegible] acted [illegible] is the [illegible] of force If the centripetal force is inversely as the distance that is directly as $\frac{1}{A}$ or $\frac{A^2}{A^3}$ n will be equal to 2 and therefore the angle between the upper and the lower apse will be $\frac{180}{\sqrt{2}}$ or 127 16 45 and hence a body revolving with such a force will by a continual repetition of this angle move alternately from the upper to the lower and from the lower to the upper apse forever So also if the centripetal force be inversely as the fourth root of the eleventh power of the altitude that is inversely as $A^{\frac{11}{4}}$ and therefore directly as $\frac{1}{A^{\frac{11}{4}}}$ or as $\frac{A^{\frac{1}{4}}}{A^3}$ n will be equal to $\frac{1}{4}$ and $\frac{180}{\sqrt{n}}$ will be equal to 360 and therefore the body parting from the upper apse and from thence continually descending will arrive at the lower apse when it has completed one entire revolution and thence ascending continually when it has completed another entire revolution it will arrive again at the upper apse and so alternately forever

EXAM 3 Taking m and n for any indices of the powers of the altitude and b and c for any given numbers suppose the centripetal force to be as $(bA^m+cA^n) \div A^3$ that is as $[b(T-X)^m-c(T-X)^n] \div A^3$ or (by the method of converging series above mentioned) as

$$[bT^m+cT^n-mbXT^{m-1}-ncXT^{n-1}+\frac{mm-m}{2}bXXT^{m-2}+\frac{nn-n}{2}cXXT^{n-2} \&c.] \div A^3$$

and comparing the terms of the numerators there will arise

$$RGG-RFF+TFF : bT^m+cT^n :: -FF : -mbT^{m-1}-ncT^{n-1}+\frac{mm-m}{2}bXT^{m-2}+\frac{nn-n}{2}cXT^{n-2} \&c.$$

And taking the last ratios that arise when the orbits come to a circular form there will come forth

$$GG : bT^{m-1}+cT^{n-1} = FF : mbT^{m-1}+ncT^{n-1}$$

and again $\quad GG : FF = bT^{m-1}+cT^{n-1} : mbT^{m-1}+ncT^{n-1}$

This proportion by expressing the greatest altitude CV or T arithmetically by unity becomes $GG : FF = b+c : mb+nc = 1 : \frac{mb+nc}{b+c}$ Whence G becomes to F that is the angle VCp to the angle VCP as 1 to $\sqrt{\frac{mb+nc}{b+c}}$ And therefore

since the angle VCP between the upper and the lower apse in a fixed ellipse is of 180°, the angle VCp between the same apsides in an orbit which a body describes with a centripetal force that is as $\frac{bA^m - cA^n}{A^3}$ will be equal to an angle of 180° $\sqrt{\frac{b-c}{mb-nc}}$. And by the same reasoning, if the centripetal force be as $\frac{bA^m - cA^n}{A}$, the angle between the apsides will be found equal to

$$180° \sqrt{\frac{b-c}{mb-nc}}$$

After the same manner the Problem is solved in more difficult cases. The ... to which the centripetal force is proportional must always be resolved

...ltitude that ...ely. That is

if the whole angular motion with which the body returns to the same apse be to the angular motion of one revolution, or 360°, as any number as m to another as n, and the altitude be called A, the force will be as the power $A^{\frac{nn}{mm}-3}$ of the altitude A, the index of which power is $\frac{nn}{mm}-3$. This appears by the second Example. Hence it is plain that the force in its recess from the centre cannot decrease in a greater than a cubed ratio of the altitude. A body revolving with

parting from the lower apse, begin to ascend ever so little, it will ascend *in infinitum*, and never come to the upper apse; but will describe the curved line

motion. But if the force in its recess from the centre either decreases in a less

force increases in the recess from the centre, or it decreases in a less than a cubed ratio of the altitude, and the sooner the body returns from one apse to

4 or 2 or 1½ to 1, and therefore $\frac{nn}{mm}-3$ be $1/64-3$ or $1/16-3$ or $1/4-3$ or $4/9$ -3, then the force will be as $A^{1/64-3}$ or $A^{1/16-3}$ or $A^{1/4-3}$ or $A^{4/9-3}$, that is, it will be

inversely as $A^{3-\frac{16}{9}}$ or $A^{3-\frac{9}{4}}$ or A^{3-9} or A^{3-16} If the body after each revolution returns to the same apse and the apse remains unmoved then m will be to n as 1 to 1 and therefore $A^{\frac{nn}{mm}-3}$ will be equal to A^{-2} or $\frac{1}{AA}$ and therefore the decrease of the forces will be in a squared ratio of the altitude as was demonstrated above If the body in three fourth parts or two thirds or one third o one fourth part of an entire revolution return to the same apse m will be to n as ¾ or ⅔ or ⅓ or ¼ to 1 and therefore $A^{\frac{nn}{mm}-3}$ is equal to $A^{\frac{16}{9}-3}$ or $A^{\frac{9}{4}-3}$ or A^{9-3} or A^{16-3} and therefore the force is either inversely as $A^{\frac{11}{9}}$ or $A^{\frac{3}{4}}$ or directly as A^6 or A^{13} Lastly if the body in its progress from the upper apse to the same upper apse again goes over one entire revolution and three degrees more and therefore that apse in each revolution of the body moves forward three degrees then m will be to n as 363 to 360 or as 121 to 120 and therefore $A^{\frac{nn}{mm}-3}$ will be equal to $A^{-\frac{29523}{14641}}$ and therefore the centripetal force will be inversely as $A^{\frac{29523}{14641}}$ or inversely as $A^{2\frac{4}{243}}$ very nearly Therefore the centripetal force decreases in a ratio something greater than the squared ratio but approaching 59¾ times nearer to the squared than the cubed

COR II Hence also if a body urged by a centripetal force which is inversely *as the square of the altitude revolves in an ellipse whose focus is in the centre* of the forces and a new and foreign force should be added to or subtracted from this centripetal force the motion of the apsides arising from that foreign force may (by the third Example) be known and conversely If the force with which the body revolves in the ellipse be as $\frac{1}{AA}$ and the foreign force as cA and therefore the remaining force as $\frac{A - cA^4}{A^3}$ then (by the third Example) b will be equal to 1 m equal to 1 and n equal to 4 and therefore the angle of revolution between the apsides is equal to $180 \sqrt{\frac{1-c}{1-4c}}$ Suppose that foreign force to be 357 45 times less than the other force with which the body revolves in the ellipse that is c to be $\frac{100}{35745}$ A or T being equal to 1 and then $180 \sqrt{\frac{1-c}{1-4c}}$ will be $180 \sqrt{\frac{35645}{35345}}$ or 180 7623 that is 180 45 44 Therefore the body parting from the upper apse will arrive at the lower apse with an angular motion of *180 45 44* and this angular motion being repeated will return to the upper apse and therefore the upper apse in each revolution will go forward 1 31 28 The apse of the moon is about twice as swift

So much for the motion of bodies in orbits whose planes pass through the centre of force It now remains to determine those motions in eccentric planes For those authors who treat of the motion of heavy bodies used to consider the

— — ndicular direction but at

the same reason we are

to centres by means of

any forces whatsoever when those bodies move in eccentric planes The e planes are supposed to be perfectly smooth and polished so as not to retard the motion of the bodies in the least Moreover in these demonstrations instead of the planes upon which those bodies roll or slide and which are therefore tangent planes to the bodies I shall use planes parallel to them in which the

centres of the bodies move and by that motion describe orbits And by the same method I afterwards determine the motions of bodies performed in curved surfaces.

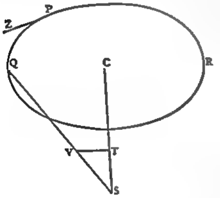
SECTION X

THE MOTION OF BODIES IN GIVEN SURFACES AND THE OSCILLATING PENDULOUS MOTION OF BODIES

PROPOSITION 46 PROBLEM 32

I near figures being allowed it is requi a w a off from a given plane with a given velocity in the direction of a given right line in that plane

Let S be the centre of force SC the least distance of that centre from the given plane P a body issuing from the place P in the direction of the right line PZ Q the same body revolving in its curve and PQR the curve itself which is required to be found described in that given plane Join CQ QS and if in QS we take SV proportional to the centripetal force with which the body is attracted towards the centre S and draw VT parallel to CQ and



other force TV coinciding with the position of the plane itself attracts the body directly towards the given point C in that plane and therefore causes the body to move in the plane in the same manner as if the force ST were taken away and the body were to re-

at any given time and lastly the velocity of the body in that place Q And conversely Q E I

PROPOSITION 47 THEOREM 15

Supposing the centripetal force to be proportional to the distance of the body from the centre all bodies revolving in any planes whatsoever will describe ellipses and complete their revolutions in equal times and those which move in right lines running backwards and forwards alternately will complete their several periods of going and returning in the same times

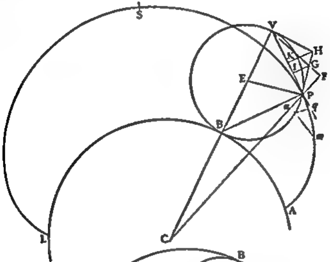
inversely as $A^{3-\frac{1}{16}}$ or $A^{3-\frac{1}{9}}$ or $A^{3-\frac{1}{4}}$ or $A^{3-\frac{4}{9}}$ If the body after each revolution returns to the same apse and the apse remains unmoved then m will be to n as 1 to 1 and therefore $A^{\frac{mm}{nn}-3}$ will be equal to A^{-2}, or $\frac{1}{AA}$ and therefore the decrease of the forces will be in a squared ratio of the altitude as was demonstrated above If the body in three fourth parts or two thirds or one third or one fourth part of an entire revolution return to the same apse, m will be to n as $\frac{3}{4}$ or $\frac{2}{3}$ or $\frac{1}{3}$ or $\frac{1}{4}$ to 1 and therefore $A^{\frac{mm}{nn}-3}$ is equal to $A^{\frac{16}{9}-3}$ or $A^{\frac{9}{4}-3}$ or A^{9-3} or A^{16-3} and therefore the force is either inversely as $A^{\frac{11}{9}}$ or $A^{\frac{3}{4}}$ or directly as A^{6} or A^{13} Lastly if the body in its progress from the upper apse to the same upper apse again goes over one entire revolution and three degrees more and therefore that apse in each revolution of the body moves forward three degrees then m will be to n as 363 to 360 or as 121 to 120 and therefore $A^{\frac{mm}{nn}-3}$ will be equal to $A^{-\frac{29523}{14641}}$ and therefore the centripetal force will be inversely as $A^{\frac{29523}{14641}}$ or inversely as $A^{2\frac{4}{243}}$ very nearly Therefore the centripetal force decreases in a ratio something greater than the squared ratio but approaching $59\frac{3}{4}$ times nearer to the squared than the cubed

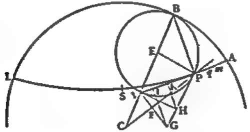
COR II Hence also if a body urged by a centripetal force which is inversely as the square of the altitude revolves in an ellipse whose focus is in the centre of the forces and a new and foreign force should be added to or subtracted from this centripetal force the motion of the apsides arising from that foreign force may (by the third Example) be known and conversely If the force with which the body revolves in the ellipse be as $\frac{1}{AA}$ and the foreign force as cA and therefore the remaining force as $\frac{A-cA^4}{A^3}$ then (by the third Example) b will be equal to 1 m equal to 1 and n equal to 4 and therefore the angle of revolution between the apsides is equal to $180\sqrt{\frac{1-c}{1-4c}}$ Suppose that foreign force to be 357 45 times less than the other force with which the body revolves in the ellipse that is c to be $\frac{1}{357.45}$ A or T being equal to 1 and then $180\sqrt{\frac{1-c}{1-4c}}$ will be $180\sqrt{\frac{356.45}{353.45}}$ or 180 7623 that is 180 45 44 Therefore the body parting from the upper apse will arrive at the lower apse with an angular motion of 180 45 44 and this angular motion being repeated will return to the upper apse and therefore the upper apse in each revolution will go forward 1 31 28 The apse of the moon is about twice as swift

So much for the motion of bodies in orbits whose planes pass through the centre of force It now remains to determine those motions in eccentric planes For those authors who treat of the motion of heavy bodies used to consider the ascent and descent of such bodies not only in a perpendicular direction but at all degrees of obliquity upon any given planes and for the same reason we are to consider in this place the motions of bodies tending to centres by means of any forces whatsoever when those bodies move in eccentric planes These

tangent planes to the bodies I shall use planes parallel to them in which the

ber and the given point P in the
ntime the curvilinear path AP
d since the wheel touched the
globe in A, and the length of this p l be to twice the versed sine of the arc ½PB as 2CE to CB For let the right line CE (produced if need be) meet the wheel in V and join CP BP EP VP produce CP and let fall thereon the perpendicular VF Let PH VH meeting in H touch the circle in P and V and let PH cut VF in G and to VP let fall the perpendiculars GI HK From





the centre C with any radius let there be described the circle *nom* cutting the right line CP in *n* the perimeter of the wheel BP in *o* and the curvilinear path with the radius V*o* let there be described a

will touch this curve in the point P Let the radius of the circle *nom* be gradually increased or diminished so that at last it becomes equal to the distance

For letting all things stand as in the foregoing Proposition the force SV
towards the
TV and CQ
are propor towards the
given point C in the plane of the orbit is as the distance CQ Therefore the forces with which bodies found in the plane PQR are attracted towards the point C are in proportion to the distances equal to the forces with which the same bodies are attracted every way towards the centre S and therefore the bodies will move in the same times and in the same figures in any plane PQR about the point C as they would do in free spaces about the centre S and therefore (by Cor II Prop 10 and Cor II Prop 38) they will in equal times either describe ellipses in that plane about the centre C or move to and fro in right lines passing through the centre C in that plane completing the same periods of time in all cases Q E D

SCHOLIUM

The ascent and descent of bodies in curved surfaces has a near relation to these motions we have been speaking of Imagine curved lines to be described
through the centre
and that the bodies
n those surfaces If
and descent their
and therefore in
nerated In
on in tho e
curved lines

PROPOSITION 48 THEOREM 16

If a wheel stands upon the outside of a globe at right angles thereto and revolving about its own axis goes forwards in a great circle the length of the curvilinear path which any point given in
that it touched the globe
cycloid) will be to double the versed sine of half the arc
touched the globe in passing over it as the sum of the diameters of the globe and the wheel to the semidiameter of the globe

PROPOSITION 49 THEOREM 17

If a wheel stands upon the inside of a concave globe at right angles thereto and
forwards in one of the great circles of the globe the
point given in the perimeter of the wheel
will be to the double of the versed sine of
half the arc which in all that time hath touched the globe in passing over it as the difference of the diameters of the globe and the wheel to the semidiameter of the globe

Let ABL be the globe C its centre BPV the wheel resting on it E the centre of the wheel B the point of contact and P the given point in the perimeter of the wheel Imagine this wheel to proceed in the great circle ABL from A through B towards L and in its progress to revolve in such a manner that the

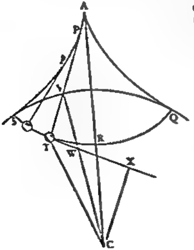
semicycloids AQ AS that as often as the pendulum part from the perpendicular AR the upper part of the thread AP may be applied to that semicycloid APS towards which the motion tends and fold itself round that curved line as if it were some solid obstacle the remaining part of the same thread PT which has not yet touched the semicycloid continuing straight. Then will the weight T oscillate in the given cycloid QRS Q.E.F

For let the thread PT meet the cycloid QRS in T and the circle QOS in V and let CV be drawn and to the rectilinear part of the thread PT from the extreme points P and T let there be erected the perpendiculars BP TW meeting the right line CV in B and W It is evident from the construction and genera[illegible] [illegible] similar figures AS SR, that those perpendiculars PB TW cut off [illegible] of the wheels OA, OR [illegible]le VBP when ½BV [illegible]A and CO CO and [illegible]A+CO to CA, or if [illegible]rop 49) the length [illegible]he arc of the cycloid [illegible] cycloid APS that is PS and the whole thread APT is a[illegible] [illegible] to the length AR And conversely if the string be always (by Cor II Prop 49) to the length AR And conversely if the string be always equal to the length AR the point T will always move in the given cycloid QRS Q.E.D

COR. The string AR is equal to the semicycloid AS and therefore has the same ratio to AC the semidiameter of the exterior globe as the like semicycloid SR has to CO the semidiameter of the interior globe

PROPOSITION 51 THEOREM 18

If a centripetal force tending on all sides to the centre C of a globe be in all places as the distance of the place from the centre and by this force alone acting upon the body T oscillate (in the manner above described) in the perimeter of the cycloid QRS I say that all the oscillations howsoever unequal in themselves will be performed in equal times



For upon the tangent TW indefinitely produced let fall the perpendicular CX and join CT Because the centripetal force with which the body T is impelled [illegible] thread PT and by the resistance the thread makes to it is totally employed producing no other effect but the other part TX, impelling the body transversely or towards X, directly accelerates the motion in the cycloid. Then it is plain that the acceleration of the body proportional to this accelerating force will be every moment as the length TX that is (because CV WV and TX TW

PV PT PG PI respectively But since VF is perpendicular to CF and VH to CV le VHG (because the equal to the angle CE will come to pass that

$$EP \quad CE = HG \quad HV \text{ or } HP = KI \quad PK$$

and by addition or subtraction

$$CB \quad CE = PI \quad PK$$

and

$$CB \quad 2CE = PI \quad PV = Pq \quad Pm$$

Therefore the decrement of the line VP that is the increment of the line BV−VP to the increment of the curved line AP is in a given ratio of CB to 2CE and therefore (by Cor Lem 4) the lengths BV−VP and AP generated by those increments are in the same ratio But if BV be radius VP is the cosine of the angle BVP or ½BEP and therefore BV−VP is the versed sine of the same angle and therefore in this wheel whose radius is ½BV BV−VP will be double the versed sine of the arc ½BP Therefore AP is to double the versed sine of the arc ½BP as 2CE to CB Q E D

The line AP in the former of these Propositions we shall name the cycloid without the globe the other in the latter Proposition the cycloid within the globe for distinction s sake

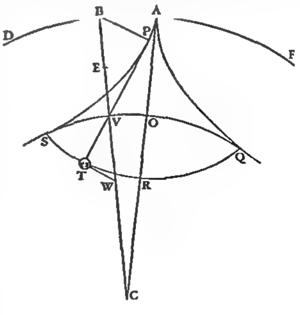
COR I Hence if there be described the entire cycloid ASL and the same be bisected in S the length of the part PS will be to the length PV (which is the double of the sine of the angle VBP when EB is radius) as 2CE to CB and therefore in a given ratio

COR II And the length of the semidiameter of the cycloid AS will be equal to a right line which is to the diameter of the wheel BV as 2CE to CB

PROPOSITION 50 PROBLEM 33

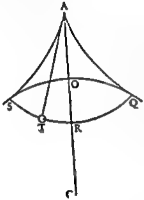
To cause a pendulous body to oscillate in a given cycloid

Let there be given within the globe QVS described with the centre C the cycloid QRS bisected in R and meeting the surface of the globe with its extreme points Q and S on either hand Let there be drawn CR bisecting the arc QS in O and let it be produced to A in such sort that CA may be to CO as CO to CR About the centre C with the radius CA let there be described an exterior globe DAF and within this globe by a wheel whose diameter is AO let there be described two semicycloids AQ AS touching the interior globe in Q and S and meeting the exterior globe in A From that point A with a thread APT in length equal to the line AR let the body T be suspended and oscillated in such manner between the two



as the ordinate LI to the radius GK or as $\sqrt{(SR - TR^2)}$ to SR. Hence since in unequal oscillations there are described in equal times arcs proportional to the entire arcs of the oscillations there are obtained from the times given both the velocities and the arcs described in all the oscillations universally Which was first required

Let now any pendulous bodies oscillate in different cycloids described within different globes whose absolute forces are also different and if the absolute force of any globe QOS be called V the accelerative force with which the pen-



body from that centre and the abso ute o the globe conjointly that is as CO V Therefore the short line HY which is as this accelerated force CO V will be described in a given time and if there be erected the perpendicular YZ meeting the circumference in Z the nascent arc HZ will denote that given time But that nascent arc HZ varies as the square root of the rectangle GH HY and therefore as $\sqrt{(GH\ CO\ V)}$ Whence the time of an entire oscillation in the cycloid QRS (it being as the semiperiphery

GH and SR are equal as $\sqrt{\frac{SR}{CO\ V}}$ or (by Cor Prop 50) as $\sqrt{\frac{}{AC\ V}}$ There-

and the centre of the globe and also inversely as the square root of the absolute force of the globe Q E I

any place to the centre and the time equal to it in which the body revolving uniformly about the centre of the globe at any distance describes an arc of a quadrant. For this time (by Case 2) is to the time of half the oscillation in any cycloid QRS as 1 to $\sqrt{\frac{AR}{AC}}$

COR II Hence also follow what Sir Christopher Wren and Mr Huygens have discovered concerning the common cycloid For if the diameter of the globe be infinitely increased its spherical surface will be changed into a plane and the centripetal force will act uniformly in the direction of lines perpendicular to that plane and our cycloid will become the same with the common cycloid But in that case the length of the arc of the cycloid between that plane

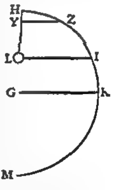
proportional to them are given) as the length TW that is (by Cor 1 Prop 39) as the length of the arc of the cycloid TR If therefore two pendulums APT Apt be unequally drawn aside from the perpendicular AR and let fall together their accelerations will be always as the arcs to be described TR tR But the parts described at the beginning of the motion are as the accelerations that is as the whole spaces that are to be described at the beginning and therefore the parts which remain to be described and the subsequent accelerations proportional to those parts are also as the whole and so on Therefore the accelerations and consequently the velocities generated and the parts described with those velocities and the parts to be described are always as the whole and therefore the parts to be described preserving a given ratio to each other will vanish together that is the two bodies oscillating will arrive together at the perpendicular AR And since on the other hand the ascent of the pendulums from the lowest place R through the same cycloidal arcs with a retrograde motion is retarded in the several places they pass through by the same forces by which their descent was accelerated it is plain that the velocities of their ascent and descent through the same arcs are equal and consequently performed in equal times and therefore since the two parts of the cycloid RS and RQ lying on either side of the perpendicular are similar and equal the two pendulums will perform as well the whole as the half of their oscillations in the same times Q E D

COR The force with which the body T is accelerated or retarded in any place T of the cycloid is to the whole weight of the same body in the highest place S or Q as the arc of the cycloid TR is to the arc SR or QR

PROPOSITION 52 PROBLEM 34

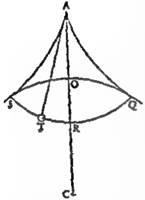
To define the velocities of pendulums in the several places and the times in which both the entire oscillations and their several parts are performed

About any centre G with the radius GH equal to the arc of the cycloid RS describe a semicircle HKM bisected by the semidiameter GK And if a centripetal force proportional to the distance of the places from the centre tend to the centre G and it be in the perimeter HIK equal to the centripetal force in the perimeter of the globe QOS tending towards its centre and at the same time that the pendulum T is let fall from the highest place S a body as L is let fall from H to G then because the forces which act upon the bodies are equal at the beginning and always proportional to the spaces to be described TR LG and therefore if TR and LG are equal are also equal in the places T and L it is plain that those bodies describe at the beginning equal spaces ST HL and therefore are still acted upon equally and continue to describe equal spaces Therefore by Prop 38 the time in which the body describes the arc ST is to the time of one oscillation as the arc HI the time in which the body H arrives at L to the semiperiphery HKM the time in which the body H will come to M And the velocity of the pendulous body in the place T is to its velocity in the lowest place R that is the velocity of the body H in the place L to its velocity in the place G or the momentary increment of the line HL to the momentary increment of the line HG (the arcs HI HK increasing with an uniform velocity)



as the ordinate LI to the radius GK, or as $\sqrt{(SR-TR)}$ to SR. Hence since in unequal oscillations there are described in equal times arcs proportional to the entire arcs of the oscillations, there are obtained from the times given, both the velocities and the arcs described in all the oscillations universally. Which was first required

bodies oscillate in different cycloids described within and if the absolute with which the pen cumference of this globe when it begins to move directly towards its centre will be as the distance of the pendulous body from that centre and the absolute force of the globe conjointly that is as CO $\times$. Therefore the short line HY which is as this accelerated force CO $\times$ will be described in a given time and if there be erected the perpendicular YZ meeting the circumference in Z the nascent arc HZ will denote that given time But that nascent arc HZ varies as the square root of the rectangle GH HY and therefore as $\sqrt{(GH\ CO\ \times)}$. Whence the time of an entire oscillation in the cycloid QRS (it being as the semiperiphery HKM which denotes that entire oscillation, directly and as the arc HZ, which in like manner denotes a given time inversely) will be as GH directly and $\sqrt{(GH\ CO\ \times)}$ inversely that is, because GH and SR are equal, as $\sqrt{\frac{SR}{CO\ \times}}$ or (by Cor Prop 50) as $\sqrt{\frac{AR}{AC\ \times}}$. Therefore the oscillations in all globes and cycloids, performed with any absolute forces whatever vary directly as the square root of the length of the string and inversely as the square root of the distance between the point of suspension and the centre of the globe and also inversely as the square root of the absolute force of the globe



Q E I

bodies may h which diameter centre of frequent ent from any place to the centre and the time equal to it in which the body revolving uniformly about the centre of the globe at any distance describes an arc of a quadrant. For this time (by Case 2) is to the time of half the oscillation in any cycloid QRS as 1 to $\sqrt{\frac{AR}{AC}}$

COR II. Hence also follows what Sir Christopher Wren and Mr Huygens have discovered concerning the common cycloid. For if the diameter of the globe be infinitely increased its spherical surface will be changed into a plane and the centripetal force will act uniformly in the direction of lines perpendicular to that plane and our cycloid will become the same with the common cycloid. But in that case the length of the arc of the cycloid between that plane

and the describing point will become equal to four times the versed sine of half the arc of the wheel between the same plane and the describing point as was discovered by Sir Christopher Wren And a pendulum between two such cycloids will oscillate in a similar and equal cycloid in equal times as Mr Huygens demonstrated The descent of heavy bodies also in the time of one oscillation will be the same as Mr Huygens exhibited

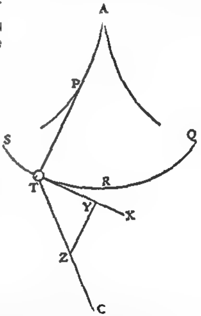
The Propositions here demonstrated are adapted to the true const t t on of th the
) and
I u lus in mines and deep caverns of the earth must oscillate in cycloids within the globe that those o cillations may be performed in equal times For gravity (as will be shown in the third book) decreases in its progress from the surface of the earth upwards as the square root of the distances from the centre of the earth downwards as these distances

PROPOSITION 53 PROBLEM 35

Granting the quadratures of curvilinear figures it is required to find the forces with which bodies moving in given curved lines may always perform their oscillations in equal times

Let the body T oscillate in any curved line STRQ whose axis is AR passing through the centre of force C Draw TX touching that curve in any place of the body T and in that tangent TX take TY equal to the arc TR The length of that arc is known from the common methods used for the quadratures of figures From the point Y draw the right line YZ perpendicular to the tangent Draw CT meeting YZ in Z and the centripetal force will be proportional to the right line TZ

Q E I

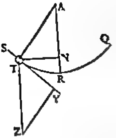


For if the force with which the body is attracted from T towards C be expressed by the right line TZ taken proportional to it that force will be resolved into two forces TY YZ of which YZ drawing the body in the direction of the length of the thr d PT do
its motion
accelerates
STRQ Therefore since that force is as the space to be described TR the accelerations or retardations of the body in describing two proportional parts (a greater and a less) of two oscillations will be always as those parts and therefore will cause those parts to be described together But bodies which continually describe in the same time parts proportional to the whole will describe the whole in the same time

Q E D

COR I Hence if the body T hanging by a rectilinear thread AT from the cent A d ribe the c l STRQ

tions will be equal For because TZ AR are parallel the triangles ATN ZTY



are similar and therefore TZ will be to AT as TY to TN that is if the uniform force of gravity be expressed by the given length AT the force TZ by which the oscil lations become isochronou will be to the force of grav ity AT as the arc TR equal to TY is to TN the sine of that arc

Co I And therefore in clocks if forces are im

al ways as a line which is obtained by ... and the radius AR by the sine TN then all the oscillations will become isochronous

PROPOSITION 54 PROBLEM 36

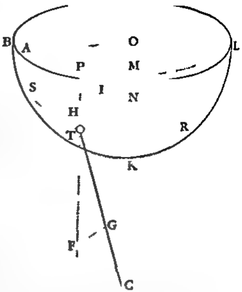
point D and because Dd is given the rectangle Dd DN that is the area DNnd will be propor ti nal to the same time Therefore if PNn be a

C

line ST and therefore that area being found the time is al o given Q E I

PROPOSITION 55 THEOREM 19

If a body move in a y curved su f ce whose axis passes through the centre of force and f om the body a perpendicular be let f ll upon the axis and a line pa allel and equal thereto be drawn from a y gi en point of the axis I say that this parallel l ne will describe an a ea proportional to the time



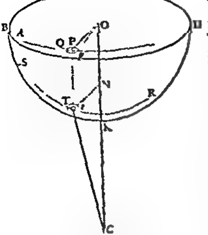
axis AP the path described by the point P in the plane AOP in which the revolving line OP is found A the beginning of that path answering to the point S TC a right line drawn from the body to the centre TG a part thereof proportional to the centripetal force with which the body tends towards the centre C TM a right line perpendicular to the curved surface TI a part thereof proportional to the force of pressure with which the body urges the surface and therefore with which it is again repelled by the surface towards M PTF a right line parallel to the axis and passing through the body and GF IH right lines let fall perpendicularly from the points G and I upon that parallel PHTF I say now that the area AOP described by the radius OP from the beginning of the motion is proportional to the time For the force TG (by Cor II of the Laws of Motion) is resolved into the forces TF FG and the force TI into the forces TH HI but the forces TF TH acting in the direction of the line PF perpendicular to the plane AOP introduce no change in the motion of the body but in a direction perpendicular to that plane Therefore its motion so far as it hath the same direction with the position of the plane that is the motion of the point P by which the projection AP of the curve is described in that plane is the same as if the forces TF TH were taken away and the body were acted on by the forces FG HI alone that is the same as if the body were to describe in the plane AOP the curve AP by means of a centripetal force tending to the centre O and equal to the sum of the forces FG and HI But with such a force as that (by Prop 1) the area AOP will be described proportional to the time Q E D

COR By the same reasoning if a body acted on by forces tending to two or more centres in the same given right line CO should describe in a free space any curved line ST the area AOP would be always proportional to the time

PROPOSITION 56 PROBLEM 37

Granting the quadratures of curvilinear figures and supposing that there are given both the law of centripetal force tending to a given centre and the curved surface whose axis passes through that centre it is required to find the curve which a body will describe in that surface when going off from a given place with a given velocity and in a given direction in that surface

The last construction remaining let the body T go from the given place S in the direction of a line given by position and turn into the curve sought STR whose orthographic projection in the plane BDO is AP And from the given velocity of the body in the altitude SC its velocity in any other altitude TC will be also given With that velocity in a given moment of time let the body describe the segment described in upon the curved su



jection of that little circle in the plane AOP be the ellipse pQ And because the magnitude of that little circle Tt and TN or PO its distance from the axis CO is also given the ellipse pQ will be given both in kind and magnitude as also its position to the right line PO And since the area POp is proportional to the time and therefore given because the time is given the angle POp will be given And thence will be given the common intersection of the ellipse and the right line Op together with the angle OPp in which the projection APp of the curve cuts the line OP But from thence (by comparing Prop 41 with its Cor II) the manner of determining the curve APp *easily appears. Then from the several points P of that projection erecting to* the plane AOP the perpendiculars PT meeting the curved surface in T there will be given the several points T of the curve Q.E.I.

SECTION XI

THE MOTIONS OF BODIES TENDING TO EACH OTHER WITH CENTRIPETAL FORCES

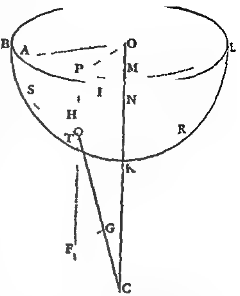
I have hitherto been treating of the attractions of bodies towards an immovable centre though very probably there is no such thing existent in nature For attractions are made towards bodies, and the actions of the bodies ... Law III so that ... ing body is truly ... it were mutually ... there be more ... racted by them ... will be so moved ... at rest ...

by a mathematical reader

PROPOSITION 57 THEOREM 20

Two bodies attracting each other mutually describe similar figures about their common centre of gravity and about each other mutually

For the distances of the bodies from their common centre of gravity are inversely as the bodies and therefore in a given ratio to each other and thence by composition of ratios in a given ratio to the whole distance between the



axis AP the path described by the point P in the plane AOP in which the revolving line OP is found A the beginning of that path answering to the point S TC a right line drawn from the body to the centre TG a part thereof proportional to the centripetal force with which the body tends towards the centre C TM a right line perpendicular to the curved surface TI a part thereof proportional to the force of pressure with which the body urges the surface and therefore with which it is again repelled by the surface towards M PTF a right line parallel to the axis and passing through the body and GF IH right lines let fall perpendicularly from the points G and I upon that parallel PHTF I say now that the area AOP described by the radius OP from the beginning of the motion is proportional to the time For the force TG (by Cor II of the Laws of Motion) is resolved into the forces TF FG and the force TI into the forces TH HI but the forces TF TH acting in the direction of the line PF perpendicular to the plane AOP introduce no change in the motion of the body but in a direction perpendicular to that plane Therefore its motion so far as it hath the same direction with the position of the plane that is the motion of the point P by which the projection AP of the curve is described in that plane is the same as if the forces TF TH were taken away and the body were acted on by the forces FG HI alone that is the same as if the body were to describe in the plane AOP the curve AP by means of a centripetal force ten[illegible] nd equal [illegible] Prop 1)

Q E D

[illegible] a body acted on by forces tending to two or more centres in the same given right line CO should describe in a free space any curved line ST the area AOP would be always proportional to the time

PROPOSITION 56 PROBLEM 37

Granting the quadratures of curvilinear figures and supposing that there are given both the law of centripetal force tending to a given centre and the curved surface whose axis passes through that centre it is required to find the curve which a body [illegible] given place with a given velocity

[illegible] T go from the given place S in the direction of a line given by position and turn into the curve sought STR whose orthographic projection in the plane BDO [illegible] AP And from the given velocity of the body in the altitude SC its velocity in any other altitude TC will be also given With that velocity in a given moment of time let the body describe the segment Tt of its curve and let Pp be the projection of that segment described in the plane AOP Join Op and a little circle being described upon the curved surface about the centre T with the radius Tt let the pro-

square root of the intervals because by Lem 10 the spaces described at the beginning of the motion are as the square of the times Suppose then the velocity of the body p to be to the velocity of the body P as the square root of the ratio of the distance sp to the distance CP so that the arcs pq PQ which are in a simple proportion to each other may be described in times that are as the square root of the distances and the bodies P p always attracted by equal forces will describe round the fixed centres C and s similar figures PQV pqv the latter of which pqv is similar and equal to the figure which the body ... round the movable body S Q E D

... of gravity together with the

space will be ...
will describe about each other the same figures as ... fore similar and equal to the figure pqv Q E D

COR I Hence two bodies attracting each other with forces proportional to their distance describe (by Prop 10) both round their common centre of gravity and round each other concentric ellipses and conversely if such figures are described the forces are proportional to the distances

COR II And two bodies whose forces are inversely proportional to the square of their distance describe (by Props 11 12 13) both round their common centre of gravity and round each other conic sections having their focus in the centre about which the figures are described And conversely if such figures are described the centripetal forces are inversely proportional to the square of the distance

COR III Any two bodies revolving round their common centre of gravity describe areas proportional to the times by radii drawn both to that centre and to each other

PROPOSITION 59 THEOREM 22

The periodic time of two bodies S and P revolving round their common centre of gravity C is to the periodic time of one of the bodies P revolving round the other S ... ng fixed and describing a figure similar and equal to those which the bodies ... each other as $\sqrt{S}$ is to $\sqrt{(S+P)}$

... which any ... sp that is ... he times in ... ole times in which the whole similar figures are desc... atio $\sqrt{S}$ to $\sqrt{(S+P)}$ Q E D

PROPOSITION 60 THEOREM 23

If two bodies S and P attracting each other with forces inversely proportional to the square of their distance revolve about their common centre of gravity I say that the principal axis of the ellipse which either of the bodies as P describes by this motion about the other S will be to the principal axis of the ellipse which the same body P may describe in the same periodic time about the other body S fixed as the sum of the two bodies S+P to the first of two mean proportionals between that sum and the other body S

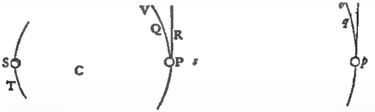
For if the ellipses described were equal to each other their periodic times by the last Theorem would be as the square root of the ratio of the body S to the

bodies Now these distances are carried round their common extremity with an uniform angular motion because lying in the same right line they never change their inclination to each other But right lines that are in a given ratio to each other and are carried round their extremities with an uniform angular motion describe upon planes which either rest together with them or are moved with any motion not angular figures entirely similar round those extremities Therefore the figures described by the revolution of these distances are similar Q E D

PROPOSITION 58 THEOREM 21

If two bodies attract each other with forces of any kind and revolve about the common centre of gravity I say that by the same forces there may be described round either body unmoved a figure similar and equal to the figures which the bodies so moving describe round each other

Let the bodies S and P revolve about their common centre of gravity C proceeding from S to T and from P to Q From the given point *s* let there be continually drawn *sp* *sq* equal and parallel to SP TQ and the curve *pqv* which the point *p* describes in its revolution round the fixed point *s* will be



similar and equal to the curves which the bodies S and P describe about each other and therefore by Theor 20 similar to the curves ST and PQV which the same bodies describe about their common centre of gravity C and that because the proportions of the lines SC CP and SP or *sp* to each other are given

CASE 1 The common centre of gravity C (by Cor IV of the Laws of Motion) is either at rest or moves uniformly in a right line Let us first suppose it at rest and in *s* and *p* let there be placed two bodies one immovable in *s* the other movable in *p* similar and equal to the bodies S and P Then let the right lines PR and *pr* touch the curves PQ and *pq* in P and *p* and produce CQ and *sq* to R and *r* And because the figures CPRQ *sprq* are similar RQ will be to *rq* as CP to *sp* and therefore in a given ratio Hence if the force with which the body P is attracted towards the body S and by consequence towards the intermediate centre C were to the force with which the body *p* is attracted towards the centre *s* in the same given ratio these forces would in equal times attract the bodies from the tangents PR *pr* to the arcs PQ *pq* through the intervals proportional to them RQ *rq* and therefore this last force (tending to *s*) would make the body *p* revolve in the curve *pqv* which would become similar to the curve PQV in which the first force obliges the body P to revolve

tances SP *sp*) mutually equal the bodies in equal times will be equally drawn from the tangents and therefore that the body *p* may be attracted through the greater interval *rq* there is required a greater time which will vary as the

PROPOSITION 63 PROBLEM 39

... ms of two bodies attracting each other with forces inversely ... from given places in

... there is given also the uniform motion of the ... d the motion of the space which moves along with this centre uniformly in a right line and also ... n of the bodies in respect of this pace Then ... ll be

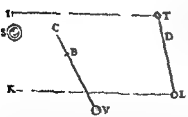
tending to that cen ...

time will be obtained the motion of the other round the same centre ... this motion compound the uniform progressive motion of the entire system of the space and the bodies revolving in it and there will be obtained the absolute motion of the bodies in immovable space Q.E.I

PROPOSITION 64 PROBLEM 40

Supposing forces with which bodies attract each other to increase in a simple ratio of their distances from the centres it is required to find the motions of several bodies among themselves

Suppose the first two bodies T and L to have their common centre of gravity in D These by Cor I Theor 21 will describe ellipses having their centres in D th m gnitudes of which ellipses are known by Prob 5



IL and therefore as the forces with which the bodies T and L attract each other added to the forces of the bodies T and L the first to the first and ...

and Cor I and VIII Prop 4) they will cause those bodies to describe ellipses as before but with a swifter motion. The remaining accelerative forces SD and DL, by the motive forces SD T and SD L, which are as the bodies attracting those bodies equally and in the

sum of the bodies S+P Let the periodic time in the latter ellipse be diminished in that ratio and the periodic times will become equal but by Prop 15 the principal axis of the ellipse will be diminished in a ratio which is the $^3/_2$th power of the former ratio that is in a ratio to which the ratio of S to S+P is the cube and therefore that axis will be to the principal axis of the other ellipse as the first of two mean proportionals between S+P and S to S+P And inversely the principal axis of the ellipse described about the movable body will be to the principal axis of that described round the immovable as S+P to the first of two mean proportionals between S+P and S Q E D

PROPOSITION 61 THEOREM 24

If two bodies attracting each other with any kind of [illegible]

[illegible] *in their common centre of gravity and the law of the attracting forces will be the same in respect of th[illegible] f the bodies from th[illegible] bodies*

[illegible] tending to the [illegible] e common centre of gravity lying directly between them and therefore are the same as if they proceeded from an intermediate body Q E D

And because there is given the ratio of the distance of either body from that common centre to the distance between the two bodies there is given of course the ratio of any power of one distance to the same power of the other distance and also the ratio of any quantity derived in any manner from one of the distances compounded in any manner with given quantities to another quantity derived in like manner from the other distance and [illegible] quant[illegible]

force [illegible]

the c[illegible] om each other or as any power of that distance or lastly as any quantity derived after any manner from that distance compounded with given quantities then will the same force with which the same body is attracted to the common centre of gravity be in like manner directly or inversely as the distance of the attracted body from the common centre or as any power of that distance or lastly as a quantity derived in like sort from that distance compounded with analogous given quantities That is the law of attracting force will be the same with respect to both distances Q E D

PROPOSITION 62 PROBLEM 38

To determine the motions of two bodies which attract each other with forces inversely proportional to the squares of the distance between them and are let fall from given places

The bodies by the last Theorem will be moved in the [illegible] f

[illegible] Laws of Motion) will be al[illegible]

[illegible] attracting each other [illegible] E I

Case 2 Let us imagine a system of lesser bodies revolving about a very great one in the manner just described or any other system of two bodies h t ach other to be moving uniformly forwards in a right line vastly

system to change its place while the p

themselves it is manifest that no change in those motions of the attracted rds the greater unless by the in the inclinations of the lines to- ittractions are made Suppo e de towards the great body to be

arising from the inequality and inclination of the lines makes the whole perturbation greater

other with proper velocities to revolve round the common centre of gravity C With such a motion the body S because the sum of the motive forces SD T and SD L is proportional to the distance CS tends to the centre C and will describe an ellipse round that centre and the point D because the lines CS and CD are proportional will describe a like ellipse over against it But the bodies T and L attracted by the motive forces SD T and SD L the first by the first and the last by the last equally and in the direction of the parallel lines TI and LK as was said before will (by Cor v and vi of the Laws of Motion) continue to describe their ellipses round the movable centre D as before Q E I

Let there be added a fourth body V and by the like reasoning it will be demonstrated that this body and the point C will describe ellipses about the common centre of gravity B the motions of the bodies T L and S round the centres D and C remaining the same as before but accelerated And by the same method one may add yet more bodies at pleasure Q E I

This would be the case though the bodies T and L should attract each other [illegible] before it will easily be concluded that all the bodies will describe different ellipses with equal periodic times about their common centre of gravity B in an immovable plane Q E I

PROPOSITION 65 THEOREM 25

Bodies whose forces decrease as the square of their distances from their centres may move among themselves in ellipses and by radii drawn to the foci may describe areas very nearly proportional to the times

In the last Proposition we demonstrated that case in which the motions will be performed exactly in ellipses The more distant the law of the forces is from the law in that case the more will the bodies disturb each other s motions neither is it possible that bodies attracting each other according to the law supposed in this Proposition should move exactly in ellipses unless by keeping a certain proportion of distances from each other However in the following cases the orbits will not much differ from ellipses

CASE 1 Imagine several lesser bodies to revolve about some very great one at different distances from it and suppose absolute forces tending to every one of the bodies proportional to each And because (by Cor iv of the Laws) the common centre of gravity of them all is either at rest or moves uniformly forwards in a right line suppose the lesser bodies so small that the great body may be never at a sensible distance from that centre and then the great body will without any sensible error be either at rest or move uniformly forwards in a right line and the lesser will revolve about that great one in ellipses and by radii drawn thereto will describe areas proportional to the times if we except the errors that may be introduced by the receding of the great body from the common centre of gravity or by the actions of the lesser bodies upon each other But the lesser bodies may be so far diminished as that this recess and the actions of the bodies on each other may become less than any assignable and therefore so as that the orbits may become ellipses and the areas answer to the times without any error that is not less than any assignable Q E O

CASE 2. Let us imagine a system of lesser bodies revolving about a very great one in the manner just described, or any other system of two bodies revolving about each other to be moving uniformly forward in a right line, and in the meantime to be impelled sideways by the force of another vastly greater body situate at a great distance. And because the equal accelerative forces with which the bodies are impelled in parallel directions do not change the situation of the bodies with respect to each other, but only oblige the whole system to change its place while the parts still retain their motions among themselves, it is manifest that no change in those motions of the attracted bodies can arise from their attractions towards the greater, unless by the inequality of the accelerative attractions, or by the inclination of the lines towards each other, in whose directions the attractions are made. Suppose, therefore, all the accelerative attractions made towards the great body to be among themselves inversely as the squares of the distances; and then, by increasing the distance of the great body till the differences of the right lines drawn from that to the others in respect of their length, and the inclinations of those lines to each other, be less than any given, the motions of the parts of the system will continue without errors that are not less than any given. And because, by the small distance of those parts from each other, the whole system is attracted as if it were but one body, it will therefore be moved by this attraction as if it were one body; that is, its centre of gravity will describe about the great body one of the conic sections (that is, a parabola or hyperbola when the attraction is but languid and an ellipse when it is more vigorous); and by radii drawn thereto, it will describe areas proportional to the times, without any errors but those which arise from the distances of the parts, and these are by the supposition exceedingly small, and may be diminished at pleasure. Q.E.D.

By a like reasoning one may proceed to more complicated cases in infinitum.

COR. I. In the second Case, the nearer the very great body approaches to the system of two or more revolving bodies, the greater will the perturbation be of the motions of the parts of the system among themselves; because the inclinations of the lines drawn from that great body to those parts become greater; and the inequality of the proportion is also greater.

COR. II. But the perturbation will be greatest of all, if we suppose the accelerative attractions of the parts of the system towards the greatest body of all are not to each other inversely as the squares of the distances from that great body; especially if the inequality of this proportion be greater than the inequality of the proportion of the distances from the great body. For if the accelerative force, acting in parallel directions and equally, causes no perturbation in the motions of the parts of the system, it must of course, when it acts unequally, cause a perturbation somewhere, which will be greater or less as the inequality is greater or less. The excess of the greater impulses acting upon some bodies, and not acting upon others, must necessarily change their situation among themselves. And this perturbation, added to the perturbation arising from the inequality and inclination of the lines, makes the whole perturbation greater.

COR. III. Hence if the parts of this system move in ellipses or circles without any remarkable perturbation, it is manifest that, if they are at all impelled by accelerative forces tending to any other bodies, the impulse is very weak, or else is impressed very near equally and in parallel directions upon all of them.

other with proper velocities to revolve round the common centre of gravity C With such a motion the body S because the sum of the motive forces SD T and SD L is proportional to the distance CS tends to the centre C and will describe an ellipse round that centre and the point D because the lines CS and CD are proportional will describe a like ellipse over against it But the bodies T and L attracted by the motive forces SD T and SD L the first by the first and the last by the last equally and in the direction of the parallel lines TI and LK as was said before will (by Cor v and vi of the Laws of Motion) continue to describe their ellipses round the movable centre D as before Q E I

Let there be added a fourth body V and by the like reasoning it will be demonstrated that this body and the point C will describe ellipses about the common centre of gravity B the motion of the bodies T L and S round the centres D and C remaining the same as before but accelerated And by the same method one may add

This would be the case with accelerative forces gr

other bodies in proportion to their distance Let all the accelerative attractions be to each other as the distances multiplied into the attracting bodies and from what has gone before it will easily be concluded that all the bodies will describe different ellipses with equal periodic times about their common centre of gravity B in an immovable plane Q E I

PROPOSITION 65 THEOREM 25

Bodies whose forces decrease as the square of their distances from their centres may move among themselves in ellipses and by radii drawn to the foci may describe areas very nearly proportional to the times

In the last Proposition we demonstrated that case in which the motions will be performed exactly in ellipses The more distant the law of the forces is

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common centre of gravity of them all is either at rest or moves uniformly

in a right line and the lesser will revolve about that great one in ellipses and by radii drawn thereto will describe areas proportional to the times if we except the errors that may be introduced by the receding of the great body from the common centre of gravity or by the actions of the lesser bodies upon each other But the lesser bodies may be so far diminished as that this recess and the actions of the bodies on each other may become less than any assignable and therefore so as that the orbits may become ellipses and the areas answer to the times without any error that is not less than any assignable Q E O

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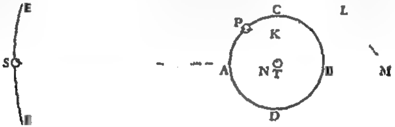
PROPOSITION 66 THEOREM 26

If three bodies whose forces decrease as the square of the distances attract each other and the accelerative attractions of any two towards the third be between themselves inversely as the squares of the distances and the two least revolve about the greatest I say that the interior of the two revolving bodies will by radii drawn to the innermost and greatest describe round that body areas more proportional to the times and a figure more approaching to that of an ellipse having its focus in the point of intersection of the radii if that great body be agitated by those attractions than it would do if that great body were not attracted at all by the lesser but remained at rest or than it would do if that great body were very much more or very much less attracted or very much more or very much less agitated by the attractions

This appears plainly enough from the demonstration of the second Corollary of the foregoing Proposition but it may be made out after this manner by a way of reasoning more distinct and m

CASE 1 Let the lesser bodies P
greatest body T the body P desc
exterior orbit ESE Let SK be the mean distance of the bodies P and S and let the accelerative attraction of the body P towards S at that mean distance be expressed by that line SK Make SL to SK as the square of SK to the square of SP and SL will be the accelerative attraction of the body P toward S at any distance SP Join PT and draw LM parallel to it meeting ST in M and



the attraction SL will be resolved (by Cor II of the Laws of Motion) into the attractions SM LM And so the body P will be urged with a threefold accelerative force One of these forces tends towards T and arises from the mutual attraction of the bodies T and P By this force alone the body P would describe round the body T by the radius PT
ellipse whose focus is in the centre of
the body T

added to and coincide with the former force and cause the areas to be still proportional to the times by Cor III Theor 21 But because it is not inversely proportional to the square of the distance PT it will compose when added to the former a force varying from that proportion this variation will be the greater by as much as the proportion of this force to the former is greater other things remaining the same Therefore since by Prop 11 and by Cor II Theor 21 the force with which the ellipse is described about the focus T ought to be directed to that focus and to be inversely proportional to the square of the distance PT that compounded force varying from that proportion will make the orbit PAB vary from the figure of an ellipse that has its focus in the

point T and so much the more by as much as the variation from that proportion is greater and in consequence by as much as the proportion of the second force LM to the first force is greater other things remaining the same But now the third force SM attracting the body P in a direction parallel to ST composes with the other forces a new force which is no longer directed from P to T and this varies so much more from this direction by as much as the proportion of this force to the other forces is greater other things remaining the same

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PAB from the elliptical figure before ... first because that force is not directed from P to T and secondly because it is not ... of the distance PT These things being

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the body P from the plane of its orbit But the other force NM acting in the direction of a line parallel to ST (and therefore when the body S is without the line of the nodes inclined to the plane of the orbit PAB) besides the perturbation of the motion just now spoken of as to longitude introduces another perturbation also as to latitude attracting the body P out of the plane of its orbit And this perturbation in any given situation of the bodies P and T to each

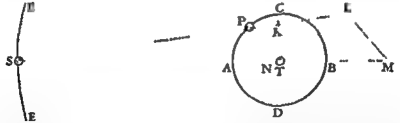
other will be as the generating force MN and therefore becomes least when the force MN is least that is (as was just now shown) where the attraction SN is not much greater nor much less than the attraction SK Q E D

COR I Hence it may be easily inferred that if several less bodies P S R, &c revolve about a very great body T the motion of the innermost revolving body P will be least disturbed by the attractions of the others when the great body is as well attracted and agitated by the rest (according to the ratio of the accelerative forces) as the rest are by each other

COR II In a system of three bodies T P S if the accelerative attractions of any two of them towards a third be to each other inversely as the squares of the distances the body P by the radius PT will describe its area about the body T swifter near the conjunction A and the opposition B than it will near the quadratures C and D For every force with which the body P is acted on and the body T is not and which does not act in the direction of the line PT, does either accelerate or retard the description of the area according as its direction is the same as or contrary to that of the motion of the body Such is the force NM This force in the passage of the body P from C to A tends in the direction in which the body is moving and therefore accelerates it then as far as D it tends in the opposite direction and retards the motion then in the direction of the body as far as B and lastly in a contrary direction as it moves from B to C

COR III And from the same reasoning it appears that the body P other things remaining the same moves more swiftly in the conjunction and opposition than in the quadratures

COR IV The orbit of the body P other things remaining the same is more curved at the quadratures than at the conjunction and opposition For the swifter bodies move the less they deflect from a rectilinear path And besides the force KL or NM at the conjunction and opposition is contrary to the force with which the body T attracts the body P and therefore diminishes that force but the body P will deflect the less from a rectilinear path the less it is impelled towards the body T



COR V Hence the body P other things remaining the same goes farther from the body T at the quadratures than at the conjunction and opposition This is said however when no account is taken of the variable eccentricity For if the orbit of the body P be eccentric its eccentricity (as will be shown presently by Cor IX) will be greatest when the apsides are in the syzygies and thence it may sometimes come to pass that the body P in its near approach to the farther apse may go farther from the body T at the syzygies than at the quadratures

COR VI Because the centripetal force of the central body T by which the body P is retained in its orbit is increased at the quadratures by the addition

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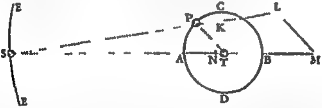
body T was diminished or increased by the increase or decrease of the action of the distant body S

Cor vii It also follows from what was before laid down that the axis of the ellipse described by the body P or the line of the apsides does as to its angular motion go forwards and backwards by turns but more forwards than backwards and by the excess of its direct motion is on the whole carried for



wards For the force with which the body P is urged to the body T at the quadratures where the force MN vanishes is compounded of the force LM and the centripetal force with which the body T attracts the body P The first force LM if the distance PT be increased is increased in nearly the same proportion with that distance and the other force decreases as the square of the ratio of the distance and therefore the sum of these two forces decreases in less than the square of the ratio of the distance PT and therefore by Cor i Prop 45 will make the line of the apsides or which is the same thing the upper apse to go backwards But at the conjunction and opposition the force with which the body P is urged toward the body T is the difference of the force KL and of the force with which the body T attracts the body P and that difference because the force KL is very nearly increased in the ratio

of these causes above the other Therefore since the force KL in the syzygies is almost twice as great as the force LM in the quadratures the excess will be on the side of the force KL and by consequence the line of the apsides will be carried forwards The truth of this and the foregoing Corollary will be more easily understood by conceiving the system of the two bodies T and P to be surrounded on every side by several bodies S S S &c disposed about the orbit ESE For by the actions of these bodies the action of the body T will be diminished on every side and decrease in more than the square of the ratio of the distance

COR VIII But since the direct or retrograde motion of the apsides depends upon the decrease of the centripetal force that is upon its being in a greater or less ratio than the square of the ratio of the distance TP in the passage of the body from the lower apse to the upper and upon a like increase in its return to the lower apse again and therefore becomes greatest where the proportion of the force at the upper apse to the force at the lower apse recedes farthest from the inverse square of the ratio of the distances it is plain that when the apsides are in the syzygies they will by reason of the subtracted force KL or NM−LM go forwards more swiftly and in the quadratures by the additional force LM go backwards more slowly Because the velocity of the progression or the slowness of the retrogression is continued for a long time this inequality becomes exceedingly great

COR IX If a body is obliged by a force inversely proportional to the square of its distance from any centre to revolve in an ellipse round that centre and afterwards in its descent from the upper apse to the lower apse that force by a continual accession of new force is increased in more than the square of the ratio of the diminished distance it is manifest that the body being impelled always towards the centre by the continual accession of this new force will incline more towards that centre than if it were urged by that force alone which decreases as the square of the diminished distance and therefore will describe an orbit interior to that elliptical orbit and at the lower apse approaching nearer to the centre than before Therefore the orbit by the accession of this new force will become more eccentric If now while the body is returning from the lower to the upper apse it should decrease by the same degrees by which it increased before the body would return to its first distance and therefore if the force decreases in a yet greater ratio the body being now less attracted than before will ascend to a still greater distance and so the eccentricity of the orbit will be increased still more Therefore if the ratio of the increase and decrease of the centripetal force be augmented with each revolution the eccentricity will be augmented also and on the contrary if that ratio decrease it will be diminished

Now therefore in the system of the bodies T P S when the apsides of the orbit PAB are in the quadratures the ratio of that increase and decrease is least of all and becomes greatest when the apsides are in the syzygies If the apsides are placed in the quadratures the ratio near the apsides is less and near the syzygies greater than the square of the ratio of the distances and from that greater ratio arises a direct motion of the line of the apsides as was just now said But if we consider the ratio of the whole increase or decrease in the progress between the apsides this is less than the square of the ratio of the distances The force in the lower is to that in the upper apse in less than the

square of the ratio of the distance of the upper apse from the focus of the ellipse to the distance of the lower apse from the same focus and conversely when the apsides are placed in the syzygies the force in the lower apse is to the force in the upper apse in a greater than the square of the ratio of the distances For the forces LM in the quadratures added to the forces of the body T compose forces in a less ratio and the forces KL in the syzygies subtracted from the forces of the body T leave the forces in a greater ratio Therefore the ratio of wh l increase and decrease in the passage between the apsides is least at

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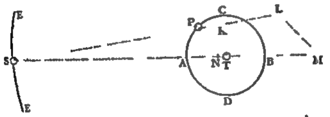

then in the passage through the next 45 degrees to the next quadrature the

nodes are in the syzygies. In their passage from the syzygies to the quadratures the inclination is diminished at each appulse of the body to the nodes and becomes least of all when the nodes are in the quadratures and the body in the syzygies then it increases by the same degrees by which it decreased before and when the nodes come to the next syzygies returns to its former magnitude

COR XI Because when the nodes are in the quadratures the body P is continually attracted from the plane of its orbit and because this attraction is made towards S in its passage from the node C through the conjunction A to the node D and in the opposite direction in its passage from the node D through the opposition B to the node C it is manifest that in its motion from the node C the body recedes continually from the former plane CD of its orbit till it comes to the next node and therefore at that node being now at its greatest distance from the first plane CD it will pass through the plane of the orbit EST not in D the other node of that plane but in a point that lies nearer to the body S which therefore becomes a new place of the node behind its former place And by a like reasoning the nodes will continue to recede in their passage from this node to the next The nodes therefore when situated in the quadratures recede continually and at the syzygies where no perturbation can be produced in the motion as to latitude are quiescent in the intermediate places they partake of both conditions and recede more slowly and therefore being always either retrograde or stationary they will be carried backwards or made to recede in each revolution

COR XII All the errors described in these Corollaries are a little greater at the conjunction of the bodies P S than at their opposition because the generating forces NM and ML are greater

COR XIII And since the causes and proportions of the errors and variations

may revolve about it And from this increase of the body S and the consequent increase of its centripetal force from which the errors of the body P arise it will follow that all these errors at equal distances will be greater in this case than in the other where the body S revolves about the system of the bodies P and T

COR XIV But since the forces NM ML when the body S is exceedingly distant are very nearly as the force SK and the ratio PT to ST conjointly that is if both the distance PT and the absolute force of the body S be given inversely as ST^3 and since those forces NM ML are the causes of all the errors and effects treated of in the foregoing Corollaries it is manifest that all those effects if the system of bodies T and P continue as before and only the distance ST and the absolute force of the body S be changed will be very nearly in a ratio compounded of the direct ratio of the absolute force of the body S and the cubed inverse ratio of the distance ST Hence if the system of bodies T and P revolve about a distant body S those forces NM ML and their effects will be (by Cor II and VI Prop 4) inversely as the square of the periodical time And thence also if the magnitude of the body S be proportional to its absolute force those forces NM ML and their effects will be directly as the cube of the apparent diameter of the distant body S viewed from T and conversely For these ratios are the same as the compounded ratio above mentioned

ratio then these forces (that is the force of the body T which obliges the body

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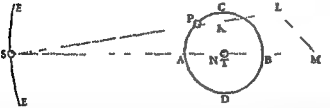
y Let these ratios be compounded with



the ratios in Cor xiv and in any system of bodies T P S where P revolves
about T very near to it and T revolves about S at a great distance the angular
errors of the body P observed from the centre T will be in each revolution of
the body P directly as the square of the periodical time of the body P and in
versely as the square of the periodical time of the body T And therefore the
...

than the radius PT let the mean quantity of the force LM be expressed by
that radius PT and then that mean force will be to the mean force SK or SN
(which may be also expressed by ST) as the length PT to the length ST But
the mean force SN or ST by which the body T is retained in the orbit it
...

periodical time of the body T about S And consequently the mean force LM
is to the force by which the body P is retained in its orbit about T (or by which

the same body P might revolve at the distance PT in the same periodical time about any immovable point T) in the same squared ratio of the periodical times The periodical times therefore being given together with the distance PT the mean force LM is also given and that force being given there is given also the force MN very nearly by the analogy of the lines PT and MN

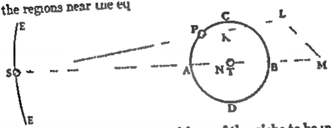
COR XVIII By the same laws by which the body P revolves about the body T let us suppose many fluid bodies to move round T at equal distances from it and to be so numerous that they may all become contiguous to each other so as to form a fluid annulus or ring of a round figure and concentric to the body T and the several parts of this ring performing their motions by the same law as the body P will draw nearer to the body T and move swifter in the conjunction and opposition of themselves and the body S than in the quadratures And the nodes of this ring or its intersections with the plane of the orbit of the body S or T will rest at the syzygies but out of the syzygies they will be carried backwards or in a retrograde direction with the greatest swiftness in the quadratures and more slowly in other places The inclination of this ring also will vary and its axis will oscillate in each revolution and when the revolution is completed will return to its former situation except only that it will be carried round a little by the precession of the nodes

COR XIX Suppose now the spherical body T consisting of some matter not fluid to be enlarged and to extend itself on every side as far as that ring and that a channel were cut all round its circumference containing water and that this sphere revolves uniformly about its own axis in the same periodical time This water being accelerated and retarded by turns (as in the last Corollary) will be swifter at the syzygies and slower at the quadratures than the surface of the globe and so will ebb and flow in its channel after the manner of the sea If the attraction of the body S were taken away the water would acquire no motion of flux and reflux by revolving round the quiescent centre of the globe The case is the same of a globe moving uniformly forwards in a right line and in the meantime revolving about its centre (by Cor V of the Laws of Motion) and of a globe uniformly attracted from its rectilinear course (by Cor VI of the same Laws) But let the body S come to act upon it and by its varying attraction the water will receive this new motion for there will be a stronger attraction upon that part of the water that is nearest to the body and a weaker upon that part which is more remote And the force LM will attract the water downwards at the quadratures and depress it as far as the syzygies and the force KL will attract it upwards in the syzygies and withhold its descent and make it rise as far as the quadratures except only so far as the motion of flux and reflux may be directed by the channel and be a little retarded by friction

COR XX If now the ring becomes hard and the globe is diminished the motion of flux and reflux will cease but the oscillating motion of the inclination and the precession of the nodes will remain Let the globe have the same axis with the ring and perform its revolutions in the same times and at its surface touch the ring within and adhere to it then the globe partaking of the motion of the ring this whole body will oscillate and the nodes will go backwards for the globe as we shall show presently is perfectly indifferent to the receiving of all impressions The greatest angle of the inclination of the ring alone is when the nodes are in the syzygies Thence in the progress of the nodes to the quadratures it endeavors to diminish its inclination and by that en

deavor impresses a motion upon the whole globe The globe retains this motion impressed till the ring by a contrary endeavor destroys that motion and impresses a new motion in a contrary direction And by this means the greatest motion of the decreasing inclination happens when the nodes are in the quadratures and the least angle of inclination in the octants after the quadratures and again the greatest motion of the reclination happens when the nodes are ... le of inclination in the octants following

matter in the regions near the eq



though we should suppose the centripetal force of this globe to be increased in any manner so that all its parts tend downwards as the parts of our earth gravitate to the centre yet the phenomena of this and the preceding Corollary would scarce be altered except that the places of the greatest and least height of the water will be different for the water is now no longer sustained and kept in its orbit by its centrifugal force but by the channel in which it flows. And besides the force LM attracts the water downwards most in the quadratures and the force KL or NM−LM attracts it upwards most in the syzygies. And

quadratures excepting only so far as the motion of ascent or descent ... e a little

quatorial

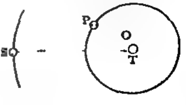
depressed or of a rarer consistence near the equator than near the poles there will arise a direct motion of the nodes

COR. XXII And thence from the motion of the nodes is known the constitution of the globe That is if the globe retains unalterably the same poles and the motion (of the nodes) is retrograde there is a redundance of the matter near the equator but if that motion is direct a deficiency Suppose a uniform and exactly spherical globe to be first at rest in a free space then by some impulse made obliquely upon its surface to be driven from its place and to receive a motion partly circular and partly straight forward Since this globe is perfectly indifferent to all the axes that pass through its centre nor has a

greater propensity to one axis or to one situation of the axis than to any other it is manifest that by its own force it will never change its axis or the inclination of its axis Let now this globe be impelled obliquely by a new impulse in the same part of its surface as before and since the effect of an impulse is not at all changed by its coming sooner or later it is manifest that these two impulses successively impressed will produce the same motion as if they had been impressed at the same time that is, the same motion as if the globe had been impelled by a simple force compounded of them both (by Cor II of the Laws) that is a simple motion about an axis of a given inclination And the case is the same if the second impulse were made upon any other place of the equator of the first motion and also if the first impulse were made upon any place in the equator of the motion which would be generated by the second impulse alone and therefore also when both impulses are made in any places whatsoever for these impulses will generate the same circular motion as if they were impressed together and at once in the place of the intersections of the equators of those motions which would be generated by each of them separately Therefore a homogeneous and perfect globe will not retain several motions distinct but will unite all those that are impressed on it and reduce them into one revolving as far as in it lies always with a simple and uniform motion about one single given axis with an inclination always invariable And the inclination of the axis or the velocity of the rotation will not be changed by centripetal force For if the globe be supposed to be divided into two hemispheres by any plane whatsoever passing through its own centre and the centre to which the force is directed that force will always urge each hemisphere equally and therefore will not incline the globe to any side with respect to its motion round its own axis But let there be added anywhere between the pole and the equator a heap of new matter like a mountain and this by its continual endeavor to recede from the centre of its motion will disturb the motion of the globe and cause its poles to wander about its surface describing circles about themselves and the points opposite to them Neither can this enormous deviation of the poles be corrected otherwise than by placing that mountain either in one of the poles in which case by Cor XXI the nodes of the equator will go forwards or in the equatorial regions in which case by Cor XX the nodes will go backwards or lastly by adding on the other side of the axis a new quantity of matter by which the mountain may be balanced in its motion and then the nodes will either go forwards or backwards as the mountain and this newly added matter happen to be nearer to the pole or to the equator

PROPOSITION 67 THEOREM 27

The same laws of attraction being supposed I say that the exterior body S *does by radii drawn to the point* O *the common centre of gravity of the interior bodies* P *and* T *describe round that centre areas more proportional to the times and an orbit more approaching to the form of an ellipse having its focus in that centre than it can describe round the innermost and greatest body* T *by radii drawn to that body*



For the attractions of the body S towards T and P compose its absolute attraction which is more directed towards O the common centre of gravity of the bodies T and P than it is to the

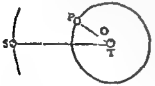
greatest body T and which approaches nearer to the inverse proportion of the square of the distance SO than of the square of the distance ST as will easily appear by a little consideration

PROPOSITION 68 THEOREM 28

The same laws of attraction supposed I say that the exterior body S will by radii ... *common centre of gravity of the interior bodies* P *and* T *describe* ... *and an orbit more approach* ... *centre if the innermost and greatest body be agitated by these attractions as well as the rest than it would do if that body either were at rest and not attracted at all or were much more or much less attracted or were much more or much less agitated*

This may be demonstrated after the same manner as Prop 66 but by a more prolix reasoning which I therefore pass over It will be sufficient to con ... manner From the demonstration of the last Proposition it ...

bodies. If this centre ... the common centre of gravity of all the three bodies were at rest the body S on one side and the common centre of gravity of the other two bodies on the other side would describe true ellipses about that quiescent common centre This appears from Cor II Prop 58 compared with what was demonstrated in Props 64 and 65 Now this accurate elliptical motion will be disturbed a little by the distance of the centre of the two bodies from the centre towards which the third body S is attracted Let ...



agitated

COR. And hence if several smaller bodies revolve about the great one it may ...

(that is if the focus of the first and innermost orbit be placed in the centre of gravity of the greatest and innermost body the focus of the second orbit in the common centre of gravity of the two innermost bodies the focus of the third orbit in the common centre of gravity of the three innermost and so on) than if the innermost body were at rest and was made the common focus of all the orbits.

PROPOSITION 69 THEOREM 29

In a system of several bodies A B C D &c *if any one of those bodies as* A *attract all the rest* B C D &c *with accelerative forces that are inversely as the squares of the distances from the attracting body and another body as* B *attracts also the rest* A C D &c *with fo*

and B *to which those forces belong*

For the accelerative attractions of all the bodies B C D towards A are by the supposition equal to each other at equal distances and in like manner the accelerative attractions of all the bodies towards B are also equal to each other at equal distances But the absolute att
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towarc ... of the body A towards B But the accelerative attraction of the body B towards A is to the accelerative attraction of the body A towards B as the mass of the body A is to the mass of the body B because the motive forces which (by the second seventh and eighth Definitions) are as the accelerative forces and the bodies attracted conjointly are here equal to one another by the third Law Therefore the absolute attractive force of the body A is to the absolute attractive force of the body B as the mass of the body A is to the mass of the body B Q E D

COR I Therefore if each of the bodies of the system A B C D &c does singly attract all the rest with accelerative forces that are inversely as the squares of the distances from the attracting body the absolute forces of all those bodies will be to each other as the bodies themselves

COR II By a like reasoning if each of the bodies of the system A B C D &c does singly attract all the rest with accelerative forces which are either inversely or directly in the ratio of any power whatever of the distances from the attracting body or which are defined by the distances from each of the attracting bodies according to any common law it is plain that the absolute forces of those bodies are as the bodies themselves

COR III In a system of bodies whose forces decrease a
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will be either accura
versely This appear ... compared with the first Corollary of this Proposition

SCHOLIUM

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SECTION XII

THE ATTRACTIVE FORCES OF SPHERICAL BODIES

PROPOSITION 70 THEOREM 30

If to every point of a spherical surface there tend equal centripetal forces decreasing as the square of the distances from those points I say that a corpuscle placed within that surface will not be attracted by those forces any way

t and P a corpuscle placed within

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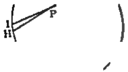


PROPOSITION 71 THEOREM 31

The same things supposed as b e I say that a corpuscle placed without the spherical su face is ttracted towa ds the centre of the sphere with a force inversely

it the centres
uate without

PROPOSITION 69 THEOREM 29

In a system of several bodies A B C D &c if any one of those bodies as A attract all the rest B C D &c with accelerative forces that are inversely as the squares of the distances from the attracting body and another body as B attracts also the rest A C D &c with forces that are inversely as the squares of the distances from the attracting body the absolute forces of the attracting bodies A and B will be to each other as those very bodies A and B to which those forces belong

For the accelerative attractions of all the bodies B C D towards A are by the supposition equal to each other at equal distances and in like manner the accelerative attractions of all the bodies towards B are also equal to each other at equal distances But the absolute attractive force of the body A is to the absolute attractive force of the body B as the accelerative attraction of all the bodies towards A is to the accelerative attraction of all the bodies towards B at equal distances and so is also the accelerative attraction of the body B *towards A to the accelerative attraction of the body A towards B But the* accelerative attraction of the body B towards A is to the accelerative attraction of the body A towards B as the mass of the body A is to the mass of the body B because the motive forces which (by the second seventh and eighth Definitions) are as the accelerative forces and the bodies attracted conjointly are here equal to one another by the third Law Therefore the absolute attractive force of the body A is to the absolute attractive force of the body B as the mass of the body A is to the mass of the body B Q E D

COR I Therefore if each of the bodies of the system A B C D &c does singly attract all the rest with accelerative forces that are inversely as the squares of the distances from the attracting body the absolute forces of all those bodies will be to each other as the bodies themselves

COR II By a like reasoning if each of the bodies of the system A B C D &c does singly attract all the rest with accelerative forces which are either inversely or directly in the ratio of any power whatever of the distances from the attracting body or which are defined by the distances from each of the attracting bodies according to any common law it is plain that the absolute forces of those bodies are as the bodies themselves

COR III In a system of bodies whose forces decrease as the square of the distances if the lesser revolve about one very great one in ellipses having their common focus in the centre of that great body and of a figure exceedingly accurate and moreover by radii drawn to that great body describe areas proportional to the times exactly the absolute forces of those bodies to each other will be either accurately or very nearly in the ratio of the bodies And so conversely This appears from Cor of Prop 68 compared with the first Corollary of this Proposition

SCHOLIUM

These Propositions naturally lead us to the analogy there is between centripetal forces and the central bodies to which those forces are usually directed, for it is reasonable to suppose that forces which are directed to bodies should depend upon the nature and quantity of those bodies as we see they do in
... when such cases occur we are to compute the
... signing to each of their particles its proper force
... hem all I here use the word *attraction* in general

PROPOSITION 72 THEOREM 32

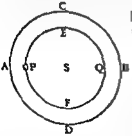
If the several points of a sphere there tend equal centripetal forces decreasing as ... and there be given both the density of ...

... ally attracted by two spheres one by ... the centres of the spheres ... the spheres ... corpuscles. ... icles of one ... alogous par ... particles ... ne ... ne ... d of ... centres of the spheres ... will be equal ... the distances will be ... appear from Cor III

... will be to each

PROPOSITION 73 THEOREM 33

If to the several points of a given sphere there tend equal centripetal forces decreasing as the square of the distances from the points I say that a corpuscle placed within the sphere is attracted by a force proportional to its distance from the centre



In the sphere ACBD described about the centre S let there be placed the corpuscle P and about the same centre S with the interval SP conceive de-

... being destroyed by contrary attractions There remains therefore only the attraction of the interior sphere PEQF And (by Prop 72) this is as the distance PS

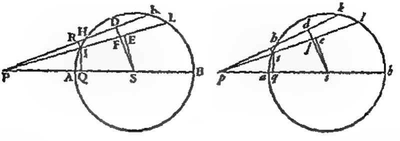
Q E D

SCHOLIUM

By the surfaces of which I here imagine the solids composed I do not mean surfaces purely mathematical but orbs so extremely thin that their thickness is as nothing that is the evanescent orbs of which the sphere will at last con

the spheres in those diameters produced Let there be drawn from th
puscles the lines PHK PIL *phk* *pil* ...
ahb the ...
SD

... Let fall al o



to the diameters the ... igles DPE *dpe*
vanish and becau ... PE PF and *pe*
pf and the short li ... be taken for equal because their last ratio
when the angles DPE *dpe* vanish together is the ratio of equality The e
things being thus determined it follows that

$$PI \quad PF = RI \quad DF$$

and
$$pf \quad pi = df \text{ or } DF \quad ri$$

Multiplying corresponding terms

$$PI\ pf \quad PF\ pi = RI \quad ri = \text{arc } HI \quad \text{arc } ih \text{ (by Cor III Lem VII)}$$

Again
$$PI \quad PS = IQ \quad SF$$

and
$$ps \quad pi = se \text{ or } SE \quad iq$$

Hence
$$PI\ ps \quad PS\ pi = IQ \quad iq$$

Multiplying together corresponding terms of this and the similarly derived preceding proportion

$$PI^2\ pf\ ps \quad pi^2\ PF\ PS = HI\ IQ \quad \ldots$$

that is as the ...
circle AKB
by the arc

... by the hypothesis as the surfaces
... res of the distances of the surfaces from
... PF PS And these forces again are to the
... parts of them which (by the ...
Laws)
pi to *pi*
to PF ... corpuscle I towards S is to
the attraction of the corpuscle *p* towards *s* as $\frac{PF\ pf\ ps}{IS}$ is to $\frac{pf\ PF\ IS}{ps}$ that
is as ps^2 to PS^2 And by a like reasoning the forces with which the surfaces described by the revolution of the arcs KL *kl* attract those corpuscles will be as ps^2 to PS^2 And in the same ratio will be the forces of all the circular surfaces into which each of the spherical surfaces may be divided by taking *sd* always equal to SD and *se* equal to SE And therefore by composition the forces of the entire spherical surfaces exerted upon those corpuscles will be in the same ratio

Q E D

h h v themselves are attracted by the others again
...ated and attracting
...g their mutual

...ing the motion
...hen an attract-
of bodies about the focus of ... and the bodies move without the sphere
ing sphere is placed in the focus and the bodies move without the sphere

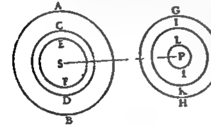
COR. IV Those things which were demonstrated before of the motion of bodies about the centre of the conic sections take place when the motions are performed within the sphere

PROPOSITION 76 THEOREM 36

... density of matter and attractive force) in ... different, but everywhere similar ... attractive ... attracted ... her will be

I say that the ... inversely proportional to the square of the distance of the centres

Imagine several concentric similar spheres AB CD EF &c the innermost of which added to the outermost may compose a matter more dense towards the centre or subtracted from them may leave the same more lax and rare Then, by Prop 75 these spheres will attract other ... similar concentric spheres GH IK, LM &c each the other with forces inversely proportional to the square of the distance SP And, by addition or subtraction the sum of all those forces or the excess of any of them above the others that is the entire force with which the whole sphere AB (composed of any ... h ... sphere GH ... ratio Let ... that the



density of the matter together with the attr... the progress ... increase or decrease according to any ... not attractive let the deficient density acquire any form desired and the other will be still, by the former reasoning in the same inverse ratio of the square of the distance Q.E.D

COR. I Hence if many spheres of this kind, similar in all respects, attract ...

and attracted spheres that is as the products arising from multiplying the spheres into each other

sist when the number of the orbs is increased and their thickness diminished without end In like manner by the points of which lines surfaces and solids are said to be composed are to be understood equal particles whose magnitude is perfectly inconsiderable

PROPOSITION 74 THEOREM 34

The same things supposed I say that a corpuscle situated without the sphere is attracted with a force inversely proportional to the square of its distance from the centre

For suppose the sphere to be divided into innumerable concentric spherical surfaces and the attractions of the corpuscle arising from the several surfaces will be inversely proportional to the square of the distance of the corpuscle from the centre of the sphere (by Prop 71) And by composition the sum of those attractions that is the attraction of the corpuscle towards the entire sphere will be in the same ratio Q E D

COR I Hence the attractions of homogeneous spheres at equal distances from the centres will be as the spheres themselves For (by Prop 72) if the distances be proportional to the diameters of the spheres the forces will be as the diameters Let the greater distance be diminished in that ratio and the distances now being equal the attraction will be increased as the square of that ratio and therefore will be to the other attraction as the cube of that ratio that is in the ratio of the spheres

COR II At any distances whatever the attractions are as the spheres applied to the squares of the distances

COR III If a corpuscle placed without an homogeneous sphere is attracted by a force inversely proportional to the square of its distance from the centre and the sphere consists of attractive particles the force of every particle will decrease as the square of the distance from each particle

PROPOSITION 75 THEOREM 35

If to the several points of a given sphere there tend equal centripetal forces decreasing as the square of the distances from the point I say that another similar sphere will be attracted by it with a force inversely proportional to the square of the distance of the centres

For the attraction of every particle is inversely as the square of its distance from the centre of the attracting sphere (by Prop 74) and is therefore the same as if that whole attracting force issued from one single corpuscle placed in the centre of this sphere But this attraction is as great as on the other hand the attraction of the same corpuscle would be if that were itself attracted by the several particles of the attracted sphere with the same force with which they are attracted by it But that attraction of the corpuscle would be (by Prop 74) inversely proportional to the square of its distance from the centre of the sphere therefore the attraction of the sphere equal thereto is also in the same ratio Q E D

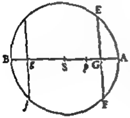
COR II The case is the same when the attracted sphere does also attract For the several points of the one attract the several points of the other with

on each side from the centre of the phere are as the um of those planes mul tipl ed by the di tance PS that is as the whole sphere and the distance PS conjointly Q E D

CASE 2 Let now the corpuscle P attract the phere AEBF And by the same rea onin it will appear that the force with wl ich tl e sphere i attracted is as the di tance PS Q E D

C SE 3 Imagine another phere composed of innumerable corpuscles P and because the force with which every corpuscle is attracted is as the di tance of ... first phere and as the same sphere con

... attracted will be the ... of the first phere and is there ... e ... entres of the pheres Q E D ... her and the force will be doubled but tl e proportion will remain Q E D ... placed within the phere AEBF and because ... hat ... ld ... the ... e of ... anes multiplied by half the difference of ... nces that is, as that sum multiplied by pS the di tance of the corpuscle from the centre of the sphere And by a l ke reasoning the attraction of all the planes EF ef throu hout the hole phere that i the attraction of the whole pl ere is conjointly a the sum of



cles u h as p ituated within the first phere A... ... before that the attraction whether sin le of one phere towards the other or mutual of both to ards each other will be as the distance pS of the centres Q E D

PROPOSITION 78 THEOREM 38

If spheres in the progress from the cent e to the ci cumference be however d ssimilar and ... eq ble but similar on every side round about at all given di tances from ... the distance of the attracted

This ... demon trated from the fore oing Proposition in the same manner as Prop 76 was demon trated from Prop 75

COR Those things that were above demonstrated in Props. 10 and 64 of th moti n of bodies round the centres of conic sections take place when all the attractions are made by the force of spherical bodies of tl e condition above described nd the attracted bodies are pl eres of the same kind

COR IV And at unequal distances directly as those products and inversely as the squares of the distances between the centres

COR V These proportions hold true also when the attraction arises from the attractive power of both spheres exerted upon each other For the attraction is only doubled by the conjunction of the forces the proportions remaining as before

COR VI If spheres of this kind revolve about others at rest each about each and the distances between the centres of the quiescent and revolving bodies are proportional to the diameters of the quiescent bodies the periodic times will be equal

COR VII And again if the periodic times are equal the distances will be proportional to the diameters

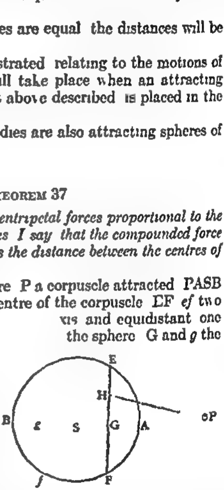
COR VIII All those truths above demonstrated relating to the motions of bodies about the foci of conic sections will take place when an attracting sphere of any form and condition like that above described is placed in the focus

COR IX And also when the revolving bodies are also attracting spheres of any condition like that above described

PROPOSITION 77 THEOREM 37

If to the several points of spheres there tend centripetal forces proportional to the distances of the points from the attracted bodies I say that the compounded force with which two spheres attract each other is as the distance between the centres of the spheres

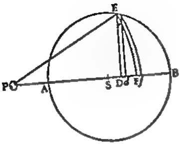
CASE 1 Let AEBF be a sphere S its centre P a corpuscle attracted PASB the axis of the sphere passing through the centre of the corpuscle EF *ef* two planes cutting th... ...xis and equidistant one on one side ... the sphere G and *g* the intersection ... and H any point in the plane EF The centripetal force of the point H upon the corpuscle P exerted in the direction of the line PH is as the distance PH and (by Cor II of the Laws) the same exerted in the direction of the line PG or towards the centre S is as the length PG Therefore the force of all the points in the plane EF



(that is of that whole plane) by which the corpuscle P is attracted towards the centre S is as the distance PG multiplied by the number of those point that is as the solid contained under that plane EF and the distance PG And ... ...e corpuscle P is attracted ... its distance P*g* or as the ... the sum of the forces of ... of the distances PG+P*g*

... the forces of all the planes in the whole sphere equidistant

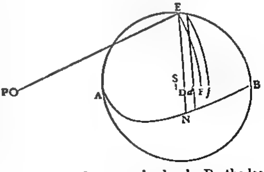
f the arc rE the same ylinder And or Pr ituated face will be as ... nular surface ... that is as the ... the ... the ... orce ... end ... ratio ... Dd



PD to PE and therefore ... Suppose now the line DF to be divided into innumerable little equal particles each of which call Dd and then the surface FE will be divided into o many equal annuli whose forces will be as the um ... les PD Dd that is as $\frac{1}{2}PF^2-\frac{1}{2}PD$ and therefore as DE ... Ff ... d the force of the ... force ... at the

PROPOSITION 80 THEOREM 40

... entre S there tend ... of the sphere AB ... culars DE ... eet ... be taken as the quantity $\frac{DE ...}{PE}$ and as the force which a particle ... re situated in the axis exerts at the dist nce PE upon the corpuscle P conjointly I say that the whole force with which the corpuscle P is attracted towards the sphere is as the area ANB comprehended under the axis of the sphere AB and the curved line ANB the locus of the point N



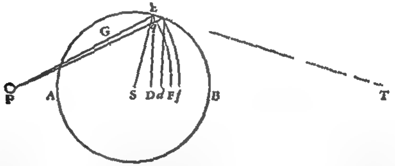
For supposing the construction in the last Lemma and Theorem to stand conceive the axis of the sphere AB to be divided into innumerable equal particles Dd and the whole sphere to be divided into so many spherical concavo convex laminæ EFfe and erect the perpendicular dn By the last Theorem the force with which the laminæ EFfe attract the corpuscle P is as DE^2 Ff and the force of one particle exerted at the distance PE or PF conjointly But (by the last Lemma) Dd is to Ff as PE to PS and therefore Ff is

SCHOLIUM

I have now explained the two principal cases of attractions to wit when the centripetal forces decrease as the square of the ratio of the distances or increase in a simple ratio of the distances causing the bodies in both cases to revolve in conic sections and composing spherical bodies whose centripetal forces observe the same law of increase or decrease in the recess from the centre as the forces of the particles themselves do which is very remarkable It would be tedious to run over the other cas
elegant
comprel

LEMMA 29

If about the centre S there be described any circle a AEB and there h
PS *in*



the distance of the arcs EF *ef be supposed to be infinitely diminished the last ratio of the evanescent line* Dd *to the evanescent line* Ff *is the same as that of the line* PE *to the line* PS

For if the line Pe cut the arc EF in q and the right line Ee which coincides with the evanescent arc Ee be produced and meet the right line PS in T and there be let fall from S to PE the perpendicular SG then because of the like triangles DTE dTe DES

$$Dd : Ee = DT : TE = DE : ES$$

and because the triangles Eeq ESG (by Lem 8 and Cor III Lem 7) are similar $Ee : eq$ or $Ff = Es : SG$

Multiplying together corresponding terms of the two proportions

$$Dd : Ff = DE : SG = PE : PS$$

(because of the similar triangles PDE PGS) Q E D

PROPOSITION 79 THEOREM 39

Suppose a surface as EFfe *to have its breadth infinitely diminished and to be just vanishing and that the same surface by its revolution round the axis* PS

in the place I *would attract the same*

corpuscle

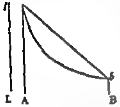
h is gen
the line

the quantity $\frac{DE^2 \cdot PS}{PE \cdot V}$ which (by Cor. [illegible] of the foregoing Prop.) is as the length of the ordinate DN, will now resolve itself into three parts

$$\frac{2SLD \cdot PS}{PE \cdot V} - \frac{LD \cdot PS}{PE \cdot V} - \frac{ALB \cdot PS}{PE \cdot V}$$

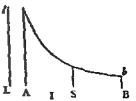
where if instead of V we write the inverse ratio of the centripetal force, and instead of PE the mean proportional between PS and 2LD, those three parts will become ordinates to so many curved lines, whose areas are discovered by the common methods. Q.E.D.

EXAM. 1. If the centripetal force tending to the several particles of the sphere be inversely as the distance, instead of V write PE the distance, then 2PS · LD for PE^2, and DN will become as $SL - \frac{1}{2}LD - \frac{LA \cdot LB}{2LD}$. Suppose DN equal to its double $2SL - LD - \frac{LA \cdot LB}{LD}$ … drawn into the



… in its motion one way or another, it may either by increasing or decreasing remain always equal to the length LD, will describe the area $\frac{LB^2 - LA^2}{2}$, that is, the area SL · AB; which taken from the former area 2SL · AB leaves the area SL · AB. But the third part $\frac{LA \cdot LB}{LD}$ drawn after the …

… cube applied to any given plane, write $\frac{PE^3}{2AS}$ for V, and 2PS · LD for PE, and DN will become as



$$\frac{SL \cdot AS}{PS \cdot LD} - \frac{AS}{2PS} - \frac{LA \cdot LB \cdot AS^2}{2PS \cdot LD}$$

that is (because PS, AS, SI are continually proportional) as

$$\frac{LSI}{LD} - \frac{1}{2}SI - \frac{LA \cdot LB \cdot SI}{2LD}$$

If we draw then these three parts into the length AB, the first $\frac{SL \cdot SI}{LD}$ will generate the area of an hyperbola; the second $\frac{1}{2}SI$ the area $\frac{1}{2}AB \cdot SI$; the third $\frac{LA \cdot LB \cdot SI}{2LD}$ the area $\frac{LA \cdot LB \cdot SI}{2LA} - \frac{LA \cdot LB \cdot SI}{2LB}$, that is $\frac{1}{2}AB \cdot SI$. From the first subtract the sum of the second and third, and there will remain ANB the area

equal to $\frac{PS \cdot Dd}{PE}$, and DE · Ff is equal to Dd $\frac{DE^2 \cdot PS}{IL}$ and therefore the force of the lamina EFfe is as Dd $\frac{DE^2 \cdot PS}{PE}$ and the force of a particle exerted at the distance PF conjointly that

[illegible]

always the same at [illegible] be made as $\frac{\quad}{PE}$ the whole force with which the corpuscle is attracted by the sphere is as the area ANB

COR. II If the centripetal force of the particles be inversely as the distance of the corpuscle attracted by it and DN be made as $\frac{DE^2 \cdot PS}{IL^2}$ the force with which the corpuscle P is attracted by the whole sphere will be as the area ANB

COR. III If the centripetal force of the particles be inversely as the cube of the distance of the corpuscle attracted by it and DN be made as $\frac{DE \cdot PS}{IL^4}$ the force with which the corpuscle is attracted by the whole sphere will be as the area ANB

COR. IV And [illegible] ticles of the [illegible] made as $\frac{DI}{PL \cdot V}$ [illegible] with which a corpuscle is attracted by the whole sphere will be as the area ANB

PROPOSITION 81 PROBLEM 41

The things remaining as above it is required to measure the area ANB

From the point P let there be drawn [illegible]

II [illegible]

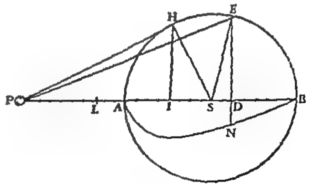
(by [illegible]

But [illegible]

SHI are alike SL or SH² is equal to the rectangle PS IS Therefore PE is equal to the rectangle contained under PS and PS+SI+2SD that is under PS and 2LS+2SD that is under PS and 2LD Moreover DE is equal to SE²−SD² or

SE −LS +2LS LD−LD²

that is

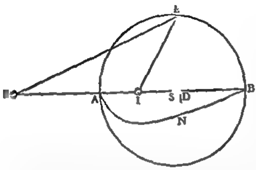


$$2LS \cdot LD - LD^2 - LA \cdot LB$$

For LS −SE² or LS²−SA² (by Prop 5 Book II *Elements* of Euclid) is equal to the rectangle LA LB Therefore if instead of DE² we write

$$2LS \cdot LD - LD^2 - LA \cdot LB$$

of the distance SI to the distance SP and the square root of the ratio of the ... f om any particle in the centre to the ...



of ratios compose ... f equality and therefore the attractions in I and P produced by the whole sphere are equal By the like calculation if the forces of the particles of the sphere are inversely as the square of the ratio of the distance it will be found that the attraction in I is to the attraction in P as the di tance SP to the semidiameter SA of the sphere If those forces are inversely as the cube of the ratio of the di tances the attractions in I and P will be to each other as SP^2 to SA if as the fourth power of the ratio as SP^3 to SA Therefore since ... f und in this last case to be inversely as PS PI the at ... because SA^3 i given in ... in infinitum The demonstration of this Theorem is as follows

The things remaining as above constructed and a corpuscle being in any place P the ordinate DN was found to be as $\frac{DE^2 \cdot PS}{PE \cdot V}$ Therefore if IE be drawn that ordinate for any other place of the corpuscle as I will become (other things being equal) as $\frac{DE^2 \cdot IS}{IE \cdot V}$ Suppose the centripetal forces flowing from any po nt of the sphere as E to be to each other at the distances IE and PE as PE^n to IE^n (where the number n denotes the index of the powers of PE and IE) and those ordinates will become as $\frac{DE^2 \cdot PS}{PE \cdot IE^n}$ and $\frac{DE^2 \cdot IS}{IE \cdot IE^n}$ whose ratio ...

which the ordinates describe and the attractions proportional to them are in a ratio compounded of the square root of those ratios Q E D

PROPOSITION 83 PROBLEM 42

To find the force with which a corpuscle placed in the centre of a sphere is attracted toward any segment of that sphere whatsoever

sought Whence arises this construction of the Problem At the points L A S B erect the perpendiculars Ll Aa Ss Bb of which suppose Ss equal to SI and thro

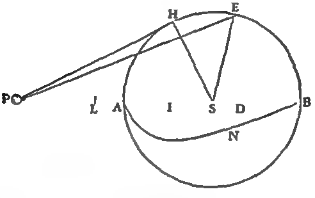
meeting

tracted f

EXAM

spheres decrease as the fourth power of the distance from the particles write $\frac{PE^4}{2AS^3}$ for V then $\sqrt{(2PS+LD)}$ for PE and DN will become as

$$\frac{SI^2\,SL}{\sqrt{2SI}}\,\frac{1}{\sqrt{LD^3}}-\frac{SI^2}{2\sqrt{2SI}}\,\frac{1}{\sqrt{LD}}-\frac{SI^2\,LA\,LB}{2\sqrt{2SI}}\,\frac{1}{\sqrt{LD^5}}$$



These three parts drawn into the length AB produce so many areas viz $\frac{2SI^2\,SL}{\sqrt{2SI}}$ into $\left(\frac{1}{\sqrt{LA}}-\frac{1}{\sqrt{LB}}\right)$ $\frac{SI^2}{\sqrt{2SI}}$ into $\sqrt{(LB-\sqrt{LA})}$ and $\frac{SI\ \ LA\ LB}{3\sqrt{2SI}}$ into $\left(\frac{1}{\sqrt{LA^3}}-\frac{1}{\sqrt{LB^3}}\right)$ And these after due reduction come forth $\frac{2SI^2\,SL}{LI}$ SI and $SI^2+\frac{2SI^3}{3LI}$ And these by subtracting the last from the first become $\frac{4SI^3}{3LI}$ Therefore the entire force with which the corpuscle P is attracted towards the centre of the sphere is as $\frac{SI^3}{PI}$ that is inversely as PS^3 PI Q E I

By the same method one may determine the attraction of a corpuscle situated within the sphere but more expeditiously by the following Theorem

PROPOSITION 82 THEOREM 41

In a sphere described about the centre S with the radius SA if there be taken SI SA SI continually proportional I say that the attraction of a corpuscle within the … *place P in a ratio* … *es from the centre* … *the centre in those* *places I a a*

As if the centripetal forces of the particles of the sphere be inversely as the distances of the corpuscle attracted by them the force with which the corpuscle situated in I is attracted by the entire sphere will be to the force with which it is attracted in P in a ratio compounded of the square root of the ratio

very small interval the forces of the particles of the attracting body decrease in the rec[illegible] body attracted in more than the squared ratio of the distance of the p[illegible]

[illegible]

acting external bodies ie placed within them ies of those orbs are (by Prop [illegible]0) destroyed by [illegible] fore have no effect even [illegible] if from these spheres and spherical orbs we take [illegible] part anywhere [illegible] it th part added or taken away being [illegible] ill cause no remarkable excess of the attraction arising from the contact of the two bodies. Therefore the Proposition holds good in bodies of all figures Q.E.D

PROPOSITION 86 THEOREM 43

If the forces of the particles of which an attractive body is composed decrease in the [illegible] of the attracting body as the third or more than the third power of the distance [illegible] the particles the attraction will be vastly stronger in the point of contact than when the attracting and attracted bodies are separated from each other though by ever so small an interval

For that the attraction is infinitely increased when the attracted corpuscle comes to touch an attracting sphere of this kind, appears, by the solution of Problem 41 exhibited in the second and third Examples. The same will also appear (by comparing those Examples and Theor 41 together) of attractions [o]f bodies made towards concavoconvex orbs whether the attracted bodies be placed [wi]thout the orbs or in the cavities within them And by adding to or [ta]king from those spheres and orbs any attractive matter anywhere without th[e] place [o]f contact so that the attractive bodies may receive any assigned fi[gu]re th[e] Proposition will hold good of all bodies universally Q.E.D

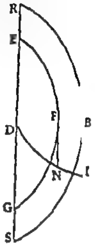
PROPOSITION 87 THEOREM 44

[illegible] small attractive [illegible] n [illegible] ill be the accelerative attractions of the corpuscles towards [illegible] ies proport[ion]al [to] the whole and similarly situated in them

[For] if the bodies are divided into particles proportional to the wholes and al[i]k[e si]tuated in them it will be as the attraction towards any particle of one [o]f th[e] [bo]dies to the attraction towards the correspondent particle in the other [b]ody so are the attractions towards the several particles of the first body to th[e a]ttractions towards the several correspondent particles of the other body [An]d by composition so is the attraction towards the first whole body to the [a]ttraction towards the second whole body Q.E.D

ment be divided into the parts BREFGS FEDG Let us suppose that segment to be not a n el m h - face having some but a perfectly ... be called ... onstrated ... suppose h ... of the sph... o be inversely as that power of the distances of which n is index and the force with which the surface EFG attracts the body P will be (by Prop 79) as $\frac{DE^2 \, O}{PF}$ that is as $\frac{2DF \, O}{PF^{n-1}} - \frac{DF^2 \, O}{PF^n}$ Let the perpendicular FN multiplied by O be proportional to this quantity and the curvilinear area BDI which the ordinate FN drawn through the len... DP... be RE ... the body P Q E I

PO

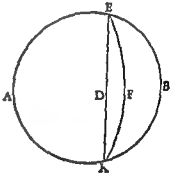


PROPOSITION 84 PROBLEM 43

To find the force with which a corpuscle placed without the centre of a sphere in the axis of any segment is attracted by that segment

Let the body P placed in the axis ADB of the segment EBK be attracted by that segment About the centre P with the radius PE let the spherical surface EFK be described and let it divide the segment into two parts EBKFE and EFKDE Find the force of the first of those parts by Prop 81 and the force of the latter part by Prop 83 and the sum of the forces will be the force of the whole segment EBKDE Q E I

PO



SCHOLIUM

The attractions of spherical bodies being now explained it comes next in order to treat of the laws of attraction in other bodies consisting in like manner of attractive particles but to treat of them particularly is not necessary to my design It will be sufficient to add some general Propositions relating to the forces of such bodies and the motions thence arising because the knowledge of these will be of some little use in philosophical inquiries

SECTION XIII

THE ATTRACTIVE FORCES OF BODIES WHICH ARE NOT SPHERICAL

PROPOSITION 85 THEOREM 42

lobe in G and
hree particles
vere placed in
may go on in
ody whatever

to put on the form of a g…

Cor. Hence the motion of the attracted body Z will be the same as if the … and therefore if that attracting body be … will move … body

PROPO…ITION 89 THEOREM …

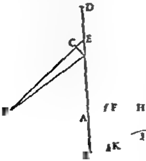
… particles whose forces are as the dis-

… ne as if the attractin bodies preserving their comm… …ould unite there and be formed into a globe And therefore if the common centre of gravity of the attracting bodies be either at rest or proceed uniformly in a right line the attracted body will move in an ellipse having its centre in the common centre of gravity of the attracting bodies

PROPOSITION 90 PROBLEM 44

If t… the several poi…ts …f any circle there tend equal centripetal forces increasing or decr…as…ng …i… any …atio …f the di…tances it is required to find the force with which a corp…scl… …m…a… acted that is situated anywhere in a right line which stands at right a…gles to the plane of the ci…cle at its centre

Suppose a circle to be described about the centre A with any radius AD in a plane to which the right line AP is perpendicular and let it be required to find the force with which a corpuscle P is attracted towards the same From any point E of the circle to the attracted corpuscle P let there be drawn the right line PE In the right line PA take PF equal to PE and make a perpendicular FK erected at F to be as the force with which the point E attracts the corpuscle P And let the curved line IKL be the locus of the point K Let that curve meet the plane of the circle in L In PA take PH equal to PD and erect the perpendicular HI meeting that curve in I and the attraction of the corpuscle P towards the circle will be as the area AHIL multiplied by the altitude AP



Q E I

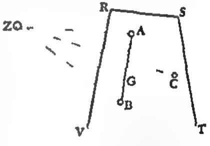
COR I Therefore if as the distances of the corpuscles attracted increase the attractive forces of the particles decrease in the ratio of any power of the distances the accelerative attractions towards the whole bodies will be directly as the bodies and inversely as those powers of the distances As if the forces of the particles decrease as the square of the distances from the corpuscles attracted and the bodies are as A^3 and B^3 and therefore both the cubic sides of the bodies and the distance of the attracted corpuscles from the bodies are as A and B the accelerative attractions towards the bodies will be as $\frac{A^3}{A^2}$ and $\frac{B^3}{B^2}$ that is as A and B *the cubic sides of those bodies If the forces of the particles* decrease as the cube of the distances from the attracted corpuscles the accelerative attractions towards the whole bodies will be as $\frac{A^3}{A^3}$ and $\frac{B^3}{B^3}$ that is equal If the forces decrease as the fourth power the attractions towards the bodies will be as $\frac{A^3}{A^4}$ and $\frac{B^3}{B^4}$ that is inversely as the cubic sides A and B And so in other cases

COR II Hence on the other hand from the forces with which like bodies attract corpuscles similarly situated may be obtained the ratio of the decrease of the attractive forces of the particles as the attracted corpuscle recedes from them if only that decrease is directly or inversely in any ratio of the distances

PROPOSITION 88 THEOREM 45

If the attractive forces of the equal particles of any body be as the distance of the places from the particles the force of the whole body will tend to its centre of gravity and will be the same with the force of a globe consisting of similar and equal matter and having its centre in the centre of gravity

Let the particles A B of the body RSTV attract any corpuscle Z with forces which supposing the particles to be equal between themselves are as the distances AZ BZ but if they are supposed unequal are as those particles and their distances AZ BZ conjointly or (if I may so speak) as those particles multiplied by their distances AZ BZ respectively And let those forces be expressed by the contents under A AZ and B BZ Join AB and let it be cut in G so that AG may be to BG as the particle B to the particle A and G will be the common centre of gravity of the particles A and B The force A AZ will (by Cor II of the Laws) be resolved into the forces A GZ and A AG and the force B BZ into the forces B GZ and B BG Now the forces A AG and B BG because A is proportional to B and BG to AG are equal and therefore having contrary directions destroy one another There remain then the forces A GZ and B GZ These tend from Z towards the centre G and compose the force (A+B) GZ that is the same force as if the attractive particles A and B were placed in their common centre of gravity G composing there a little globe



By the same reasoning if there be added a third particle C and the force of it be compounded with the force (A+B) GZ tending to the centre G the force

A B C and will be the ... for globe there and so ...

to put on the form of a globe

COR Hence the motion of the attracted body Z will be the same as if the body RSTV were spherical and therefore if that attracting body be ... ted will move ... racting body

PROPOSITION 89 THEOREM 46

... f equal particles whose forces are as the dis-

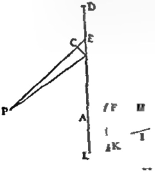
centre of gravity should unite there and ...

... manner as the foregoing Proposition ... will be the same as if the ... e of gravity should unite there and be formed into a globe And ... if the common centre of gravity of the attracting bodies be either at rest or proceed uniformly in a right line the attracted body will move in an ellipse having its centre in the common centre of gravity of the attracting bodies

PROPOSITION 90 PROBLEM 44

right angles to the plane of the circle at its centre

Suppose a circle to be described about the centre A with any radius AD in a plane to which the right line AP is perpendicular and let it be required to find the force with which a corpuscle P is attracted towards the same From any point E of the circle to the attracted corpuscle P let there be drawn the right line PE In the right line PA ...



the curved line IKL be the locus of the point K Let that curve meet the plane of the circle in L In PA take PH equal to PD and erect the perpendicular HI meeting that curve in I and the attraction of the corpuscle P towards the

plane attracts to itself the body P is supposed to be as FK and therefore the force with which that point attracts the body P towards A is as $\frac{AP \cdot FK}{PE}$ and the force with which the whole ring attracts the body P towards A is as the ring and $\frac{AP \cdot FK}{PE}$ conjointly and that ring also is as the rectangle under the radius AE and the breadth Ee and this rectangle (because PE and AE Ee and CE are proportional) is equal to the rectangle PE CE or PE Ff the force with which that ring attracts the body P towards A will be as PE Ff and $\frac{AP \cdot FK}{PE}$

ul
in
P
D

COR I Hence if the forces of the points decrease as the square of the distances that is if FK be as $\frac{1}{PF}$ and therefore the area AHIKL as $\frac{1}{PA}-\frac{1}{PH}$ the attraction of the corpuscle P towards the circle will be as

$$1-\frac{PA}{PH} \text{ that is as } \frac{AH}{PH}$$

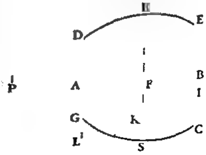
COR II And universally if the forces of the points at the distances D be inversely as any pow n D of the distances that is if FK be as $\frac{1}{D}$ and therefore the area AHIKL as $\frac{1}{PA^{-1}}-\frac{1}{PH^{-1}}$ the attraction of the corpuscle P towards the circle will be as $\frac{1}{PA^{-2}}-\frac{PA}{PH^{-1}}$

COR III And if the diameter of the circle be increased *in infinitum* and the number n be greater than unity the attraction of the corpuscle P toward the whole infinite plane will be inversely as PA^{-2} because the other term $\frac{PA}{PH^{-1}}$ vanishes

PROPOSITION 91 PROBLEM 45

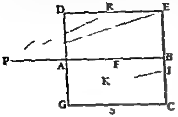
To find the attraction of a corpuscle situated in the axis of a round solid to whose several points there tend equal centripetal forces decreasing in any ratio of the distances whatsoever

Let the corpuscle P situated in the axis AB of the solid DECG be attracted towards that solid Let the solid be cut by any circle as RFS perpendicular to the axis and in its semidiameter FS in any plane PALKB passing through the axis let there be taken (by Prop 90) the length FK proportional to the force with which the corpuscle P is attracted



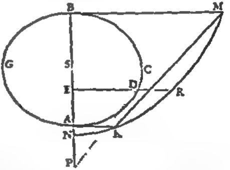
LI and the attraction of the corpuscle P towards the solid will be as the area LABI Q E I

COR. I. Hence if the solid be a cylinder described by the parallelogram ADEB revolved about the axis AB and the centripetal forces tending to the several points be inversely as the squares of the distances from the points the attraction of the corpuscle P towards this cylinder will be as AB−PE+PD For the ordinate FK (by Cor I Prop 90) will be as $1-\frac{PF}{PR}$ The part 1 of this quantity multiplied by the length AB describes the area 1 AB and the other part $\frac{PF}{PR}$ multiplied by the length PB describe the area 1 (PE−AD) (as may be easily shown from the quad

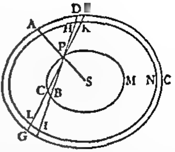


there will remain the area LABI equal to ... force being proportional to this area, i.e. as AB−PE+PD

COR II Hence also is known the force by which a spheroid AGBC attracts any body P situate externally in its axis AB Let NKRM be a conic section whose ordinate ER perpendicular to PE may be always equal to the length of the line PD continually drawn to the point D in which that ordinate cuts the spheroid. From the vertices A B of the spheroid let there be erected to its axis AB the perpendiculars AK, BM respectively equal to AP BP and therefore meeting the conic section in K and M and join KM cutting off from it the segment KMRK. Let S be the centre of the spheroid, and SC its greatest semidiameter and the force with which the spheroid attracts the body P will be to the force with which a sphere described with the diameter AB attracts the same body as $\frac{AS \cdot CS^2 - PS \cdot KMRK}{PS^2+CS^2-AS^2}$ is to $\frac{AS^3}{3PS^2}$ And by a calculation founded on the same principles may be found the forces of the segments of the spheroid



the exterior the first of which passes through the body P and cuts the right lines DE, FG in B and C the latter cuts the same right lines in H and I K and



L Let the spheroids have all one common axis and the parts of the right lines intercepted on both sides DP and BE FP and CG DH and IE FK and LG will be mutually equal because the right lines DE PB and HI are bisected in the same point as are also the right lines FG PC and KL Conceive now DPF EPG to represent opposite cones described with the infinitely small vertical angles DPF EPG and the lines DH EI to be infinitely small also Then the particles of the cones DHKF GLIE cut off by the spheroidal surfaces by reason of the equality of the lines DH and EI will be to one another as the squares of the distances from the body P and will therefore attract that corpuscle equally And by a like reasoning if the spaces DPF EGCB be divided into particles by the surfaces of innumerable similar spheroids concentric to the former and having one common axis all these particles will equally attract on both sides the body P towards contrary parts Therefore the forces of the cone DPF and of the conic segment EGCB are equal and by their opposed actions destroy each other And the case is the same of the forces of all the matter that lies without the interior spheroid PCBM Therefore the body P is attracted by the interior spheroid PCBM alone and therefore (by Cor III Prop 72) its attraction is to the force with which the body A is attracted by the whole spheroid AGOD as the distance PS is to the distance AS

Q E D

PROPOSITION 92 PROBLEM 46

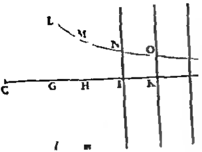
An attracting body being given it is required to find the ratio of the decrease of the centripetal forces tending to its several points

The body given must be formed into a sphere a cylinder or some regular figure whose law of attraction answering to any ratio of decrease may be found by Props 80 81 and 91 Then by experiments the force of the attractions must be found at several distances and the law of attraction towards the whole made known by that means will give the ratio of the decrease of the forces of the several parts which was to be found

PROPOSITION 93 THEOREM 47

If a solid be plane on one side and infinitely extended on all other sides and consist of equal particles equally attractive whose forces decrease in receding from the solid in the ratio of any power greater than the square of the distances and a corpuscle placed towards either part of the plane is attracted by the force of the whole solid I say that the attractive force of the whole solid in receding from its plane surface will decrease in the ratio of a power whose side is the distance of the corpuscle from the plane and its index less by 3 than the index of the power of the distances

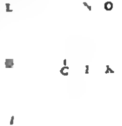
CASE 1 Let LG*l* be the plane by which the solid is terminated Let the solid lie on that side of the plane that is towards I and let it be resolved into innumerable planes *m*HM *n*IN *o*KO &c parallel to GL And first let the attracted



not less than 3 Therefore (by Cor III Prop 90) the force with which any plane mHM attracts the point C is inversely as CH^{n-2} In the plane mHM take the length HM inversely proportional to CH^{n-2} and that force will be as HM In like manner in the several planes lGL nIN oKO &c take the lengths GL IN KO &c inversely proportional to CG^{n-2} CI^{n-2} CK^{n-2} &c and the forces of those planes will be as the lengths so taken and therefore the sum of the forces as the sum of ... of the whole solid as the area GLOK produced

CG^{n-3}

CASE 2 Let the corpuscle C be now placed on that side of the plane lGL that is within the solid and take the distance CK equal to the distance CG And the part of the solid $LGloKO$ terminated by the parallel planes lGL oKO will attract the corpuscle C situated in the middle neither one way nor another the contrary actions of the opposite point destroying one another by reason of their equality Therefore the corpuscle C is attracted by the force only of the solid situated beyond the plane OK But this force (by Case 1) is inversely as CK^{n-3} that is (because CG CK are equal) inversely as CG^{n-3}



Q E D

COR I Hence if the solid LGIN be terminated on each side by two infinite ... from the

... compared with the attraction of the nearer part is inconsiderable the attraction of that nearer part will as the distance increases decrease nearly in the ratio of the power CG^{n-3}

COR III And hence if any finite body plane on one side attract a corpuscle situated over against the middle of that plane and the distance between the corpuscle and the plane compared with the dimensions of the attracting body be extremely small and the attracting body consist of homogeneous particles whose attractive forces decrease in the ratio of any power of the distances greater than the fourth the attractive force of the whole body will decrease very nearly in the ratio of a power whose side is that very small distance and the index less by 3 than the index of the former power This assertion does not hold good however of a body consisting of particles whose attractive forces decrease in the ratio of the third power of the distances because in that case the attraction of the remoter part of the infinite body in the second Corollary is always infinitely greater than the attraction of the nearer part

SCHOLIUM

If a body

(by Cor. II of the Laws) compounding that motion with an uniform motion performed in the direction of lines parallel to that plane And on the contrary if there be required the law of the attraction tending towards the plane in perpendicular directions by which the body may be caused to move in any given curved line the Problem will be solved by working after the manner of the third Problem

But

verging

angle and that length be as any power of the base $A^{\frac{m}{n}}$ and there be sought the force with which a body either attracted towards the base

to be increased by a very small part O and I resolve the ordinate $(A+O)^{\frac{m}{n}}$ into an infinite series

$$A^{\frac{m}{n}}+\frac{m}{n}OA^{\frac{m-n}{n}}+\frac{mm-mn}{2nn}OOA^{\frac{m-2n}{n}}\ \&c$$

and I suppose the force proportional to the term of this series in which O is of two dimensions that is to the term $\frac{mm-mn}{2nn}OOA^{\frac{m-2n}{n}}$ Therefore the force sought is as $\frac{mm-mn}{nn}A^{\frac{m-2n}{n}}$ or which is the same thing as $\frac{mm-mn}{nn}B^{\frac{m-2n}{m}}$ As if

2 and $n=1$ the force will be as

Therefore with a given force the

as Galileo hath demonstrated If the ordinate

$=0-1$ and $n=1$ the force will be as $2A^{-3}$ or $2B$ and therefore a force which is as the cube of the

body

on to

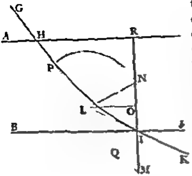
SECTION XIV

THE MOTION OF VERY SMALL BODIES WHEN AGITATED BY CENTRIPETAL FORCES TENDING TO THE SEVERAL PARTS OF ANY VERY GREAT BODY

PROPOSITION 94 THEOREM 48

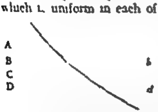
If two similar mediums be separated from each other by a space terminated on both sides by parallel planes and a body in its passage through that space be attracted or impelled perpendicularly towards either of those mediums and not agitated or hindered by any other force and the attraction be everywhere the same at equal distances from either plane taken towards the same side of the plane I say that the sine of incidence upon either plane will be to the sine of emergence from the other plane in a given ratio

CASE 1 Let Aa and Bb be two parallel planes and let the body light upon the first plane Aa in the direction of the line GH and in its whole passage through



the intermediate space let it be attracted or impelled toward the medium of incidence and by that action let it be made to describe a curved line HI and let it emerge in the direction of the line IK Let there be erected IM perpendicular to Bb the plane of emergence and meeting the line of incidence GH prolonged in M and the plane of incidence Aa in R and let the line of emergence KI be produced and meet HM in L About the centre L with the radius LI let a circle be described cutting both HM in P and Q and MI produced in N ... HI (by ... st of ... re of ... re be let fall the perpendicular LO then ... qual lines ON OI the wholes MN IR will be equal also Therefore since IR is given MN is also given and the rectangle MI MN is to the rectangle under the latus rectum and IM that is to HM in a given ratio But the rectangle MI MN is equal to the rectangle MP MQ that is to the difference of the ... d PI or LI² and HM hath a given ratio to its fourth part

Case Let now the body pass ... nated with parallel planes AaBb BbCc &c and let it be acted on by a force which is uniform in each of them separately but different in the different spaces and by what was just demonstrated the sine of the angle of incidence on the first plane Aa is to the sine of emergence from the second plane Bb in a given ratio and this sine of incidence upon the second plane Bb will be to the sine of emergence from the third plane Cc in a given ratio and this sine to the sine of emergence from the fourth plane Dd in a given ratio and so on in infinitum and by multiplication of equals the sine of incidence on the first plane is to the sine of emergence from the last plane in a given ratio Let now the intervals of the planes be diminished and their number be infinitely increased, so that the ...



then also

Q.E.D

SCHOLIUM

If a body is attracted perpendicularly towards a given plane and from the [illegible] red the Problem will [illegible] descending in a right [illegible] plane and (by Cor II of the Laws) compounding that motion with an uniform motion performed in the direction of lines parallel to that plane And on the contrary if there be required the law of the attraction tending towards the plane in perpendicular directions by which the body may be caused to move in any given curved line the Problem will be solved by working after the manner of the third Problem

But the operations may be contracted by resolving the ordinates into converging series As if to a base A the length B be ordinately applied in any given angle and that length be as any power of the base $A^{\frac{m}{n}}$ and there be sought the force with which a body either attracted towards the base or driven from it in the direction of that ordinate may be caused to move in the curved line which that ordinate always describes with its superior extremity I suppose the base to be increased by a very small part O and I resolve the ordinate $(A+O)^{\frac{m}{n}}$ into an infinite series

$$A^{\frac{m}{n}}+\frac{m}{n}OA^{\frac{m-n}{n}}+\frac{mm-mn}{2nn}OOA^{\frac{m-2n}{n}} \text{ \&c}$$

and I suppose the force proportional to the term of this series in which O is of two dimensions that is to the term $\frac{mm-mn}{2nn}OOA^{\frac{m-2n}{n}}$ Therefore the force sought is as $\frac{mm-mn}{nn}A^{\frac{m-2n}{n}}$ or which is the same thing as $\frac{mm-mn}{nn}B^{\frac{m-2n}{m}}$ As if the ordinate describe a parabola m being $=2$ and $n=1$ the force will be as the given quantity 2B and therefore is given Therefore with a given force the body will move in a parabola as Galileo hath demonstrated If the ordinate describe an hyperbola m being $=0-1$ and $n=1$ the force will be as $2A^{-3}$ or $2B^3$ and therefore a force which is as the cube of the ordinate will cause the body to move in an hyperbola But leaving Propositions of this kind I shall go on to some others relating to motion which I have not yet touched upon

SECTION XIV

THE MOTION OF VERY SMALL BODIES WHEN AGITATED BY CENTRIPETAL FORCES TENDING TO THE SEVERAL PARTS OF ANY VERY GREAT BODY

PROPOSITION 94 THEOREM 48

If two similar mediums be separated from each other by a space terminated on both sides by parallel planes and a body in its passage through that space be attracted or impelled perpendicularly towards either of those mediums and not agitated or hindered by any other force and the attraction be everywhere the same at equal distances from either plane taken towards the same side of the plane I say that the sine of incidence upon either plane will be to the sine of emergence from the other

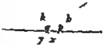
as before in P H &c and will emerge at last with the same obliquity at h
... that plane at H Conceive now the intervals of
... and the number
exerted accord
le of emergence
remainin all along equ... ual to the same
also at last Q E D

SCHOLIUM

... the reflections and refractions
s was discovered by Snell and
a given ratio of the sines as was

exhibited by Descartes For it is now certain from the phenomena of Jupiter s satellites confirmed by the observations of different astronomers that light is propagated in succession and requires about seven or eight minutes to travel from the sun to the earth Moreover the rays of light that are in our air (as lately was discovered by Grimaldi by the admission of light into a dark room through ... which I have also tried) in their passage near the angles of ... circular and rectangular
... r broken pieces of stone or glass) are bent or inflect... ... if they were attracted to them and those rays which in their passage come nearest to the bodies are the most inflected as if they were most attracted which thing I myself have ... which pass at greater distances are less in

whi... done partly in the air before ... the glass partly (if I mistake not) within the glass after they have entered it as is represented in the ... $ckzc$ $biyb$ $ahxa$ falling upon ... and in flected between k and ... and y h and x Therefore

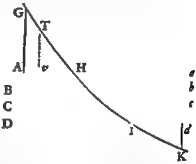


rays of light
ung the curves

PROPOSITION 95 THEOREM 49

The same things being supposed I say that the velocity of the body before its incidence is to its velocity after emergence as the sine of emergence to the sine of incidence

Make AH and Id equal and erect the perpendiculars AG dK meeting the lines of incidence and emergence GH IK in G and K In GH take TH equal to IK and to the plane Aa let fall a perpendicular Tv And (by Cor II of the Laws of Motion) let the motion of the body be resolved into two one perpendicular to the planes Aa Bb Cc &c and another parallel to them The force of attraction or impulse acting in directions perpendicular to those planes does not at all alter the motion in parallel directions and therefore the body proceeding with this motion will in equal times go through those equal parallel intervals that lie between the line AG and the point H and



between the point I and the line dK that is they will describe the lines GH IK in equal times Therefore the velocity before incidence is to the velocity after emergence as GH to IK or TH that is as AH or Id to vH that is (supposing TH or IK radius) as the sine of emergence to the sine of incidence Q E D

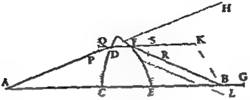
PROPOSITION 96 THEOREM 50

The same things being supposed and that the motion before incidence is swifter than afterwards I say that if the line of incidence be inclined continually the body will be at last reflected and the angle of reflection will be equal to the angle of incidence

For conceive the body passing between the parallel planes Aa Bb Cc &c to describe parabolic arcs as above and let those arcs be HP PQ QR &c And let the obliquity of the line of incidence GH to the first plane Aa be such that the sine of incidence may be to the radius of the circle whose sine it is in the same ratio which the same sine of incidence hath to the sine of emergence from the plane Dd into the space DdeE and because the sine of emergence is now become equal to the radius the angle of emergence will be a right one and therefore the line of emergence will coincide with the plane Dd Let the body come to this plane in the point R and because the line of emergence coincides with that plane it is manifest that the body can proceed no farther towards the

plane Ee But neither can it proceed in the line of emergence Rd because it is perpetually attracted or impelled towards the medium of incidence It will return therefore between the planes Cc Dd describing an arc of a parabola QRq whose principal vertex (by what Galileo hath demonstrated) is in R

that BG may be to CE as M−N to N and AD to H so that AH may be equal to AG and DF to K so that DK may be to DH as N to M Join KB and about the centre D with the radius DH describe a circle meeting KB produced in L,



F will touch the line EF which

urface sought Q E F

perpendicular to AD DF

— QS to be always

and there-

H−FD−

(because

But (be-

G L to CE

d therefore

n it in the

direction DF to go on in the line FR to the place Q E D

SCHOLIUM

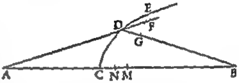
In the same manner one may go on to three or more surfaces But of all figures the spherical is the most proper for optical uses If the object glasses of telescopes were made of two glasses of a spherical figure containing water between them it is not unlikely that the errors of the refractions made in the

ease and accuracy but because the pencils of rays situated without the axis of the glass would be more accurately refracted by them But the different refrangibility of different rays is the real obstacle that hinders optics from being made perfect by spherical or any other figures Unless the errors thence arising can be corrected all the labor spent in correcting the others is quite thrown away

PROPOSITION 97 PROBL... 47

Supp... of cr ...

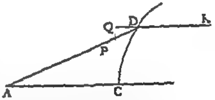
... place from ...
they ...
axis
EF ... perpendiculars let
paths of the bodies AD DB Let the point D approach to and coalesce with the point F and the ultimate ratio of the line DF by which AD is increased to the line DG by which DB is diminished will be the same as that of the sine of incidence to the sine of emergence Therefore the ratio of the increment of the line AD to the decrement of the line DB is given and therefore if in the axis AB there be taken anywhere the point C through which the curve CDE must pass and CM the increment of AC be taken in that given ratio to CN the decrement of BC and from the centres A B with the radii AM BN there be described two circles cutting each other in D that point D will touch the curve sought CDE and by touching it anywhere at pleasure will determine that curve Q E I



COR I By causing the point A or B to go off sometimes *in inf...* ... sometimes to move toward ot... ...

... body lighting on
CD ...

... another right line DK and from the point C there be drawn curved lines CP CQ always perpendicular to AD DK the increments of the lines PD QD and therefore the lines themselves PD QD generated by those increments will be as the sines of incidence and emergence to each other and conversely



PROPOSITION 98 PROBLEM 48

The same things supposed if round the axis AB any attractive surface be described as CD regular or irregular through which the bodies issuing from the given place A must pass it is required to find a second attractive surface EF which may make those bodies converge to a given place B

Let a line joining AB cut the first surface in C and the second in E the point D being taken in any manner at pleasure And supposing the sine of incidence on the first surface to the sine of emergence from the same and the sine of emergence from the second surface to the sine of incidence on the same to be as any given quantity M to another given quantity N then produce AB to G so

BOOK TWO

THE MOTION OF BODIES

IN RESISTING MEDIUMS

SECTION I

THE MOTION OF BODIES THAT ARE RESISTED IN THE RATIO OF THE VELOCITY

PROPOSITION 1 THEOREM 1

— ... by resistance is as the

is as the velocity
— ... y composition the
as the whole pace gone over Q E D
te of all gravity move by its innate force
only in free spaces and there be given both its whole motion at the beginning and al o the motion remaining after some part of the way is gone over there will be gi en also the whole pace which the body can describe in an infinite time For that space will be to the pace now described as the whole motion at th beginning ... to the part lost of that motion

LEMMA 1

Quantities proportional to their differences are continually proportional.

Let A A−B=B B−C=C C−D=&c
then by subtraction

A B=B C=C D=&c QED

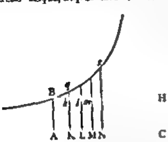
PROPOSITION 2 THEOREM 2

— ... d m ... by its inertia only ... s in the ... spaces described in each of the times are as the velocities

CASE 1 Let the time be divided into equal intervals and if at the very begin

their differences, and therefore (by Lem 1 Book II) continually proportional Therefore if out of an equal number of intervals there be compounded any equal portions of time the velocities at the beginning of those times will be as terms in a continued progression which are taken by jumps omitting every where an equal number of intermediate terms But the ratios of these terms are

force of gravity to the resistance in the beginning of the second time then from the force of gravity subtract the resistances and ABHC KkHC LlHC Mm HC &c will be as the absolute forces with which the body is acted upon in the beginnin of each of the times and therefore (by Law 1) as the increments of the velocities that is, as the rectangles Al Kl Lm Mn &c and therefore (by Lem 1 Book II) in a geometrical progression Therefore if the right lines Kk Ll Mm Nn &c are produced so as to meet the hyperbola in q r s t &c the areas ABqK, KqrL LrsM MstN &c will be equal and therefore analogous to



the equal times and equal gravitating forces. But the area ABqK (by Cor III Lem 7 and 8 Book I) is to the area Bkq as Kq to ½kq or AC to ½AK that is as the force of gravity to the resistance in the middle of the first time And by the like reasoning the areas qKLr rLMs sMNt &c are to the areas qklr rlms smnt &c as the gravitating forces to the resistances in the middle of the second third fourth time and so on Therefore since the equal areas BAKq qKLr rLMs sMNt &c are analogous to the gravitatin forces the areas Bkq qklr rlms smnt &c will be analogous to the resistances in the middle of each of the times that is (by supposition) to the velocities and so to the paces described Take the sums of the analogou

scribe the pace Bl and in the time LrtN the pace rlnt Q.E.D And the like demonstration holds in ascending motion

COR. I Therefore the greatest velocity that the body can acquire by falling is to the elocity acquired in any given time as the given force of gravity which continually acts upon it to the resisting force which opposes it at the end of that time

COR II But the time bein augmented in an arithmetical progression the sum of that greatest velocity and the velocity in the ascent and also their difference in the descent decreases in a geometrical progression

COR III Also the differences of the spaces which are described in equal differences of the times decrease in the same geometrical progression.

COR IV The pace described by the body is the difference of two paces wh reof one is as the time taken from the beginning of the descent and the ther as the elocity which [spaces] also at the beginning of the descent are equal among themselves

PROPOSITION 4 PROBLEM 2

Supposing the force of gravity in any homogeneous medium to be uniform and to tend perpen ... d ...

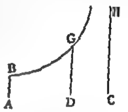
th

I

... may be to AC as the v tical component of the

compounded of the equal ratios of the intermediate terms equally repeated and therefore are equal Therefore the velocities being proportional to those terms are in geometrical progression Let those equal intervals of time be diminished and their number increased *in infinitum* so that the impulse of resistance may become continual and the velocities at the beginnings of equal times always continually proportional will be also in this case continually proportional Q E D

CASE 2 And by division the differences of the velocities that is the parts of the velocities lost in each of the times are as the wholes but the spaces described in each of the times are as the lost parts of the velocities (by Prop 1 Book I) and therefore are also as the wholes Q E D

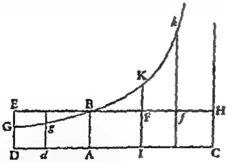


COR Hence if to the rectangular asymptotes AC CH the hyperbola BG is described and AB DG be drawn perpendicular to the asymptote AC and both the velocity of the body and the resistance of the medium at the very beginning of the motion be expressed by any given line AC and after some time is elapsed by the indefinite line DC the time may be expressed by the area ABGD and the space described in that time by the line AD For if that area by the motion of the point D be uniformly increased in the same manner as the time the right line DC will decrease in a geometrical ratio in the same manner as the velocity and the parts of the right line AC described in equal times will decrease in the same ratio

PROPOSITION 3 PROBLEM 1

To define the motion of a body which in an homogeneous medium ascends or descends in a right line and is resisted in the ratio of its velocity and acted upon by an uniform force of gravity

The body ascending let the gravity be represented by any give ... BACH and the resistance of the ...



... through the point B with the rectangular asymptotes AC CH describe an hyperbola cutting the perpendiculars DE de in G g and the body ascending will in the time DGgd describe the space EGge in the time DGBA the space of the whole ascent EGB in the time ABKI the space of descent BFK and in the time IKki the space of descent KFfk and the velocities of the bodies (proportional to the resistance of the medium) in these periods of time will be ABED ABed o ABFI ABfi respectively and the greatest velocity which the body can acquire by descending will be BACH

For let the ...

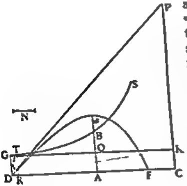
Lm M

... ABHC to ABKK as the

produced to V so that RX may be equal to $\frac{DR \cdot AB}{V}$ that is if the parallelogram ACPY be completed and DY cutting CP in Z be drawn and RT be produced till it meets DY in X Xr will be equal to $\frac{RDGT}{V}$ and therefore proportional to the time

COR. II Whence if innumerable lines CR or which is the same innumerable lines ZX be taken in a geometrical progression there will be as many lines Xr in an arithmetical progression And hence the curve DraF is easily delineated by the table of logarithms

COR. III If a parabola be constructed to the vertex D and the diameter DG ... 2DP as the whole resistance at ... the velocity with which ... ion of the right line DP so ... m resisting medium to describe the curve DraF will be the same as that with which it ought to go from the same place D in the direction of the same right line DP so as to describe a parabola in a nonresisting medium For the latus rectum of this parabola at the very beginning of the motion, is $\frac{DV^2}{Vr}$ and Vr is $\frac{tGT}{N}$ or $\frac{DR \cdot Tt}{2N}$ But a right line which if drawn would touch the hyperbola GTS in G is parallel to DK and therefore Tt is $\frac{CK \cdot DP}{DC}$ and N is $\frac{QB \cdot DC}{CP}$ And therefore Vr is equal to $\frac{DR^2 \cdot CK \cdot CP}{2DC^2 \cdot QB}$ that is (because DR and DC DV and DP are proportionals) to $\frac{DV^2 \cdot CK \cdot CP}{2DP^2 \cdot QB}$ and the latus rectum $\frac{DV^2}{Vr}$ comes out $\frac{2DP^2 \cdot QB}{CK \cdot CP}$ that is (because QB and CK DA and AC are proportionals) $\frac{2DP^2 \cdot DA}{AC \cdot CP}$ and therefore to 2DP as DP·DA to CP·AC that is as the resistance to the gravity Q.E.D.

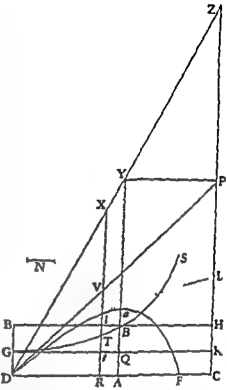


COR. IV Hence if a body be projected from any place D with a given velocity ...

tum of the parabola is given as is well known And taking 2DP to that latus rectum as the force of gravity to the resisting force DP is also given. Then cutting DC in A so that CP·AC may be to DP·DA in the same ratio of the gravity to the resistance the point A will be given And hence the curve DraF is also given

COR. V And conversely if the curve DraF be given there will be given both the velocity of the body and the resistance of the medium in each of the places r F For the ratio of CP·AC to DP·DA being given there is given both the resistance of the medium at the beginning of the motion and the latus rectum of

resistance of the medium arising from the motion upwards at the beginning to the force of gravity or (which comes to the same) so that the rectangle under DA and DP may be to that under AC and CP as the whole resistance at the beginning of the motion to the force of gravity With the asymptotes DC CP describe any hyperbola GTBS cutting the perpendiculars DG AB in G and B complete the parallelogram DGKC and let its side GK cut AB in Q Take a line N in the same ratio to QB as DC is in to CP and from any point R of the right line DC erect RT perpendicular to it meeting the hyperbola in T and the right lines EH GK DP in I t and V in that perpendicular take Vr equal to $\frac{tGT}{N}$ or which is the same thing take Rr equal to $\frac{GTIE}{N}$ and the projectile in the time DRTG will arrive at the point r describing the curved line DraF the locus of



the point r thence it will come to its greatest height a in the perpendicular AB … asymptote PC And its velocity in any … e curve Q E I

Fo … $= \text{DR} \quad \text{RV}$

and therefore RV is equal to $\frac{\text{DR} \quad \text{QB}}{N}$ and Rr

$\left(\text{that is } \text{RV} - \text{V}r \text{ or } \frac{\text{DR} \quad \text{QB} - t\text{GT}}{N}\right)$ is equal to $\frac{\text{DR} \quad \text{AB} - \text{RDGT}}{N}$ Now let

the … ws Cor II) distinguish the … the other lateral And since … also be distinguished into two parts proportional and contrary to the parts of the motion and therefore the length described by the lateral motion will be (by Prop ■ Book II) as the line DR and the height (by Prop 3 Book II) as the area DR AB − RDGT that is as the line Rr But in the very beginning of the motion the area RDGT is equal

to the rectangle DR AQ and therefore that line Rr $\left(\text{or } \frac{\text{DR} \quad \text{AB} - \text{DR} \quad \text{AQ}}{N}\right)$

will then be to DR as AB − AQ or QB to N that is as CP to DC and therefore as the motion upwards … fore Rr is always as the …

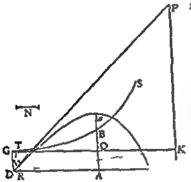
Q E D

COR I Therefore Rr is equal to $\frac{\text{DR} \quad \text{AB}}{N} - \frac{\text{RDGT}}{N}$ and therefore if RT be

produced to Y so that RX may be equal to $\frac{DR \cdot AB}{N}$ that is if the parallelogram ACPY be completed and DY cutting CP in Z be drawn and RT be produced till it meets DY in X Xr will be equal to $\frac{RDGT}{N}$ and therefore proportional to the time

Cor. II Whence if innumerable lines CR or which is the same innumerable lines ZX be taken in a geometrical progression there will be as many lines Xr in an arithmetical progression And hence the curve DraF is easily delineated by the table of logarithms

... be constructed to the vertex D and the diameter DG ... tance at



P as in an ...

right line DP so as to describe a parabola in a nonresisting medium For the latus rectum of this parabola at the very beginning of the motion is $\frac{DV^2}{Vr}$ and Vr is $\frac{tGT}{N}$ or $\frac{DR \cdot Tt}{2N}$ But a right line which ... GTS

$\frac{CK \cdot DP}{DC}$ and N is $\frac{QB \cdot DC}{CP}$

(because DR and DC DV and DP are proportionals) to $\frac{DV^2 \cdot CK \cdot CP}{2DP^2 \cdot QB}$ and the latus rectum $\frac{DV^2}{Vr}$ comes out $\frac{2DP^2 \cdot QB}{CK \cdot CP}$ that is (because QB and CK DA and AC are proportionals) $\frac{2DP^2 \cdot DA}{AC \cdot CP}$ and therefore is to 2DP as DP DA to CP AC that is as the resistance to the gravity Q.E.D.

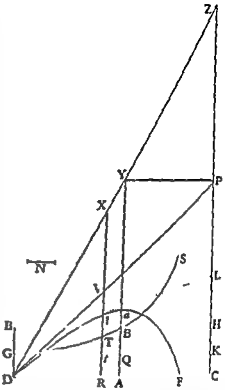
Cor. III Hence if a body be projected from any place D with a given velocity in the direction of a right line DP given by position and the resistance of the ...

cutting DC in A so that CP AC may be to DP DA in the same ratio of the gravity to the resistance the point A will be given And hence the curve DraF is also given

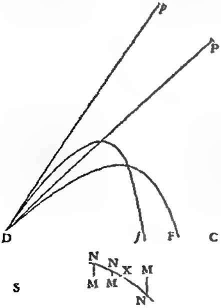
Cor. IV And conversely if the curve DraF be given there will be given both the velocity of the body and the resistance of the medium in each of the places r For the ratio of CP AC to DP DA being given there is given both the resistance of the medium at the beginning of the motion and the latus rectum of

the parabola and thence the velocity at the beginning of the motion is given also Then from the length of the tangent rL there is given both the velocity proportional to it and the resistance proportional to the velocity in any place r

COR VI But since the length 2DP is to the latus rectum of the parabola as the gravity to the resistance in D and from the velocity augmented the resistance is augmented in the same ratio but the latus rectum of the parabola is augmented as the square of that ratio it is plain that the length 2DP is augmented in that simple ratio only and is therefore always proportional to the velocity nor will it be augmented or diminished by the change of the angle CDP unless the velocity be also changed



COR VII Hence appears the meth[illegible] of [illegible] [illegible] and velocity with which the body is projected Let two similar and equal bodies be projected with the same velocity from the place D in different angles CDP CDp and let the places F f where they fall upon the horizontal plane DC be known Then taking any length for DP or Dp suppose the resistance in D to be to the gravity in any ratio whatsoever and let that ratio be represented by any length SM Then by computation from that assumed length DP find the lengths DF Df and from the ratio $\frac{Ff}{DF}$ found by calculation subtract the same ratio as found by experiment and let the difference be represented by the perpendicular MN Repeat the same a second and a third time by assuming always a new ratio SM of the resistance to the gravity and collecting a new difference MN Draw the positive differences on one side of the right line SM and the negative on the other [illegible] N the be t[illegible] [illegible] to the gravity which was to be found From this ratio the length DF is to be found by calculation and a length which is to the assumed length DP as the length DF known by experiment to the length DF



just now found, will be the true length DP This being known you will have both the curved line DraF which the body describes and also the velocity and resistance of the bod in each place

SCHOLIUM

h th resistance of bodies is in the ratio of the velocity is more void of all tenac velocities For by the action of a swifter body to a greater veloc ity is communicated to the same quantity of the medium in a less time and in an equal time by reason of a greater quantity of the disturbed medium a motion is communicated as the square of the ratio greater and the resi tance (by Laws II and III) is as the motion communicated Let us therefore see what motions arise from this law of resistance

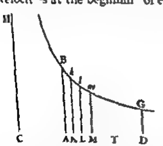
SECTION II

THE MOTION OF BODIES THAT ARE RESISTED AS THE SQUARE OF THEIR VELOCITIES

PROPOSITION 5 THEOREM 3

If a body is resisted as the square of its velocity and moves by its innate force only through an homogeneous medium and the times be taken in a geometrical progression proceeding from less to greater terms I say that the velocities at the beginning of each of the times are in the same geometrical progression inversely and that the spaces are equal which are described in each of the times

For since the resistance of the medium is proportional to the square of the velocity and the decrement of the velocity is proportional to the resistance if the time be divided into innumerable equal intervals the squares of the velocities at the beginning of each of the times will be proportional to the differences of the same velocities Let those inter



AB—Kk to Kk as Ak to CA and alternately AB—Kk to Ak as Kk to CA and therefore as AB Kk to AB CA Therefore since Ak and AB CA are given AB—Kk will be as AB Kk and lastly when AB and Kk coincide as AB And by the like reasoning Kk—Ll Ll—Mm &c will be as Kk Ll &c Therefore the squares of the lines AB Kk Ll Mm &c are as their differences and therefore since the squares of the velocities were shown above to be as their differences the progression of both will be alike This being demonstrated it follows also that the areas described by these lines are in a like progression with the spaces described by these velocities Therefore if the velocity at the beginning of the first time Ak be represented by the line AB and

the velocity at the beginning of the second time KL by the line Kk and the length described in the first time by the area AKkB all the following velocities will be represented by the following lines Ll Mm &c and the lengths described by the areas Kl Lm &c And by composition if the whole time be represented by AM the sum of its parts the whole length described will be represented by AMmB the sum of its parts Now conceive the time AM to be divided into the parts AK KL LM &c so that CA CK CL CM &c may be in a geometrical progression and those parts will be in the same progression and the velocities AB Kk Ll Mm &c will be in the same progression inversely and the spaces described Ak Kl Lm &c will be equal

COR I Hence it appears that if the asymptote and the velocity in AB the and

the first velocity by the rectangle AB AD

COR II Hence the space described in a resisting medium is given by taking it to the space described with the uniform velocity AB in a nonresisting medium as the hyperbolic area ABGD to the rectangle AB AD

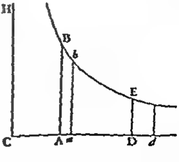
COR III The resistance of the medium is also given by making it equal in the very beginning force which could genera the velocity AB in the tim touching the hyperbola in B and meeting the asymptote in T the right line AT will be equal to AC and will express the time in which the first resistance uniformly continued may take away the whole velocity AB

COR IV And thence is also given the proportion of this resistance to the force of gravity or any other given centripetal force

COR V And conversely if there is given the proportion of the resistance to any given centripetal force the time AC is also given in which a centripetal force equal to the resistance as AB and thence is CH CD for its asymptote which a body by beginning with that velocity AB can describe in any time AD in an homogeneous resisting medium

PROPOSITION 6 THEOREM 4

Homogeneous and equal spherical bodies opposed by resistances that are as the square of the velocities and moving on by their innate force only will in times which are inversely as the velocities at the beginning describe equal spaces and lose parts of their velocities proportional to the wholes



To the rectangular asymptotes CD CH describe any hyperbola BbEe cutting the perpendiculars AB ab DE

AI by the lines Aa Dd Therefore as Aa is to Dd so (by the hypothesis) is DE to AB and so (from the nature of the hy

perbola) is CA to CD and by composition so is Ca to Cd Therefore the areas ABba DEed that is the paces described are equal among themselves and th first velocities AB DE are proportional to the last ab de and therefore by subtraction proportional to the parts of the velocities lost AB−ab DE−de
Q E D

PROPOSITION 7 THEOREM 5

of their motions proportional to the u
to the product of those times and the first velocities

For the parts of the motions lost are as the product of the resistances and times Therefore that those parts may be proportional to the whole the product of the resistance and time ought to be as the motion Therefore the ... the motion directly and the resistance inversely Therefore the ... lose parts ... velocities ... en ratio of ... duct of the first velocities and the times
Q E D

COR. I Therefore if bodies equally swift are resisted as the quare of their diameters homogeneous globes moving with any velocities whatsoever by describing paces proportional to their diameters will lose parts of their mo- ... wholes For the motion of each globe will be as the

COR IV Now if the globes are not homogeneous the pace described by the d nse globe must be augmented in the ratio of the density For the motion with n equal veloc ty is greater in the ratio of the density and the time (by th Proposition) is augmented in the ratio of motion directly and the pace described in the ratio of the time

the velocity at the beginning of the second time KL by the line Kk and the length described in the first time by the area AKkB all the following velocities will be represented by the following lines Ll Mm &c and the lengths described by the areas Kl Lm &c And by composition if the whole time be represented by AM the sum of its parts the whole length described will be represented by AMmB the sum of its parts Now conceive the time AM to be divided into the parts AK KL LM &c so that CA CK CL CM &c may be in a geometrical progression and those parts will be in the same progression and the velocities AB Kk Ll Mm &c will be in the same progression inversely and the spaces described Ak Kl Lm &c will be equal Q E D

COR I Hence it appears that if the time be represented by any part AD of the asymptote and the velocity in the beginning of the time by the ordinate AB the [illegible]

AB in a nonresisting medium by the rectangle AB AD

COR II Hence the space described in a resisting medium is given by taking it to the space described with the uniform velocity AB in a nonresisting medium as the hyperbolic area ABGD to the rectangle AB AD

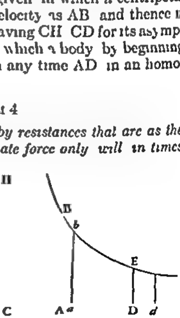
COR III The resistance of the medium is also given by making it equal in the very beginning of the motion to an uniform centripetal force which could generate in a body falling through a nonresisting medium the velocity AB in the time AC For if BT be drawn touching the hyperbola in B and meeting the asymptote in T the right line AT will be equal to AC and will express the time in which the first resistance uniformly continued may take away the whole velocity AB

COR IV And thence is also given the proportion of this resistance to the force of gravity or any other given centripetal force

COR V And conversely if there is given the proportion of the resistance to any given centripetal force the time AC is also given in which a centripetal force equal to the resistance may generate any velocity as AB and thence is given the point B through which the hyperbola having CH CD for its asymptotes is to be described as also the space ABGD which a body by beginning its motion with that velocity AB can describe in any time AD in an homogeneous resisting medium

PROPOSITION 6 THEOREM 4

Homogeneous and equal spherical bodies opposed by resistances that are as the square of the velocities and moving on by their innate force only will in times which are inversely as the velocities at the beginning describe equal spaces and lose parts of their velocities proportional to the wholes



To the rectangular asymptotes CD CH describe any hyperbola BbEe cutting the perpendiculars AB ab DE de in B b E e let the initial velocities be represented by the perpendiculars AB DE and the times by the lines Aa Dd Therefore as Aa is to Dd so (by the hypothesis) is DE to AB and so (from the nature of the hy

fore with the whole increments a and b of the ides the increment $aB+bA$ of the rectangle is generated Q E D

CASE 2 Suppose AB always equal to G and then the moment of the content ABC or GC (by CASE 1) will be $gC+cG$ that is (putting AB and $aB+bA$ for G and g) $aBC+bAC+cAB$ And the reasoning is the same for contents under e er o many ides Q E D

[illegible] des A B and C to be always equal among them [illegible] will be $2aA$ [illegible] will be $3aA^2$ [illegible] Q E D

CASE 4 Therefore since $\frac{1}{A}$ into A is 1 the moment of $\frac{1}{A}$ multiplied by A together with $\frac{1}{A}$ multiplied by a will be the moment of 1 that is nothing Therefore the moment of $\frac{1}{A}$ or of A^{-1} is $\frac{-a}{A^2}$ And generally since $\frac{1}{A^n}$ into A^n is 1 the moment of $\frac{1}{A^n}$ multiplied by A^n together with $\frac{1}{A^n}$ into naA^{n-1} will be nothing And therefore the moment of $\frac{1}{A^n}$ or A^{-n} will be $-\frac{na}{A^{n+1}}$ Q E D

CASE 5 And since $A^{1/2}$ into $A^{1/2}$ is A the moment of $A^{1/2}$ multiplied by $2A^{1/2}$ will be a (by Case 3) and therefore the moment of $A^{1/2}$ will be $\frac{a}{2A^{1/2}}$ [illegible] $A^{-1/2}$ And generally putting $A^{\frac{m}{n}}$ equal to B then A^m will be equal to B^n and therefore maA^{m-1} equal to nbB^{n-1} and maA^{-1} equal to nbB^{-1} or [illegible] and therefore $\frac{m}{n}aA^{\frac{m-n}{n}}$ is equal to b that is equal to the moment of $A^{\frac{m}{n}}$ Q E D

CASE 6 Therefore the moment of any generated quantity A^mB^n is the moment of A^m multiplied by B^n together with the moment of B^n multiplied by A^m that is $maA^{m-1}B^n+nbB^{n-1}A^m$ and that whether the indices m and n [illegible] affirmative or negative And the [illegible] Q E D

[illegible] one term is given [illegible]

[illegible]

COR II And if in four proportionals the two means are given the moments of the extremes will be as those extremes The same is to be understood of the sides of any given rectangle

COR III And if the sum or difference of two squares is given the moments of the sides will be inversely as the sides

SCHOLIUM

In a letter of mine to Mr J Collins dated December 10 16[illegible] having described a method of tangents [illegible] I suspected to be the same with Sluse s method which at that time was not made public I added these words *This is one particular or rather a Corollary of a general method which extends itself*

Cor. v And if the globes move in different mediums the space in a medium

ished in the ratio of the augmented resistance and the space

the time

Lemma 2

The moment of any genitum is equal to the moments of each of the generating sides multiplied by the indices of the powers of those sides and by their coefficients continually

I call any quantity a *genitum* which is not made by addition or subtraction of divers parts but is generated or produced in arithmetic by the multiplication division or extraction of the root of any terms whatsoever in geometry by the finding of contents and sides or of the extremes and means of proportionals Quantities of this kind are products quotients roots rectangles squares cubes square and cubic sides and the like These quantities I here consider as variable and indetermined and increasing or decreasing as it were by a continual motion or flux and I understand their momentary increments or decrements by the name of moments so that the increments may be esteemed as added or affirmative moments and the decrements as subtracted or negative ones But take care not to look upon finite particles as such Finite particles are not moments but the very quantities generated by the moments We are to conceive them as the just nascent principles of finite magnitudes Nor do we in this Lemma regard the magnitude of the moments but their first proportion as nascent It will be the same thing if instead of moments we use either the velocities of the increments and decrements (which may also be called the motions mutations and fluxions of quantities) or any finite quantities *proportional to those velocities The coefficient of any generating side is the quantity which arises by applying the genitum to that side*

Wherefore the sense of the Lemma is that if the moments of any quantities A B C &c increasing or decreasing by a continual flux or the velocities of the mutations which are proportional to them be called a b c &c the moment or mutation of the generated rectangle AB will be $aB+bA$ the moment of the generated content ABC will be $aBC+bAC+cAB$ and the moments of the generated powers A^2 A^3 A^4 $A^{1/2}$ $A^{3/2}$ $A^{1/3}$ $A^{2/3}$ A^{-1} A^{-2} $A^{-1/2}$ will be $2aA$ $3aA^2$ $4aA^3$ $\frac{1}{2}aA^{-1/2}$ $\frac{3}{2}aA^{1/2}$ $\frac{1}{3}aA^{-2/3}$ $\frac{2}{3}aA^{-1/3}$ $-aA^{-2}$ $-2aA^{-3}$ $-\frac{1}{2}aA^{-3/2}$ respectively and, in general that the moment of any power $A^{\frac{n}{m}}$ will be $\frac{n}{m}aA^{\frac{n-m}{m}}$ Also that the moment of the generated quantity A^2B will be $2aAB+bA^2$ the moment of the generated quantity $A^3B^4C^2$ will be $3aA^2B^4C^2+4bA^3B^3C^2+2cA^3B^4C$ and the moment of the generated quantity $\frac{A^3}{B^2}$ or A^3B^{-2} will be $3aA^2B^{-2}-2bA^3B^{-3}$ and so on The Lemma is thus demonstrated

Case 1 Any rectangle as AB augmented by a continual flux when as yet there wanted of the sides A and B half their moments $\frac{1}{2}a$ and $\frac{1}{2}b$ was $A-\frac{1}{2}a$ into $B-\frac{1}{2}b$ or $AB-\frac{1}{2}aB-\frac{1}{2}bA+\frac{1}{4}ab$ but as soon as the sides A and B are augmented by the other half moments the rectangle becomes $A+\frac{1}{2}a$ into $B+\frac{1}{2}b$ or $AB+\frac{1}{2}aB+\frac{1}{2}bA+\frac{1}{4}ab$ From this rectangle subtract the former rectangle and there will remain the excess $aB+bA$ There

medium may be represented by the lines AC AP and AK respectively and conversely

... test velocity which the body can ever acquire in an ... im answering to any given ... ound by taking it to that

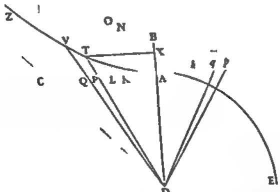
velocity be known the greatest ... given velocity as the square root of the ratio which the force of gravity bears to that known resistance of the medium

PROPOSITION 9 THEOREM 7

Supposing what is above dem n trated I a y that if the tangents of the angles of ... f an hyperbola be taken proportional to the velocities ... highest place will ... the highest place

as the sector of the hyperbola

To the right line AC which expresses the force of gravity let AD be drawn perpendicular and equal From the centre D with the semidiameter AD describe as well the quadrant AtE of a circle as the rectangular hyperbola AVZ whose axis is AK principal vertex A and asymptote DC Let Dp DP



be drawn and the circular sector AtD will be as all the time of the ascent to the highest place and the hyperbolic sector ATD as all the time of descent from the highest place if so be that the tangents Ap AP of those sectors be as the velocities

C r 1 Draw Dvq cuttin off the moments or least intervals tDv and qDp

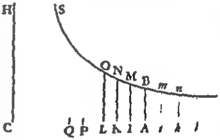
the interval tDv will be as $\frac{}{pD}$ that is (because tD is given) as $\frac{}{pD}$ But pD^2 is $AD^2 + Ap$ that is $AD + AD$ ... or AD Ck and qDp is $\frac{1}{2}$ AD pq Therefore tDv the interval of the sector is as $\frac{pq}{Ck}$ that is directly as the least decrement pq of the velocity and inversely as the force Ck which diminishes the velocity and therefore as the interval of time answering to the decrement

without any troublesome calculation not only to the dra ... curved lines whether ... lines or other curves ... about the crookednes ... of gravity of curves &c nor is it (as Hudden s method de maximis et minimis) limited to equations which are free from surd quantities This method I have interwoven with that other of working in equations by reducing them to infinite series So far that letter And these last words relate to a treatise I composed on that subject in the year 1671 The foundation of that general method is contained in the preceding Lemma

PROPOSITION 8 THEOREM 6

If a body in an uniform medium being uniformly acted upon by the force of gravity ascends or descends in a right line and the whole space described be divided into equal parts and in the beginning of each of the parts (by adding or subtracting the resisting force of the medium to or from the force of gravity when the body ascends or descends) you derive the absolute forces I say that those absolute forces are in a geometrical progression

Let the force of gravity be represented by the given line AC the force of resistance by the indefinite line AK the absolute force in the descent of the body by the difference KC the velocity of the body by a line AP which shall be a mean proportional between AK and AC and therefore as the square root of the resistance the increment of the resistance made in a given interval of time by the short line KL and the contemporaneous increment of the velocity by the short line PQ and with the centre C and rectangular asymptotes CA CH describe any hyperbola BNS meeting the erected ... and KL ... PQ of the other that is as AP KC for the ... rement PQ of the velocity is (by Law II) ... force KC Let the ratio of KL be multiplied by t... KL KN will become as AP KC KN that is (b... the rectangle KC KN is given) as AP But the ultimate ratio of the hyperbolic area KNOL to the rectangle KL KN becomes when the points K and L coincide the ratio of equality Therefore that hyperbolic evanescent area is as AP Therefore the whole hyperbolic area ABOL is composed of intervals KNOL which are always proportional to the velocity AP and therefore is itself proportional to the space described with that velocity Let that area be now divided into equal parts as ABMI IMNK KNOL &c and the absolute forces AC IC KC LC &c will be in a geometrical progression Q E D And by a like reasoning in the ascent of the body taking on the contrary side of the point A ... and ... C IC KC LC &c will be continually proportional Q E D



COR I Hence if the space described be represented by the hyperbolic area ABNK the force of gravity the velocity of the body and the resistance of the

medium may be represented by the lines AC AP and AK respectively and conversely

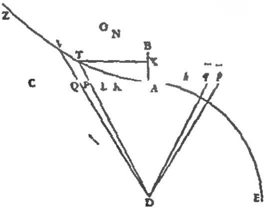
COR II And the greatest velocity which the body can ever acquire in an infinite descent will be represented by the line AC

COR. III. Therefore if the resistance of the medium answering to any given velocity be known the greatest velocity will be found by taking it to that given velocity as the square root of the ratio which the force of gravity bears to that known resistance of the medium

PROPOSITION 9 THEOREM

Supposing what is above demonstrated I say that if the tangents of the angles of the sector of a circle and of an hyperbola be taken proportional to the velocities *f the ascent to the highest place will* *hest place*

be drawn perpendicular and equal From the centre D neter AD describe as well the quadrant AtE of a circle as the rectangular hyperbola AVZ whose axis is AK principal vertex A and asymptote DC Let Dp DP

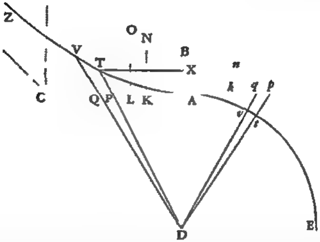


elocities.

pD² is AD² + Ap² that is AD + AD Ak or AD Ck and qDp is ½ AD pq Therefore tDv the interval of the sector is as $\frac{pq}{Ck}$ that is directly as the least decrement pq of the velocity and inversely as the force Ck which diminishes the velocity and therefore as the interval of time answering to the decrement

of the velocity And by composition the sum of all the intervals tDv in the sector ADt will be as the sum of the intervals of time answering to each of the lost intervals pq of the decreasing velocity Ap till that velocity being diminished into nothing vanishes that is the whole sector ADt is as the whole time of ascent to the highest place Q E D

CASE 2 Draw DQV cutting off the least intervals TDV and PDQ of the sector DAV and of the triangle DAQ and these intervals will be to each other as DT to DP that is (if TX and AP are parallel) as DX^2 to DA^2 or TX^2 to



AP^2 and by subtraction as DX^2-TX^2 to DA^2-AP^2 But from the nature of the hyperbola DX^2-TX^2 is AD^2 and by the supposition AP^2 is AD AK Therefore the intervals are to each other as AD^2 to $AD^2-AD\ AK$ that is as AD to AD−AK or AC to CK and therefore the interval TDV of the sector is $\frac{PDQ\ AC}{CK}$ and therefore (because AC and AD are given) as $\frac{PQ}{CK}$ that is directly as the increment of the velocity and inversely as the force generating the increment and therefore as the interval of the time answering to the increment And by composition the sum of the intervals of time in which all the intervals PQ of the velocity AP are generated will be as the sum of the intervals of the sector ATD that is the whole time will be as the whole sector Q E D

COR 1 Hence if AB be equal to a fourth part of AC the space which a body will describe by falling in any time will be to the space which the body could describe by moving uniformly on in the same time with its greatest velocity AC as the area ABNK which expresses the space described in falling to the area ATD which expresses the time For since

$$AC\ \ AP=AP\ \ AK$$

and by Cor 1 Lem 2 of this Book

$$LK\ \ PQ=2AK\ \ AP=2AP\ \ AC$$

therefore $\quad LK\ \ \frac{1}{2}PQ=AP\ \ \frac{1}{4}AC \text{ or } AB$

and since $\quad KN\ \ AC \text{ or } AD=AD\ \ CK$

multiplying together corresponding terms

$$LKNO\ \ DPQ=AP\ \ CK$$

As shown above

$$DPQ \quad DTV = CK \quad AC$$

Hence

$$LKNO \quad DTV = AP \quad AC$$

that is as the velocity of the falling body to the greatest velocity which the body by falling can acquire. Since therefore the moments LKNO and DTV of the areas ABNK and ATD are as the velocities, all the parts of those areas generated in the same time will be as the spaces described in the same time and therefore the whole areas ABNK and ADT generated from the beginning w the whole spaces described from the beginning of the descent Q E D

h pace described in the ascent That is to described in the same time with the is to the sector ADt

me ATD is to the velocity ting space as the triangle in a nonresisting medium um is as AP that is as the

triangle ig of the descent are equal among themselves as well as those areas PD

COR IV By the same argument the velocity in the ascent is to the velocity with which the body in the same time in a nonresisting space would lose all it motion of ascent as the triangle ApD to the circular sector AtD or as the right line Ap to the arc At

COR. V Therefore the time in which a body by falling in a resisting medium would acquire the velocity AP is to the time in which it would acquire its greatest velocity AC by falling in a nonresisting space as the sector ADT to the triangle ADC and the time in which it would lose its velocity Ap by ascending in a resisting medium is to the time in which it would lose the same

end n a nonresistin space as the arc At to its tangent Ap

pace described in the ading in infinitum e the time is given nonresisting pace ratio of the given or Ap and is the space

ght to that which would in th scribed with

the given pace of ascent or descent time ADt or ADT

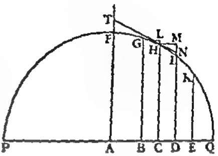
PROPOSITION 10 PROBLEM 3

ppose the un form force f gravity to tend d rectly to the plane of the horizon and the re nce to be as the product of the density of the medium and the square of th veloc y It is proposed to find the density of the med um in each place which shall mak the body or in a y given curved line the velocity of the body and the res nce f the med um in each place

Let PQ be a plane perpend cular to the plane of the scheme itself PFHQ a curved line meeting that plane in the points P and Q G H I K four places of the body going on in this curve from F to Q and GB HC ID KE four

n

rigl
cur ... and meeting the ordinates CH, DI produced upwards in L and N; and complete the parallelogram HCDM. And the times in which the body describes the arcs GH, HI will be as the square root of the altitudes LH, NI which the bodies would describe in those times by falling from the tangents; and the velocities will be directly as the lengths described



GH, HI and inversely as the times. Let the times be represented by T and t, and the velocities by $\frac{GH}{T}$ and $\frac{HI}{t}$; and the decrement of the velocity produced in the time t will be represented by $\frac{GH}{T}-\frac{HI}{t}$. This decrement arises from the resistance which retards the body, and from the gravity which accelerates it. Gravity, in a falling body, which in its fall describes the space NI, produces a velocity with which it would be able to describe twice that space in the same time, as Galileo hath demonstrated; that is, the velocity $\frac{2NI}{t}$: but if the body describes the arc HI, it augments that arc only by the length HI$-$HN, or $\frac{MI \cdot NI}{HI}$; and therefore generates only the velocity $\frac{2MI \cdot NI}{t \cdot HI}$. Le ...
be added ...

of the v ... $\frac{GH}{T}-\frac{HI}{t}+\frac{2MI \cdot NI}{t \cdot HI}$

Therefore since in the same time the action of gravity generates in a falling body the velocity $\frac{2NI}{t}$, the resistance will be to the gravity as

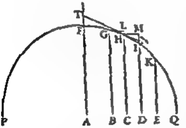
$$\frac{GH}{T}-\frac{HI}{t}+\frac{2MI \cdot NI}{t \cdot HI} \text{ to } \frac{2NI}{t}, \text{ or as } \frac{t \cdot GH}{T}-HI+\frac{2MI \cdot NI}{HI} \text{ to } 2NI.$$

Now for the abscissas CB, CD, CE put $-o$, o, $2o$. For the ordinate CH put P; and for MI put any series $Qo+Ro^2+So^3+$, &c. And all the terms of the series after the first, that is, Ro^2+So^3+, &c. will be NI; and the ordinates DI, EK, and BG will be $P-Qo-Ro^2-So^3-$, &c. $P-2Qo-4Ro^2-8So^3-$, &c. and $P+Qo-Ro^2+So^3-$, &c. respectively. And by squaring the differences of the ordinates BG$-$CH and CH$-$DI, and to the squares thence produced adding the squares of BC and CD themselves, you will have $oo+QQoo-2QRo^3+$, &c. and $oo+QQoo+2QRo^3+$, &c. the squares of the arcs GH, HI; whose roots $o\sqrt{(1+QQ)}-\frac{QRoo}{\sqrt{(1+QQ)}}$ and $o\sqrt{(1+QQ)}+\frac{QRoo}{\sqrt{(1+QQ)}}$ are the arcs GH and HI. Moreover, if from the ordinate CH there be subtracted half the sum of the ordinates BG and DI, and from the ordinate DI there be subtracted half the sum of the ordinates CH and EK, th ...
$Roo+3So^3$ the ...
to the short line

small times T and t and thence the ratio $\frac{t}{T}$ varies as the square root of $\frac{R+3So}{R}$ or $\frac{R-3So}{R}$ and $\frac{t \times GH}{T} - HI + \frac{2MI \times NI}{HI}$ by substituting the values of $\frac{t}{T}$ GH HI MI and NI just found becomes $\frac{3Soo}{2R}\sqrt{(1+QQ)}$ And since 2NI is 2Roo the resistance will be now to the gravity as $\frac{3Soo}{2R}\sqrt{(1+QQ)}$ to 2Roo that is as $3S\sqrt{(1+QQ)}$ to 4RR

A th velocity will be such that a body going off therewith from any place ... whe in a vacuum a parabola ... or $\frac{1+QQ}{R}$

And the resistance is as the product of the density of the medium and the square of the velocity and therefore the density of the medium is directly as the resistance and inversely as the square of the velocity that is directly as $\frac{3S\sqrt{(1+QQ)}}{4RR}$ and inversely as $\frac{1+QQ}{R}$ that is as $\frac{S}{R\sqrt{(1+QQ)}}$ Q E I



COR. 1 If the tangent HN be produced both ways so as to meet any ordinate AF in T $\frac{HT}{AC}$ will be equal to $\sqrt{(1+QQ)}$ and therefore in what has gone before may be put for $\sqrt{(1+QQ)}$ By this means the resistance will be to the gravity as 3S HT to 4RR AC the velocity will be as $\frac{HT}{AC\sqrt{I}}$ and the density of the medium will be as $\frac{S\ AC}{R\ HT}$

... the relation be-

PQ to find the density of the medium that shall make a projectile move in that l n

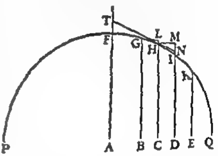
Bisect the diameter PQ in A and call AQ n AC a CH e and CD o then DI ... AQ − AD $= nn - aa - 2ao - oo$ or $ee - 2ao - oo$ and the root being extracted by our method will give

$$DI = e - \frac{ao}{e} - \frac{oo}{2e} - \frac{aaoo}{2e^3} - \frac{ao^3}{2e^3} - \frac{a^3o^3}{2e^5} \ \&c$$

Here put nn for $ee + aa$ and DI will become $= e - \frac{ao}{e} - \frac{nnoo}{2e^3} - \frac{anno^3}{2e^5} - \&c$

In such a series I distinguish the successive terms after this manner I call that the first term in which the infinitely small quantity o is not found the second in which that quantity is of one dimension only the third in which it arises to two dimensions the fourth in which it is of three and so ad infinitum And the first term which here is e will always denote the length of

the ordinate CH erected at the starting point of the indefinite quantity o The second term which here is $\frac{ao}{e}$ will denote the difference between CH and DN that is the short line MN which is cut off by completing the parallelogram HCDM and therefore always determines the position of the tangent HN as in this case by taking MN HM $= \frac{ao}{e}$ $o = a$ e The third term which here is $\frac{nnoo}{2e^3}$ will represent the short line IN which lies between the tangent and the curve and therefore determines the angle of contact IHN or the curvature which the curved line has in H If that short line IN is of a finite magnitude it will be expressed by the third term together with those that follow *in infinitum* But if that short line be diminished *in infinitum* the terms following become infinitely less than the third term and therefore may be neglected The fourth term determines the variation of the curvature the fifth the variation of the variation and so on From this by the way appears the use not to be disdained which may be made of these series in the solution of problems that depend upon tangents, and the curvature of curves



Now compare the series

$$e - \frac{ao}{e} - \frac{nnoo}{2e^3} - \frac{anno^3}{2e^5} - \&c$$

with the series

$$P - Qo - Roo - So^3 - \&c$$

and for P Q R and S put e $\frac{a}{e}$ $\frac{nn}{2e^3}$ and $\frac{ann}{2e^5}$ and for $\sqrt{(1+QQ)}$ put $\sqrt{\left(1+\frac{aa}{ee}\right)}$ or $\frac{n}{e}$ and the density of the medium will come out as $\frac{a}{ne}$ that is (because n is given) as $\frac{a}{e}$ or $\frac{AC}{CH}$ that is as that length of the tangent HT which is terminated at the semidiameter AF standing perpendicularly on PQ and the resistance will be to the gravity as $3a$ to $2n$ that is as 3AC to the diameter PQ of the circle and the velocity will be as $\sqrt{CH}$ Therefore if the body goes from the place F with a due velocity in the direction of a line parallel to PQ and the density of the medium in each of the places H is as the length of the tangent HT and the resistance also in any place H is to the force of gravity as 3AC to PQ that body will describe the quadrant FHQ of a circle Q E I

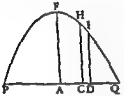
But if the same body should go from the place P in the direction of a line perpendicular to PQ and should begin to move in an arc of the semicircle PFQ we must take AC or a on the contrary side of the centre A and therefore its sign must be changed and we must put $-a$ for $+a$ Then the density of the medium would come out as $-\frac{a}{e}$ But Nature does not admit of a negative density that is a density which accelerates the motion of bodies and therefore it cannot naturally come to pass that a body by ascending from P should

describe the quadrant PF of a circle To produce uch an effect a body ought
[illegible] by an impelling medium and not impeded by a re i ting one
AF perpendicular
will make a pro-

From the nature of the parabo [illegible] is equal to the rectangle under the ordinate DI and some given right line that is if that right line be called b PC a PQ c CH e and CD o the rectan le

$$(a+o)(c-a-o)=ac-aa-2ao+co-oo=b\ \mathrm{DI}$$

therefore $\mathrm{DI}=\frac{ac-aa}{b}+\frac{c-2a}{b}o-\frac{oo}{b}$



Now the second term $\frac{c-2a}{b}o$ of this series is to be put for Qo and the third term $\frac{oo}{b}$ for Roo But ince there are no more terms the coefficient S of the fourth term will vani h and therefore the quantity $\frac{S}{R\sqrt{(1+QQ)}}$ to which the density of the medium is proportional will be nothing Therefore where the medium is of no density the projectile will move in a parabola [illegible]

Q E I
er
nat

Let MX be the other asymptote meetin the ordin ite DG [illegible] and from the nature of the hyperbola the rectan le of XY into YG will be [illegible]

or DN be $\frac{\ }{}$ Then DN will be equal to $a-o$ NG equal to $\frac{oo}{a-o}$ VZ equal to $\frac{m}{n}$ $(a-\ )$ and GD or NX−VZ−VG equal to

$$c-\frac{m}{n}a+\frac{m}{n}o-\frac{bb}{a-o}$$

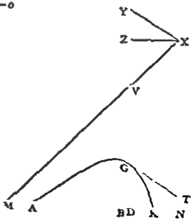
Let the term $\frac{bb}{a-o}$ be resolved into the con erging eries

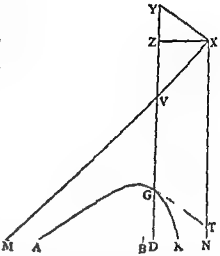
$$\frac{bb}{a}+\frac{bb}{aa}\ +\frac{bb}{a}oo+\frac{bb}{a}o^3\ \&c$$

and GD will become equal to

$$c-\frac{m}{n}a-\frac{bb}{a}+\frac{m}{n}o-\frac{bb}{aa}o-\frac{bb}{a^3}o^2-\frac{bb}{a^4}o^3\ \&c.$$

The second term $\frac{m}{n}o-\frac{bb}{aa}o$ of thi series is to be used for Qo the third $\frac{bb}{a^3}o^2$ with its sign hanged f r Roo and the fourth $\frac{bb}{a}o^3$ with





its sign changed also for So^3 and their coefficients $\frac{m}{n}-\frac{bb}{aa}$, $\frac{bb}{a^3}$ and $\frac{bb}{a^4}$ are to be put for Q, R and S in the former rule. Which being done, the density of the medium will come out as

$$\frac{\frac{bb}{a^4}}{\frac{bb}{a^3}\sqrt{\left(1+\frac{mm}{nn}-\frac{2mbb}{naa}+\frac{b^4}{a^4}\right)}}$$

or

$$\frac{1}{\sqrt{\left(aa+\frac{mm}{nn}aa-\frac{2mbb}{n}+\frac{b^4}{aa}\right)}}$$

that is, if in VZ you take VY equal to VG as $\frac{1}{XY}$. For aa and $\frac{m^2}{n^2}a^2-\frac{2mbb}{n}+\frac{b^4}{aa}$ are the squares of XZ and ZY. But the ratio of the resistance to gravity is found to be that of 3XY to 2YG, and the velocity is that with which the body would describe a parabola, whose vertex is G, diameter DG, latus rectum $\frac{XY}{VG}$. Suppose therefore that the densities of the medium in each of the places

C ... e n ... he resistance in any place G is

... o from the place A with a due

Q E I

...

I

I

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For BN, BD, NX put A, O, C respectively, and let VZ be to XZ or DN as d to e, and VG be equal to $\frac{bb}{DN}$; then DN will be equal to A − O, $VG=\frac{bb}{(A-O)}$, $VZ=\frac{d}{e}(A-O)$ and GD or NX − VZ − VG equal to

$$C-\frac{d}{e}A+\frac{d}{e}O-\frac{bb}{(A-O)}$$

Let the term $\frac{bb}{(A-O)}$ be resolved into an infinite series

$$\frac{bb}{A}+\frac{nbb}{A^{n+1}}O+\frac{nn+n}{2A^{n+2}}bbO^2+\frac{n^3+3nn+2n}{6A^{n+3}}bbO^3, \&c$$

and GD will be equal to

$$C-\frac{d}{e}A-\frac{bb}{A}+\frac{d}{e}O-\frac{nbb}{A^{n+1}}O-\frac{+nn+n}{2A^{n+2}}bbO^2-\frac{+n^3+3nn+2n}{6A^{n+3}}bbO^3, \&c$$

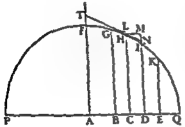
The second term $\frac{d}{e}O-\frac{nbb}{A^{n+1}}O$ of this series is to be used for Qo, the third $\frac{nn+n}{2A^{n+2}}bbO^2$ for Roo, the fourth $\frac{n^3+3nn+2n}{6A^{n+3}}bbO^3$ for So^3. And thence the density

of the medium $\frac{S}{R\sqrt{(1+QQ)}}$ in any place G will be

$$\frac{n+2}{2\sqrt{\left(A+\frac{dd}{ee}A-\frac{2dnbb}{eA}A+\frac{nnb}{A}\right)}}$$

and therefore if in XZ you take VY equal to n·VG, that density is reciprocally as XY. For A^2 and $\frac{dd}{ee}A-\frac{2dnbb}{eA}A+\frac{nnb}{A}$ are the squares of XZ and ZY. But the resistance in the same place G is to the force of gravity as $3S\frac{XY}{A}$ to $4RR$, that is, as XY to $\frac{2nn+2n}{n+2}VG$. And the velocity there is the same wherewith the projected body would move in a parabola, whose vertex is G, diameter GD, and latus rectum $\frac{1+QQ}{R}$ or $\frac{2XY}{(nn+n)VG}$. Q.E.I.

SCHOLIUM

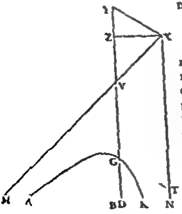


In the same manner that the density of the medium comes out to be as $\frac{S\times AC}{R\times HT}$, in Cor. 1, if the resistance is put as any power V^n of the velocity V, the density of the medium will come out to be as

$$\frac{S}{R^{\frac{4-n}{2}}}\times\left(\frac{AC}{HT}\right)^{n-1}$$

And therefore if a curve can be found such that the ratio of $\frac{S}{R^{\frac{4-n}{2}}}$ to $\left(\frac{HT}{AC}\right)^{n-1}$, or of $\frac{S^2}{R^{4-n}}$ to $(1+QQ)^{n-1}$ may be given, the body, in an uniform medium whose resistance is as the power V^n of the velocity V, [illegible]. But let us return to more simple curves.

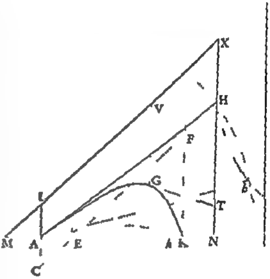
it is e[illegible]nt that the line which a projec[illegible] [illegible] medium approaches nearer to these hyper[illegible]



in the parts remote from the vertex draws nearer to them than these hyperbolas here described. The difference however is not so great between the one and the other but that these latter may be commodiously enough used in practice instead of the former. And perhaps these may prove more useful than an hyperbola that is more accurate and at the same time more complex. They may be made use of then in this manner

Complete the parallelogram XYGT and the right line GT will touch the hyperbola in G and therefore the density of the medium in G is inversely as the tangent GT and the velocity there as $\sqrt{\frac{GT^2}{GV}}$ and the resistance is to the force of gravity as GT to $\frac{2nn+2n}{n+2}$ GV

Therefore if a body projected from the place A in the direction of the right line AH describes the hyperbola AGK and AH produced meets the asymptote NX in H and AI drawn parallel to it meets the other asymptote MX in I the density of the medium in A will be inversely as AH and the velocity of the body as $\sqrt{\frac{AH^2}{AI}}$ and the resistance there to the force of gravity as AH to $\frac{2nn+2n}{n+2}$ AI Hence the following Rules are deduced



RULE 1 If the density of the medium at A and the velocity with which the body is projected remain the same and the angle NAH be changed the lengths AH AI HX will remain Therefore if those lengths in any one case are found the hyperbola may afterwards be easily determined from any given angle NAH

RULE 2 If the angle NAH and the density of the medium at A remain the same and the velocity with which the body is projected be changed the length AH will continue the ame and AI will be changed inversely as the square of the velocity

RULE 3 If the angle NAH the

ı ui the above mentioned parabola remaining the same and also the length $\frac{AH^2}{AI}$ proportional to it and therefore AH will be diminished in the same ratio and AI will be diminished as the square of that ratio But the proportion of

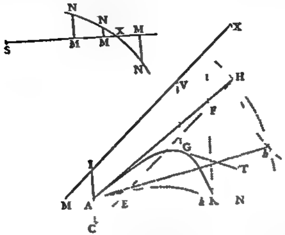
mented

equal or ui which by diminishing the resistance becomes diminished in a less ratio than the weight

RULE 4 Because the density of the me

the figure AGK is to be de-

scribed produce HN to Y so that HX may be to AI as $n+1$ to 1 and with the centre X, and the asymptote MX NX describe an hyperbola through the point A, such that AI may be to any of the lines VG as XV to XI



RULE 6 By how much the greater the number n is so much the more accurate are these hyperbolas in the ascent of the body from A and less accurate

Th hyperbola keeps a mean ratio

if the hyperbola be

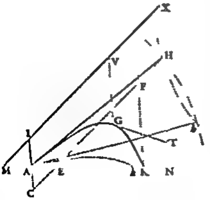
projected body falls

upon any right line A passing produced meet the asymptotes MX NX in M and N and take NK equal to AM

RULE 7 And hence appears an expeditious method of determining this hy

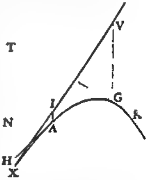
had or be pro ed

it be as d to e Then erecting a perpendicular AI of any length assume any length AH or Ah and thence graphically or by scale and compass collect the lengths AK Ak (by Rule 6) If the ratio of AK to Ak be the same with that of d to e the length of AH was rightly assumed If not, take on the indefinite right line SM the length SM equal to the assumed AH and erect a perpendicular MN equal to the difference $\frac{AK}{Ak}-\frac{d}{e}$ of the ratios multiplied by any given right line By the like method from several assumed lengths AH you may find several points N and draw through them all a regular curve NNXN cutting the right line SMMM in X.



Lastly assume AH equal to the abscissa SX and thence find again the length AK and the lengths which are to the assumed length AI and this last AH as the length AK known by experiment to the length AK last found will be the true lengths AI and AH which were to be found But these being given there will be given also the resisting force of the medium in the place A it being to the force of gravity as AH to $\frac{4}{3}$AI Let the density of the medium be increased by Rule 4 and if the resisting force just found be increased in the same ratio it will become still more accurate

RULE 8 The lengths AH HX being found let there be now required the position of the line AH according to which a projectile thrown with that given velocity shall fall upon any point K At the points A and K erect the lines AC KF perpendicular to the horizon whereof let AC be drawn downwards and be equal to AI or $\frac{1}{2}$HX With the asymptotes AK KF describe an hyperbola ... centre A with ... point H then ... l fall upon the point K Q E I For the point H because of the given length AH must be somewhere in the circumference of the described circle Draw CH meeting AK and KF in E and F and because CH MX are parallel and AC AI equal AE will be equal to AM and therefore also equal to KN But CE is to AE as FH to KN and therefore CE and FH are equal Therefore the point H falls upon the hyperbolic curve described with the asymptotes AK KF whose conjugate passes through the point C and is therefore found in the common intersection of this hyperbolic curve and the circumference of the described circle Q E D It is to be ob erved that this operation is the same whether the right line AKN be parallel to the horizon or inclined thereto in any angle and that from two intersections H *h* there arise two angles NAH NA*h* and that in *mechanical* practice it is sufficient once to describe a circle then to apply a ruler CH of an indeterminate length so to the point C that its part FH intercepted between the circle and the right line FK may be equal to its part CE placed between the point C and the right line AK



What has been said of hyperbolas may be easily applied to parabolas For if a parabola be represented by XAGK touched by a right line XV in the vertex X and the ordinates IA VG be as any powers XI XV of the abscissas XI XV draw XT GT AH whereof let XT be parallel to VG and let GT AH touch the parabola in G and A and a body projected from any place A in the direction of the right line AH with a due velocity will describe this parabola if the density of the medium in each of the places G be inversely as the tangent GT In that case the velocity in G will be the same as would cause a body moving in a nonresisting space to describe a conic parabola having G for its vertex VG produced downwards for its diameter and $\frac{2GT^2}{(nn-n)\ VG}$ for its latus rectum And the resisting force in G will be to the force of gravity as GT to $\frac{2nn-2n}{n-2}$ VG Therefore if NAK represent an horizontal line and both the density of the medium at A and the

[illegible] AH [illegible] AH, HX [illegible] VG [illegible] G is the [illegible]

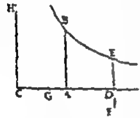
SECTION III

The motion of bodies which are resisted partly in the ratio of the velocities, and partly in the square of the same ratio

Proposition XI. Theorem 8

If a body be resisted partly in the ratio and partly in the squared ratio of its velocity, and moves in a similar medium by its inertia only, and the times be taken in arithmetical progression, then quantities inversely proportional to the velocities, increased by a certain given quantity, will be in geometrical progression.

With the centre C, and the rectangular asymptotes CADd and CH, describe an hyperbola BEe, and let AB, DE, de be parallel to the asymptote CH. In the asymptote CD let A, G be given points; and if the time be represented by the hyperbolic area ABED uniformly increasing, I say, that the velocity may be expressed by the length DF, whose reciprocal GD, together with the given line CG, compose the length CD increasing in a geometrical progression.



For let the small area DEed be the least given increment of the time, and Dd will be inversely as DE, and therefore directly as CD. Therefore the decrement of $\frac{1}{GD}$, which (by Lem. II, Book II) is $\frac{Dd}{GD^2}$, will be also as $\frac{CD}{GD^2}$ or $\frac{CG+GD}{GD^2}$, that is, as $\frac{1}{GD}+\frac{CG}{GD^2}$. Therefore the time ABED uniformly increasing by the addition of the given intervals EDde, it follows that $\frac{1}{GD}$ decreases [illegible] the velocity. For the decrement of the velocity [illegible]

the decrement of $\frac{1}{GD}$ is as the sum of the quantities $\frac{1}{GD}$ and $\frac{CG}{GD^2}$ [illegible] first is as $\frac{1}{GD}$ itself, and the last $\frac{CG}{GD^2}$ is as $\frac{1}{GD^2}$; therefore $\frac{1}{GD}$ is as the velocity, the decrements of both being analogous. And if the quantity GD inversely proportional to $\frac{1}{GD}$ be augmented by the given quantity CG, the sum CD, the time ABED uniformly increasing, will increase in a geometrical progression.

Q.E.D.

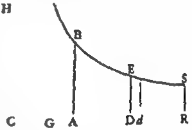
Cor. I. Therefore, if, having the points A and G given, the time be represented by the hyperbolic area ABED, the velocity may be represented by $\frac{1}{GD}$, the reciprocal of GD.

COR II And by taking GA to GD as the reciprocal of the velocity at the beginning to the reciprocal of the velocity at the end of any time ABED the point G will be found And that point being found the velocity may be found from any other time given

PROPOSITION 12 THEOREM 9

The same things being supposed I say that if the spaces described are taken in arithmetical progression the velocities augmented by a certain given quantity will be in geometrical progression

In the asymptote CD let there be given the point R and erecting the perpendicular RS meeting the hyperbola in S let the space described be represented by the hyperbolic area RSED and the velocity will be as the length GD which together with the given line CG composes a length CD decreasing



... se the increment EDde of the space is given the short line Dd which is the decrement of GD will be reciprocally as ED and therefore directly as CD that is as the sum of the same GD and the given length CG But the decrement of the velocity in a time inversely proportional thereto in which the given interval of space DdeE is described is as the resistance and the time conjointly that is directly as the sum of two quantities whereof one is as the velocity the other as the square of the velocity and inversely as the velocity and therefore ...

I

... y be represented by the length CD the ... will be ...

C

by t

spac

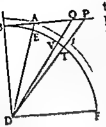
give

COR III Whence since (by Prop 11) the velocity is given from the given time and (by this Proposition) the space is given from the given velocity the space will be given from the given time and conversely

PROPOSITION 13 THEOREM 10

Supposing that a body attracted downwards by an uniform gravity ascends or descends in a right line and that the same is resisted partly in the ratio of its velocity and partly as the square of the ratio thereof I say that if right lines parallel to the diameters of a circle and an hyperbola be drawn through the ends of the conjugate diameters and the velocities be as some segments of those parallels drawn from a given point the times will be as the sectors of the areas cut off by right lines drawn from the centre to the ends of the segments and conversely

CASE 1 Suppose first that the body is ascending and from the centre D with any semidiameter DB describe a quadrant BETF of a circle and through



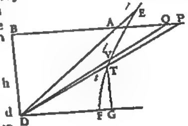
the end B of the semidiameter DB draw the indefinite line BAP parallel to the semidiameter DF In that line let there be given the point A and take the segment AP proportional to the velocity And since one part of the resistance is as the velocity and another part as the square of the velocity let the whole resistance be as $AP^2+2BA \cdot AP$ Join DA DP cutting the circle in E and T and let the gravity be represented by DA so that the gravity shall be to the resistance in P as DA^2 to $AP^2+2BA \cdot AP$ and the time of the whole ascent will be as the sector EDT of the circle

For draw DVQ cutting off the moment PQ of the velocity AP and the DET moment to a given moment of time and ... of gravity ... Elements ... P^2 and the ... given quantity ... to the rate of the

DT^2 Therefore the area ... and is therefore proportional to the future time by subtraction of given intervals DTV ... tional to the time of the whole ascent

Q E D

CASE 2 If the velocity in the ascent of the body be represented by the length AP as before and the resistance be made as ... BA AP and if the force of gravity be ... BD



whose conjugate semidiameter ... DF and which cuts DA in E and DP DQ in ... of the whole ascent will be as the hyperbolic sector TDE ... l of time ... B −BD ... to DP^2 ... $-DF^2$ to ... $-BD$ Therefore DTV will be as the uniformly in each of ... en intervals DTV

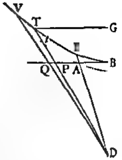
Q E D

... therefore proportional to the time ...

CASE 3 Let AP be the velocity in the descent of the body and AP^2+2BA ... resistance and BD − AB the force of gravity the angle DBA ... vertex B there ... P and DQ produced ... be as the whole time of descent.

... increment PQ of the velocity and the area DPQ proportional to it is as the excess of the gravity above the resistance that is as

$BD^2-AB^2-2BA \cdot AP-AP^2$



or $BD^2 - BP^2$ And the area DTV is to the area DPQ as DT^2 to DP and therefore as GT^2 or $GD^2 - BD^2$ to BP and as GD^2 to BD^2 and by subtraction as BD^2 to $BD - BP$ Therefore since the area DPQ is as $BD^2 - BP^2$ the area DTV will be as the given quantity BD^2 Therefore the area EDT increases uniformly in the several equal intervals of time by the addition of as many given intervals DTV and therefore is proportional to the time of the descent Q E D

COR If with the centre D and the semidiameter DA there be drawn through the vertex A an arc At similar to the arc ET and similarly subtending the angle ADT the velocity AP will be to the velocity which the body in the time EDT in a nonresisting space can lose in its ascent or acquire in its descent as the area of the triangle DAP to the area of the sector DAt and therefore is given from the time given For the velocity in a nonresisting medium is proportional to the time and therefore to this sector in a resisting medium it is as the triangle and in both mediums where it is least it approaches to the ratio of equality as the sector and triangle do

SCHOLIUM

One may demonstrate also that case in the ascent of the body where the force of gravity is less than can be expressed by DA^2 or $AB^2 + BD^2$ and greater than can be expressed by $AB - DB$ and must be expressed by AB^2 But I hasten to other things

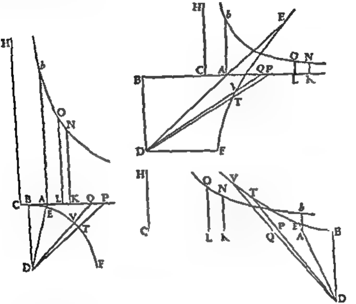
PROPOSITION 14 THEOREM 11

The same things being supposed I say that the space described in the ascent or descent is as the difference of the area by which the time is expressed and of some other area which is augmented or diminished in an arithmetical progression if the forces compounded of the resistance and the gravity be taken in a geometrical progression

Take AC (in these three figures) proportional to the gravity and AK to the resistance but take them on the same side of the point A if the body is descending otherwise on the contrary Erect Ab which make to DB as DB^2 to 4BA CA and to the rectangular asymptotes CK CH describe the hyperbola bN and erecting KN perpendicular to CK the area $AbNK$ will be augmented or diminished in an arithmetical progression while the forces CK are taken in a geometrical progression I say therefore that the distance of the body from its greatest altitude is as the excess of the area $AbNK$ above the area DET

For since AK is as the resistance that is as AP^2 2BA AP assume any given quantity Z and put AK equal to $\frac{AP^2 + 2BA\ AP}{Z}$ then (by Lem 2 of this book) the moment KL of AK will be equal to $\frac{2PQ\ AP + 2BA\ PQ}{Z}$ or $\frac{2PQ\ BP}{Z}$ and the moment KLON of the area $AbNK$ will be equal to $\frac{2PQ\ BP\ LO}{Z}$ or $\frac{PQ\ BP\ BD^3}{2Z\ CK\ AB}$

CASE 1. Now if the body ascends, and the gravity be as $AB^2 + BD^2$, BET being a circle, the line AC, which is proportional to the gravity, will be $\frac{AB^2 + BD^2}{Z}$, and DP^2 or $AP^2 + 2BA \cdot AP + AB^2 + BD^2$ will be $AK \cdot Z + AC \cdot Z$ or $CK \cdot Z$; and therefore the area DTV will be to the area DPQ as DT^2 or DB^2 to $CK \cdot Z$.



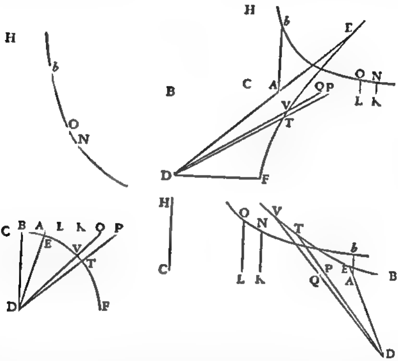
CASE 2. If the body ascends, and the gravity be as $AB^2 - BD^2$, the line AC will be $\frac{AB^2 - BD^2}{Z}$, and DT^2 will be to DP^2 as DF^2 or DB^2 to $BP^2 - BD^2$ or $AP^2 + 2BA \cdot AP + AB^2 - BD^2$, that is, to $AK \cdot Z + AC \cdot Z$ or $CK \cdot Z$. And therefore the area DTV will be to the area DPQ as DB^2 to $CK \cdot Z$.

CASE 3. And by the same reasoning, if the body descend, and therefore the gravity is as $BD^2 - AB^2$, and the line AC becomes equal to $\frac{BD^2 - AB^2}{Z}$, the area DTV will be to the area DPQ as DB^2 to $CK \cdot Z$, as above.

Since, therefore, these areas are always in this ratio, if for the area DTV, by which the moment of the time, always equal to itself, is expressed, there be put any determinate rectangle, as $BD \cdot m$, the area DPQ, that is, $\frac{1}{2}BD \cdot PQ$, will be to $BD \cdot m$ as $CK \cdot Z$ to BD^2. And thence $PQ \cdot BD^3$ becomes equal to $2BD \cdot m \cdot CK \cdot Z$, and the moment KLON of the area AbNK, found before, becomes $\frac{BP \cdot BD \cdot m}{AB}$. From the area DET subtract its moment DTV or $BD \cdot m$, and there will remain $\frac{AP \cdot BD \cdot m}{AB}$. Therefore the difference of the moments, that is, the moment of the difference of the areas, is equal to $\frac{AP \cdot BD \cdot m}{AB}$; and

therefore (because of the given quantity $\frac{BD\ m}{AB}$) as the velocity AP that is as the moment of the space which the body describes in its ascent or descent And therefore the difference of the areas and that space increasing or decreasing by proportional moments and beginning together or vanishing together are proportional

Q E D



COR If the length which arises by applying the area DET to the line BD be called M and another length V be taken in that ratio to the length M which the line DA has to the line DE the space which a body in a resisting medium describes in its whole ascent or descent will be to the space which a body in a nonresisting medium falling from rest can describe in the same time as the difference of the aforesaid areas to $\frac{BD\ V^2}{AB}$ and therefore is given from the time given For the space in a nonresisting medium is as the square of the time or as V^2 and because BD and AB are given as $\frac{BD\ V^2}{AB}$ This area is equal to the area $\frac{DA^2\ BD\ M^2}{DE^2\ AB}$ and the moment of M is m and therefore the moment of this area is $\frac{DA^2\ BD\ 2M\ m}{DE^2\ AB}$ But this moment is to the moment of the difference of the aforesaid areas DET and AbNK viz to $\frac{AP\ BD\ m}{AB}$ as $\frac{DA^2\ BD\ M}{DE^2}$ to $\frac{1}{2}BD\ AP$ or as $\frac{DA^2}{DE^2}$ into DET to DAP and therefore when the areas DET and DAP are least in the ratio of equality Therefore the area $\frac{BD\ V^2}{AB}$ and the difference of the areas DET and AbNK when all these areas

are least have equal moment and are therefore equal Therefore since the velocities, and therefore also the spaces in both mediums described together in the beginning of the descent or the end of the ascent approach to equality and therefore are then one to another as the area $\frac{BD \times V^2}{AB}$ and the difference of the areas DET and AbNK and moreover since the space in a nonresisting medium is continually as $\frac{BD \times V^2}{AB}$ and the space in a resisting medium is continually as the difference of the areas DET and AbNK it necessarily follows, that the spaces, in both mediums, described in any equal times, are one to another as that area $\frac{BD \times V^2}{AB}$ and the difference of the areas DET and AbNK. Q.E.D

SCHOLIUM

The resistance of spherical bodies in fluids arises partly from the tenacity ... And that

the fluid is uniform or as the moment of the time and therefore ... no proceed to the motion of bodies, which are resisted partly by an uniform force or in the ratio of the moments of the time and partly as the square of the velocity But it is sufficient to have cleared the way to this speculation in Props. 8 and 9 foregoing and their Corollaries. For in those Propositions, instead of the uniform resistance made to an ascending body arising from its ... uniform resistance which arises from the te-

ratio of the velocity and in part as the square of the same velocity And I

things.

SECTION IV

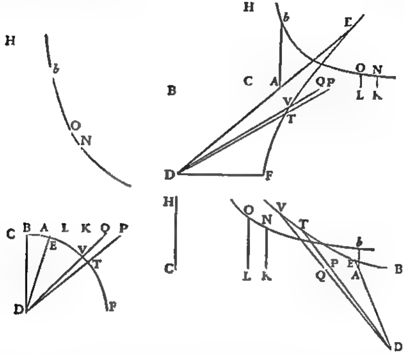
THE CIRCULAR MOTION OF BODIES IN RESISTING MEDIUMS

LEMMA 3

Let PQR be a spiral cutting all the radii SP SQ SR &c in equal angles Draw the right line PT touching the spiral in any point P and cutting the radius SQ in T draw PO QO perpendicular to the spiral and meeting in O and join SO I say that if the points P and Q approach and coincide the angle PSO will become a right angle and the ultimate ratio of the rectangle TQ 2PS to PQ will be the ratio of equality

For from the right angles OPQ OQR subtract the equal angles SPQ SQR and there will remain the equal angles OPS OQS Therefore a circle which

therefore (because of the given quantity $\frac{BD\ m}{AB}$) as the velocity AP that is as the moment of the space which the body describes in its ascent or descent And therefore the difference of the areas and that space increasing or decreasing by proportional moments and beginning together or vanishing together are proportional Q E D



Cor If the length which arises by applying the area DET to the line BD be called M and another length V be taken in that ratio to the length M which the line DA has to the line DE the space which a body in a resisting medium describes in its whole ascent or descent will be to the space which a body in a nonresisting medium falling from rest can describe in the same time as the difference of the aforesaid areas to $\frac{BD\ V^2}{AB}$ and therefore is given from the time given For the space in a nonresisting medium is as the square of the time or as V^2 and because BD and AB are given as $\frac{BD\ V^2}{AB}$ This area is equal to the area $\frac{DA^2\ BD\ M^2}{DE^2\ AB}$ and the moment of M is m and therefore the moment of this area is $\frac{DA^2\ BD\ 2M\ m}{DE^2\ AB}$ But this moment is to the moment of the difference of the aforesaid areas DET and A*b*NK viz to $\frac{AP\ BD\ m}{AB}$ as $\frac{DA\ BD\ M}{DE}$ to ½BD AP or as $\frac{DA^2}{DE^2}$ into DET to DAP and therefore when the areas DET and DAP are least in the ratio of equality Therefore the area $\frac{BD\ V^2}{AB}$ and the difference of the areas DET and A*b*NK when all these areas

are least have equal moments and are therefore equal Therefore since the velocities and therefore also the spaces in both mediums described together in the beginning of the descent or the end of the ascent approach to equality and therefore are then one to another as the area $\frac{BD \cdot V^2}{AB}$ and the difference of the areas DET and AbNK and moreover since the space in a nonresisting medium is continually as $\frac{BD \cdot V}{AB}$ and the space in a resisting medium is continually as the difference of the areas DET and AbNK it necessarily follows that the spaces in both mediums described in any equal times are one to another as that area $\frac{BD \cdot V^2}{AB}$ and the difference of the areas DET and AbNK

Q E D

SCHOLIUM

The resistance of spherical bodies in fluid arises partly from the tenacity ... f ... th d n ity of the medium And that ... of the fluid is as I said as ... arises from the tenacity of ... e and therefore we might now proceed to the motion of bodies which are resisted partly by an uniform force or in the ratio of the moments of the time and partly as the square of the velocity But it is sufficient to have cleared the way to this speculation in Props 8 and 9 foregoing and their Corollaries For in those Propositions instead of the uniform resistance made to an ascending body arising from its ... h ... ce which arises from the te-

SECTION IV

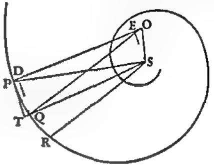
THE CIRCULAR MOTION OF BODIES IN RESISTING MEDIUMS

LEMMA 3

Let PQR be a spiral cutting all the radii SP SQ SR &c in equal angles Draw the right line PT touching the spiral in any point P and cutting the radius SQ in T draw PO QO perpendicular to the spiral and meeting in O and join SO I say that if the points P and Q approach and coincide the angle PSO will become a right angle and the ultimate ratio of the rectangle TQ 2PS to PQ will be the ratio of equality

For from the right angles OPQ OQR subtract the equal angles SPQ SQR and there will remain the equal angles OPS OQS Therefore a circle which

passes through the points OSP will pass also through the point Q Let the points P and Q coincide and this circle will touch the spiral in the place of coincidence PQ and will therefore cut the right line OP perpendicularly Therefore OP will become a diameter of this circle and the angle OSP being in a semicircle becomes a right one Q E D



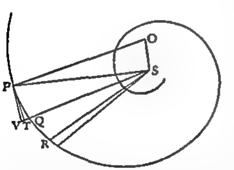
Draw QD SE perpendicular to OP and the ultimate ratios of the lines will be as follows

TQ PD = TS or PS PE = 2PO 2PS
and PD PQ = PQ 2PO
multiplying together corresponding terms of equal ratios
TQ PQ = PQ 2PS
Whence PQ becomes equal to TQ 2PS
Q E D

PROPOSITION 15 THEOREM 12

If the density of a medium in each place thereof be inversely as the distance of the places from an immovable centre and the centripetal force be as the square of the density I say that a body may revolve in a spiral which cuts all the radii drawn from that centre in a given angle

Suppose everything to be as in the foregoing Lemma and produce SQ to V so that SV may be equal to SP In any time let a body in a resisting medium describe the least arc PQ and in double the time the least arc PR and the decrements of those arcs arising from the resistance or their differences from the arcs which would be described in a non resisting medium in the same times will be to each other as the squares of the times in which they are generated therefore the decrement of the arc PQ is the fourth part of the decrement of the arc PR Whence also if the area QSr be taken equal to the area PSQ the decrement of the arc PQ will be equal to half the short line Rr and therefore the force of resistance and the centripetal force are to each other as the short line



½Rr and TQ which they generate in the same time Because the centripetal force with which the body is urged in P is inversely as SP and (by Lem 10 Book 1) the short line TQ which is generated by that force is in a ratio compounded of the ratio of this force and the squared ratio of the time in which the arc PQ is described (for in this case I neglect the resistance as being infinitely

by the last

the time is

is described

in that time as $\frac{TQ}{PQ\ \sqrt{SP}}$ or $\frac{1}{\sqrt{SP}}$ that is inversely as the square root of SP And by a like reasoning the velocity with which the arc QR is described is inversely as the square root of SQ Now those arcs PQ and QR are as the describing velocities to each other that is as the square root of the ratio of

SO to SP or as SQ to $\sqrt{(SP\ SQ)}$ and because of the equal angles SPQ SQr PQ is to the arc Qr as SQ to SP Take the ... the arc ... ing the ... nce the ... is as the ... ll be as ... comes as

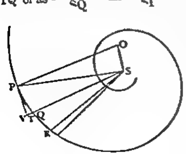
$\frac{Rr}{PQ\cdot SP}$ But PQ was to Rr as SQ to ½ ...

$\frac{\frac{1}{2}VQ}{PQ\ SP\ SQ}$ or as $\frac{\frac{1}{2}OS}{OP\ SP^2}$ For the points P and Q coinciding SP and SQ coincide also and the angle PVQ becomes a right one and because of the similar triangles PVQ PSO PQ becomes to ½ VQ as OP to ½OS Therefore $\frac{OS}{OP\ SP^2}$ is as the resistance that is in the ratio of the density of the medium in P and the squared ratio of the velocity conjointly Subtract the squared ratio of the velocity namely the ratio $\frac{1}{SP}$ and there will remain the density of the medium in P as $\frac{OS}{OP\ SP}$ Let the spiral be given and because of the given ratio of OS to OP the density of the medium in P will be as $\frac{1}{SP}$ Therefore in a medium whose density is inversely as SP the distance from the centre a body will revolve in this spiral Q E D

COR. I. The velocity in any place P is always the same wherewith a body in a nonresisting medium with the same centripetal force would revolve in a circle at the same distance SP from the centre

COR. II. The density of the medium if the distance SP be given is as $\frac{OS}{OP}$ but if that distance is not given as $\frac{OS}{OP\ SP}$ And thence a spiral may be fitted to any density of the medium

COR. III The force of the resistance in any place P is to the centripetal force in the same place as ½OS to OP For those forces are to each other as ½Rr and TQ or as $\frac{\frac{1}{2}VQ\ PQ}{SQ}$ and $\frac{\frac{1}{2}PQ^2}{SP}$ that is as ½VQ and PQ or ½OS and OP The spiral therefore being given there is given the proportion of the resistance to the centripetal force and conversely from that proportion given the spiral is given



COR IV Therefore the body cannot revolve in this spiral except where the force of resistance is less than half the centripetal force Let the resistance be made equal to half the centripetal force and the spiral will coincide with the right line PS and in

passes through the points OSP will pass also through the point Q Let the points P and Q coincide and this circle will touch the spiral in the place of coincidence PQ and will therefore cut the right line OP perpendicularly Therefore OP will become a diameter of this circle and the angle OSP being in a semicircle becomes a right one

Q E D

Draw QD SE perpendicular to OP and the ultimate ratios of the lines will be as follows

TQ PD=TS or PS PE=2PO 2PS

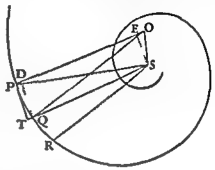
and PD PQ=PQ 2PO

multiplying together corresponding terms of equal ratios

TQ PQ=PQ 2PS

Whence PQ^2 becomes equal to TQ 2PS

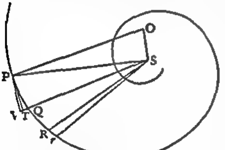
Q E D



PROPOSITION 15 THEOREM 12

If the density of a medium in each place thereof be inversely as the distance of the places from an immovable centre and the centripetal force be as the square of the density I say that a body may revolve in a spiral which cuts all the radii drawn from that centre in a given angle

Suppose everything to be as in the foregoing Lemma and produce SQ to V so that SV may be equal to SP In any time let a body in a resisting medium describe the least arc PQ and in double the time the least arc PR and the decrements of those arcs arising from the resistance or their differences from the arcs which would be described in a non resisting medium in the same times will be to each other as the squares of the times in which they are generated therefore the decrement of the arc PQ is the fourth part of the decrement of the arc PR Whence also if the area QSr be taken equal to the area PSQ the decrement of the arc PQ will be equal to half the short line Rr and therefore the force of resistance and the centripetal force are to each other as the short line ½Rr and TQ which they generate in the same time Because the centripetal force with which the body is urged in P is inversely as SP^2 and (by Lem 10 Book 1) the short line TQ which is generated by that force is in a ratio com



Lemma) ½PQ^2 SP will be as the square of the time and therefore the time is as PQ $\sqrt{SP}$ and the velocity of the body with which the arc PQ is described in that time as $\frac{PQ}{PQ\ \sqrt{SP}}$ or $\frac{1}{\sqrt{SP}}$ that is inversely as the square root of SP And by a like reasoning the velocity with which the arc QR is described is inversely as the square root of SQ Now those arcs PQ and QR are as the describing velocities to each other that is as the square root of the ratio of

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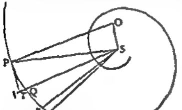
PROPOSITION 16 THEOREM 13

power of the same distance I say that the body
all the radii drawn from that centre in a given angle

This demonstrated in the same manner as the foregoing Proposition For if the centripetal force in P be inversely as any power SP of the distance SP whose index is $n+1$ it will be concluded as above that the time in which the body describes any arc PQ will be as PQ PS ^{n} and the resistance in P as $\frac{Pr}{IQ^2\ SI}$ or as $\frac{(1-\frac{1}{2}n)\ VQ}{PQ\ SP\ SQ}$ and therefore as $\frac{(1-\frac{1}{2}n)\ OS}{OI\ SP}$ that is (because $\frac{(1-\frac{1}{2}n)\ OS}{OP}$ is a given quantity) inversely as SP^{n+1} And therefore since the velocity is inversely as $SP^{\frac{1}{2}n}$ the density in P will be reciprocally as SP



COR. I. The resistance is to the centripetal force as $(1-\frac{1}{2}n)$ OS to OP

COR. II. If the centripetal force be inversely as SP^3 $1-\frac{1}{2}n$ will be $=0$ and therefore the resistance and density of the medium will be nothing, as in Prop 9 Book I

COR. III If the centripetal force be inversely as any power of the radius SP whose index is greater than the number 3 the positive resistance will be changed into a negative

SCHOLIUM

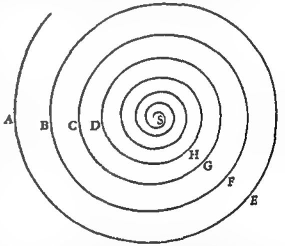
This Proposition and the former which relate to mediums of unequal density are to be understood as applying only to the motion of bodies that are so small that the greater density of the medium on one side of the body above that on the other is not to be considered. I suppose also the resistance other

in a nonresisting medium as the square root of the ratio of unity to the number 2 And the times of the descent will be here inversely as the velocities and therefore given

COR V And because at equal distances from the centre the velocity is the same in the spiral PQR as it is in the right line SP and the length of the spiral is to the length of the right line PS in a given ratio namely in the ratio of OP to OS the time of the descent in the spiral will be to the time of the descent in the right line SP in the same given ratio and therefore given

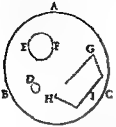
COR VI If from the centre S with any two given radii two circles are described and these circles remaining the angle which the spiral makes with the radius PS be changed in any manner the number of revolutions which the body can complete in the space between the circumferences of those circles going round in the spiral from one circumference to another will be as $\frac{PS}{OS}$ or as the tangent of the angle which the spiral makes with the radius PS and the time of the same revolutions will be as $\frac{OP}{OS}$ that is as the secant of the same angle or inversely as the density of the medium

COR VII If a body in a medium whose density is inversely as the distances of places from the centre revolves in any curve AEB about that centre and cuts the first radius AS in the same angle in B as it did before in A and that with a velocity that shall be to its first velocity in A inversely as the square root of the distances from the centre (that is as AS to a mean proportional between AS and BS) that body will continue to describe innumerable similar revolutions BFC CGD &c and by its intersections will divide the radius AS into parts AS BS CS DS &c that are continually proportional But the times of the revolutions will be directly as the perimeters of the orbits AFB BFC CGD &c and inversely as the velocities at the beginnings A B C of those orbits that is as $AS^{3/2}$ $BS^{3/2}$ $CS^{3/2}$ And the whole time in which the body will arrive at the centre will be to the time of the first revolution as the sum of all the continued proportionals $AS^{3/2}$ $BS^{3/2}$ $CS^{3/2}$ going on *ad infinitum* is to the first term $AS^{3/2}$ that is as the first term $AS^{3/2}$ is to the difference of the two first $AS^{3/2} - BS^{3/2}$ or as $\frac{2}{3}$AS is to AB very nearly Whence the whole time may be easily found



COR VIII From hence also may be deduced near enough the motions of bodies in mediums whose density is either uniform or observes any other assigned law From the centre S with radii SA SB SC &c continually proportional describe as many circles and suppose the time of the revolutions between the perimeters of any two of those circles in the medium whereof we treated to be to the time of the revolutions between the same in the medium

sure For if any part as D be moved all such parts at the same distance from the centre on every sic ... ly be moved at the same time by a like ... and not all of ... ensed ... f they ... wards ... uppo- ... on re- ... or the same reason, they may move in ... ection



but the same part cannot be moved contrary ways at the same time Therefore no part of the fluid will be moved *from its place* Q E D

CASE 2 I say now that all the spherical parts of this fluid are equally pressed on every side For let EF be a spherical part of the fluid if this be not pressed equally on every side augment the lesser pressure till it be pressed equally on every side and its parts (by Case 1) *will remain in their places. But before the increase of the pressure* they would remain in their places (by Case 1) and by the addition of a new pressure they will be moved by the definition of a fluid from those places Now these two conclusions contradict each other Therefore it was false to say that the sphere EF was not pressed equally on every side Q E.D

CASE 3 I say besides, that different spherical parts have equal pressures. For th contiguous spherical parts press each other mutually and equally in the ... pressed on every sid

force

CASE 4 I say now that all the parts of the fluid are everywhere pressed equally For any two parts may be touched by spherical parts in any points whatever and there they will equally press those spherical parts (by Case 3) and are in reaction equall pressed by them (by Law III) Q E.D

CASE 5 Since therefore any part GHI of the fluid is inclosed by the rest of th fluid as in a vessel and is equally pressed on every side and also it parts equally press one an ther and are at rest among themselves it is manifest that all the parts of any fluid as GHI which is pressed equally on every side do press each other mutually and equally and are at rest among themselves. Q E.D

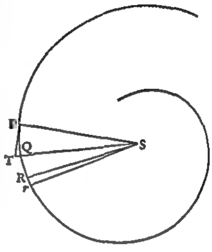
CASE 6 Therefore if that fluid be included in a vessel of a yielding substance o that is not rigid, and be not equally pressed on every side the same will give

the fluid as soon as t endeavors to recede from the part that is most pressed is withstood by the resistance of the vessel on the opposite side the pressure

things being equal to be proportional to its density Hence in mediums whose force of resistance is not as the density the density must be so much augmented or diminished that either the excess of the resistance may be taken away or the defect supplied

PROPOSITION 17 PROBLEM 4

To find the centripetal force and the resisting force of the medium by which a body the law of the velocity being given shall revolve in a given spiral



Let that spiral be PQR From the velocity with which the body goes over the very small arc PQ the time will be given and from the altitude TQ which is as the centripetal force and the square of the time that force will be given Then from the difference RSr of the areas PSQ and QSR described in equal intervals of time the retardation of the body will be given and from the retardation will be found the resisting force and density of the medium

PROPOSITION 18 PROBLEM 5

The law of centripetal force being given to find the density of the medium in each of the places thereof by which a body may describe a given spiral

From the centripetal force the velocity in each place must be found then from the retardation of the velocity the density of the medium is found as in the foregoing Proposition

But I have explained the method of managing these Problems in the tenth Proposition and second Lemma of this book and will no longer detain the reader in these complicated investigations I shall now add some things relating to the forces of progressive bodies and to the density and resistance of those mediums in which the motions hitherto discussed and those akin to them are performed

SECTION V

THE DENSITY AND COMPRESSION OF FLUIDS HYDROSTATICS

THE DEFINITION OF A FLUID

A FLUID IS ANY BODY WHOSE PARTS YIELD TO ANY FORCE IMPRESSED ON IT AND BY YIELDING ARE EASILY MOVED AMONG THEMSELVES

PROPOSITION 19 THEOREM 14

All the parts of an homogeneous and unmoved fluid included in any unmoved

perpendicular or oblique or whether the fluid continued upwards from the compressed surface rises perpendicularly in a rectilinear direction or creeps obliquely through crooked cavities and canals whether those passages be regular or irregular wide or narrow That the pressure is not altered by any of these circumstances may be inferred by applying the demonstration of this

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COR IV And therefore if another body of the same specific gravity is capable of condensation be immersed in this fluid it will acquire no motion by the pressure of the incumbent weight it will neither descend nor ascend nor change its figure If it be spherical it will remain so notwithstanding the pressure

ll remain square and that whether it be soft or fluid

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figure and that because be at rest and retain

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COR. Therefore bodies placed in fluids have a twofold gravity the one absolute the other apparent common and comparative Absolute

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mpared with one another they do not preponderate but hindering one an the endeavor to descend remain in their proper places as if they were not hea

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also the weight of the air Hence also commonly those things are called light which are less heavy and by yielding to the preponderating air mount

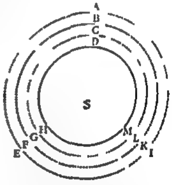
will on every side be reduced to equality in a moment of time without any local motion and from thence the parts of the fluid (by Case 5) will press each other mutually and equally and be at rest among themselves Q E D

COR Hence neither will a motion of the parts of the fluid among themselves be changed by a pressure communicated to the external surface except so far as either the figure of the surface may be somewhere altered or that all the parts of the fluid by pressing one another more intensely or remissly may slide with more or less difficulty among themselves

PROPOSITION 20 THEOREM 15

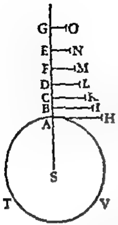
If all the parts of a spherical fluid homogeneous at equal distances from the centre lying on a spherical concentric bottom gravitate towards the centre of the whole the bottom will sustain the weight of a cylinder whose base is equal to the surface of the bottom and whose altitude is the same with that of the incumbent fluid

Let DHM be the surface of the bottom and AEI the upper surface of the fluid Let the fluid be divided into concentric orbs of equal thickness by the innumerable spherical surfaces BFK CGL and conceive the force of gravity to act only in the upper surface of every orb and the actions to be equal on the equal parts of the surfaces Therefore the upper surface AE is pressed by the single force of its own gravity by which all the parts of the upper orb and the second surface BFK will (by Prop 19) according to its measure be equally pressed The second surface BFK is pressed likewise by the force of its own gravity which added to the former force makes the pressure double The third surface CGL is according to its measure acted on by this pressure and the force of its own gravity besides which makes its pressure triple And in like manner the fourth surface receives a quadruple pressure the fifth surface a quintuple and so on Therefore the pressure acting on every surface is not as the solid quantity of the incumbent fluid but as the number of the orbs reaching to the upper surface of the fluid and is equal to the gravity of the lowest orb multiplied by the number of orbs that is to the gravity of a solid whose ultimate ratio to the cylinder above mentioned (when the number of the orbs is increased and their thickness diminished *ad infinitum* so that the action of gravity from the lowest surface to the uppermost may become continued) is the ratio of equality Therefore the lowest surface sustains the weight of the cylinder above determined Q E D And by a like reasoning the Proposition will be evident where the gravity of the fluid decreases in any assigned ratio of the distance from the centre and also where the fluid is more rare above and denser below Q E D



COR I Therefore the bottom is not pressed by the whole weight of the incumbent fluid but only sustains that part of it which is described in the Proposition the rest of the weight being sustained archwise by the spherical figure of the fluid

COR II The quantity of the pressure is the same always at equal distances from the centre whether the surface pressed be parallel to the horizon or

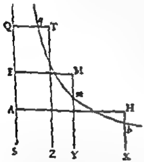


particle B as the um of all AH+BI+CK+DL in in
sum of all BI+CK+DL &c And BI
den
+DL,
&c to the sum of all these
sums are proportional to their differences AH BI CK
&c and therefore continually proportional (by Lem
1 of this book) and therefore the differences AH BI
to the sums are also continually

be continually propor
and at the distances SA SC SE continually propor
tional the densities AH CK, EM will be continually
proportional And by the same reasoning at any dis-
tances SA SD SG continually proportional the den
sities AH DL, GO will be continually proportional Let now the points A, B
C D E &c coincide so that the progression of the specific gravities from
the bottom A to the top of the fluid may be made continual and at any dis-
tances SA, SD SG continually proportional the densities AH DL, GO being
all along continually proportional will still remain continually proportional

Q.E.D

COR Hence if the density of the fluid in two places as A and E, be given its
density in any other place Q may be obtained With the centre S and the
rectangular asymptotes SQ SX, describe an hyperbola cutting the perpen
diculars AH EM QT in *a* *e* and *q* as also the
perpendiculars HX, MY TZ let fall upon the
asymptote SX, in *h* *m* and *t* Make the area Y*mt*Z
to the given area Y*mh*X as the given area E*eq*Q
to the given area E*ea*A and the line Z*t* produced
will cut off the line QT proportional to the den



obtain an other order in the series of continued proportionals the lines AH
FM QT because of the proportional hyperbolic areas will obtain the same
order in another series of quantities continually proportional

PROPOSITION 22 THEOREM 1

Let the density of any fluid be proportional to the compression and its parts be

upwards But these are only comparatively light and not truly so because they descend in a vacuum Thus in water bodies which by their greater or less gravity descend or ascend are comparatively and apparently heavy or light and their comparative and apparent gravity or levity is the excess or defect by which their true gravity either exceeds the gravity of the water or is exceeded by it But those things which neither by preponderating descend nor by yielding to the preponderating fluid ascend although by their true weight they do increase the weight of the whole yet comparatively and as commonly understood they do not gravitate in the water For these cases are alike demonstrated

COR VII These things which have been demonstrated concerning gravity take place in any other centripetal forces

COR VIII Therefore if the medium in which any body moves be acted on either by its own gravity or by any other centripetal force and the body be urged more powerfully by the same force the difference of the forces is that very motive force which in the foregoing Proposition I have considered as a centripetal force But if the body be more lightly urged by that force the difference of the forces becomes a centrifugal force and is to be considered as such

COR IX But since fluids by pressing the included bodies do not change their external figures it appears also (by Cor Prop 19) that they will not change the situation of their internal parts in relation to one another and therefore if animals were immersed therein and if all sensation did arise from the motion of their parts the fluid would neither hurt the immersed bodies nor excite any sensation unless so far as those bodies might be condensed by the compression And the case is the same of any system of bodies encompassed with a compressing fluid All the parts of the system will be agitated with the same motions as if they were placed in a vacuum and would only retain their comparative gravity unless so far as the fluid may somewhat resist their motions or be requisite to unite them by compression

PROPOSITION 21 THEOREM 16

Let the density of any fluid be proportional to the compression and its parts be attracted downwards by a centripetal force inversely proportional to the distances from the centre I say that if those distances be taken continually proportional the densities of the fluid at the same distances will be also continually proportional

Let ATV denote the spherical bottom of the fluid S the centre SA SB SC SD SE SF &c distances continually proportional Erect the perpendiculars AH BI CK DL EM FN &c which shall be as the densities of the medium in the places A B C D E F and the specific gravities in those places will be as $\frac{AH}{AS}$ $\frac{BI}{BS}$ $\frac{CK}{CS}$ &c or which is all one as $\frac{AH}{AB}$ $\frac{BI}{BC}$ $\frac{CK}{CD}$ &c Suppose first these gravities to be uniformly continued from A to B from B to C from C to D &c the decrements in the points B C D &c being taken by steps And these gravities multiplied by the altitudes AB BC CD &c will give the pressures AH BI CK &c by which the bottom ATV is acted on (by Theor 15) Therefore the particle A sustains all the pressures AH BI CK DL &c proceeding *in infinitum* and the particle B sustains the pressures of all but the first AH and the particle C all but the two first AH BI and so on and therefore the density AH of the first particle A is to the density BI of the second

FN will be found at any height SF by taking the area *thn* to that given area *thu* as the difference $Aa-Ff$ to the difference $Aa-Bb$

SCHOLIUM

— of the particles of a tre and the reciprocals of the quares of the distances SA SB &c nely $\frac{SA^2}{SA}, \frac{SA^2}{SB}, \frac{SA^2}{SC}$) be taken in an arithmetical progression the densities AH BI CK &c will be in a geometrical progression And if the gravity be diminished as the fourth power of the distances and the reciprocals of the cubes of the distances (as $\frac{SA}{SA}, \frac{SA}{SB}$ SA^4 &c) be taken in arithmetical progression the densities AH BI CK &c

of

arithmetical progression the den

Dr Halley hath found If the gravity be as the distance and the quares of the d t nces be in arithmetical progression the densities will be in geometrical progression And o in t *finitum* These things will be so when the density of the flu d condensed by compression is as the force of compression or which is m thin when the pace possessed by the fluid is inversely as this force ompress- ie ratio of

f the compressing force be as the

be inversely as the quare of the di tance the density will be inversely as the fifth po er of the di tance Suppose the compressing force to be as the square f the density and the gravity inversely as the quare of the di tance then the d nsity ll be in ersely as the distance To run over all the cases that might be ffered ould be tedious But as to our own air this is certain from experi at l ast as the com the f the mercury in the barometer

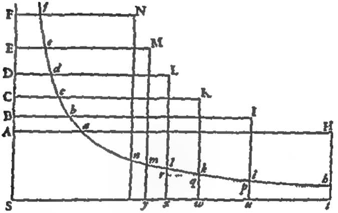
PROPOSITION 23 THEOREM 18

thereof in those places will be as $\frac{AH}{SA^2}$ $\frac{BI}{SB^2}$ $\frac{CK}{SC^2}$ &c Suppose these gravitie to be uniformly continued the first f om ...

... proportional

to those altitudes will give $\frac{AH}{SA}$ $\frac{BI}{SB}$ $\frac{CK}{SC}$ &c repre ent ...

... centre S and the asymptote ...



perpendiculars Ht Iu Kw let fall upon the asymptote Sx in h i k and the differences of the densities tu uw &c will be as $\frac{AH}{SA}$ $\frac{BI}{SB}$ &c And the rectangles tu th uw ui &c or tp uq &c as $\frac{AH\ th}{SA}$ $\frac{BI\ ui}{SB}$ &c that is as Aa Bb &c For by the nature of the hyperbola SA is to AH or St as th to Aa and therefore $\frac{AH\ th}{SA}$ is equal to Aa And by a like reasoning $\frac{BI\ ui}{SB}$ is equal to Bb &c But Aa Bb Cc &c are continually proportional and therefore proportional to their differences $Aa-Bb$ $Bb-Cc$ &c therefore the rectangle tp uq &c are proportional to th ... ngles $tp+uq$ or $tp+uq+wr$ ... Suppose ... will be pro ...

... proportional to those differences densities St Sx Sz that is AH BI

Q E D

... e fluid as AH and BI be given the area thiu answering to their difference tu will be given and thence the density

— — d that he next

t
h other is a
property of
rs may take

luids consisting of particles of this kind
xcasion to discuss that question

SECTION VI

THE MOTION AND RESISTANCE OF PENDULOUS BODIES

PROPOSITION 24 THEOREM 19

The quantities of matter in pendulous bodies whose centres of oscillation are equally distant f om the centre of suspension are in a ratio compounded of the rat o f the weights and the squared ratio of the times of the oscillations in a acu m

wh h a given force can generate in a given matter in a

f

t
t

oscillating describe equal arcs and those arcs are divided

since the times in which the bodies describe each of the correspondent parts of the arcs are as the times of the whole oscillation the velocities in the corresponding parts of the osc llat ons will be to each other directly as the motive

are inversely as the times and therefore the times are directly and the velocities inversely as the squares of the times and therefore the quantities of matter are as the motive forces and the squares of the times that is, as the weights nd the squares of the times Q E D

COR I Therefore if the times are equal the quantities of matter in each of th bod es are as the weight

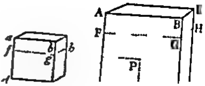
COR. II If the weights are equal the quantities of matter will be as the squares of the times

COR. III If the quantities of matter are equal the weights will be inversely as the squares of the times

COR. IV Since the squares of the times other things being equal are as the lengths of the pendulum therefore if both the times and the quantities of matter are equal the weights will be as the lengths of the pendulums

COR V And in general the quantity of matter in the pendulous body is directly as the weight and the square of the time and inversely as the length of the pendulum

greater cube ABCD take the square DP equal to the plane side db of the lesser cube and by the supposition the pres o h



h u fluid as the l

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ms of the proportion th ore multiplying togethe

in the pro
th

The number of the particles
according to the planes FGH fgh upon all are as the forces which all the which each exerts on each Therefore the forces which each exerts on each according to the plane FGH in the greater cube are to the forces which each exerts on each according to the plane fgh in the lesser cube as ab to AB that inversely as the distances of the particles from each other Q E D

And conversely if the forces of the single particles are inversely as the distances that is inversely as the sides of the cubes AB ab the sums of the forces will be in the same ratio and the pressures of the sides DB db as the sums of the forces and the pressure of the square DP to the pressure of the side DB as ab to AB And multipl ng corresponding terms of the n on DP to the pressure of

sion in the other as the density on in the one is to the in the latter in the former to the density
Q E D

SCHOLIUM

B ugal forces of the particles are inversely as the centres the cubes of the compressing f the densities If the centrifugal force be er of th

power L b ts will be th
are to be un
particles tha
example of tl
n attractive force is terminated nearly in bodies of their own kind that are next them The force of the magnet is reduced by the interposition of an iron plate and is almost terminated at it for bodies farther off are not attracted by the magnet so much as by the iron

The force with which the body D in a nonresisting medium is retarded in E is as CE, and the force with which the body d in the resisting medium is re- ... force Ce and the resistance CO that is as Oe ... s are retarded are as the arcs CB ... retarded in

... of the ... nal to ... Q E D

the whole arcs ... the swiftest motion in a resisting medium does not fall upon

Co Therefore the swiftest motion in a resisting medium ... whole arc de- ... is retarded ... from B to O

PROPOSITION 26 THEOREM 21

Pendulous bodies that are resisted in the ratio of the velocity have their oscillations in a cycloid isochronal

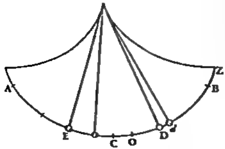
... from the centres of suspension describe ... he ... the

... decrem ... of the ...

... the motion when the

PROPOSITION 27 THEOREM 22

If pendulous bodies are resisted as the square of their velocities the differences between the times of the oscillations in a resisting medium and the times of the oscillations in a non resisting medium of the same specific gravity will be proportional to the arcs described in oscillating nearly



For let equal pendulums in a resisting medium describe the unequal arcs A B and the resistance of the body in the arc A will be to the resistance of the body in the

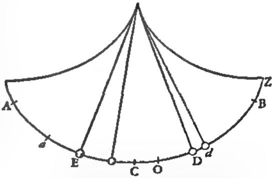
COR VI But in a nonresisting medium the quantity of matter in the pendulous body is directly as the comparative weight and the square of the time and inversely as the length of the pendulum For the comparative weight is the motive force of the body in any heavy medium as was shown above and therefore does the same thing in such a nonresisting medium as the absolute weight does in a vacuum

COR VII And hence appears a method both of comparing bodies one with another as to the quantity of matter in each and of comparing the weights of the same body in different places to know the variation of its gravity And by experiments made with the greatest accuracy I have always found the quantity of matter in bodies to be proportional to their weight

PROPOSITION 25 THEOREM 20

Pendulous bodies that are in any medium resisted in the ratio of the moments of time and pendulous bodies that move in a nonresisting medium of the same specific gravity perform their oscillations in a cycloid in the same time and describe proportional parts of arcs together

Let AB be an arc of a cycloid which a body D by vibrating in a nonre i tin medium shall describe in any time Bisect that arc in C so that C may be the lowest point thereof and the accelerative force with which the body is urged in any place D or d or E will be as the length of the arc CD or Cd or CE Let that force be expressed by that same arc and since the resistance is as the moment of the time and therefore given let it be expre ed by the given part CO of the cycloidal arc and take the arc Od in the same ratio to the arc CD that the arc OB has to the arc CB and the force with which the body in d is urged in a resisting medium being the excess of the force Cd above the re i t ance CO will be expressed by the arc Od and will therefore be to the force with which the body D is urged in a non re isting medium in the place D as the arc Od to the arc CD and therefore also in the place B as the arc OB to the arc CB Therefore if two bodies D d go from the place B and are urged by the e forces since the forces at the beginning are as the arcs CB and OB the first velocitie and arcs first de cribed will be in the same ratio Let those



e ratio Therefore the forces being propor main in the same ratio as at the beginning inue describing together arcs in the ime ratio Therefore the force and velocities and the remaining arcs CD Od will be always as the whole arcs CB OB and therefore tho e remaining arc will be described together Therefore the two bodie D and d will arrive together

at the
nce the
e cribe
nd Oc

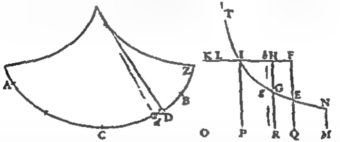
(doubling those arcs) as the whole cycloidal arc or twice the length of the pendulum is to the arc Aa

Q E D

PROPOSITION 29 PROBLEM 6

Supposing that a body oscillating in a cycloid is resisted as the square of the ... in each place

... llation C the lowest point of the ... equal to the length of the pen... ance of the body in any place D ... O S P Q so that (erecting the ... the centre O and the asymptotes ... tting the perpendiculars ST PI ... aving KF parallel to the asymptote OQ meeting the asymptote OK in ..., and the perpendiculars ST and QE in L and F) the hyperbolic area PIEQ may be to the hyperbolic area PITS as the arc BC described in the descent of the body is to the arc Ca described in the ascent and that the area IEF may be to the area ILT as OQ to OS Then



with the perpendicular MN cut off the hyperbolic area PINM and let that area be to the hyperbolic area PIEQ as the arc CZ to the arc BC described in the descent. And if the perpendicular RG cut off the hyperbolic area PIGR, which shall be to the area PIEQ as any arc CD is to the arc BC described in the whole descent the resistance in any place D will be to the force of gravity as the area $\frac{OR}{OQ}$ IEF $-$ IGH is to the area PINM

For since the forces arising from gravity with which the body is urged in the places Z B D a are as the arcs CZ CB CD Ca and those arcs are as the areas ...

between the parallels RG rg and produce rg to h so that GHhg and RGgr may be the contemporaneous ... ment GHhg $-\frac{Rr}{OQ}$ IEF ...

be to the decrement RGgr ... Rr RG of the area PIGR, as HG $-\frac{IEF}{OQ}$ is to PG and therefore as OR HG $-\frac{OR}{OQ}$ IEF is to OP GR or OP PI that is (because of the equal quantities OR HG OR HR $-$ OR GP ORHK $-$ OPIK

correspondent part of the arc B as the square of the velocities that is as AA to BB nearly If the resistance in the arc B were to the resistance in the arc A as AB to AA the times in the arcs A and B would be equal (by the last Proposition) Therefore the resistance AA in the arc A or AB in the arc B causes the excess of the time in the arc A above the time in a nonresisting medium and the resistance BB causes the excess of the time in the arc B above the time in a nonresisting medium But those excesses are as the efficient forces AB and BB nearly that is as the arcs A and B Q E D

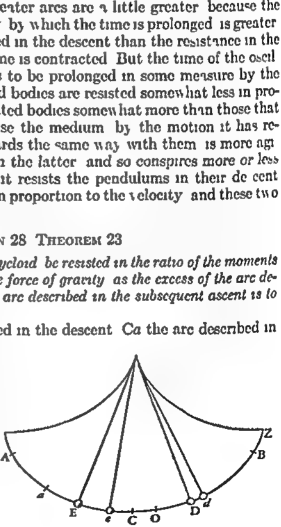
COR I Hence from the times of the oscillations in unequal arcs in a resisting medium may be known the times of the oscillations in a nonresisting medium of the same specific gravity For the difference of the times will be to the excess of the time in the shorter arc above the time in a nonresisting medium as the difference of the arcs is to the shorter arc

COR II The shorter oscillations are more isochronal and very short ones are performed nearly in the same times as in a nonresisting medium But the times of those which are performed in greater arcs are a little greater because the resistance in the descent of the body by which the time is prolonged is greater in proportion to the length described in the descent than the resistance in the subsequent ascent by which the time is contracted But the time of the oscillations both short and long seems to be prolonged in some measure by the motion of the medium For retarded bodies are resisted somewhat less in proportion to the velocity and accelerated bodies somewhat more than those that proceed uniformly forwards because the medium by the motion it has received from the bodies going forwards the same way with them is more agitated in the former case and less in the latter and so conspires more or less with the bodies moved Therefore it resists the pendulums in their descent more and in their ascent less than in proportion to the velocity and these two causes concurring prolong the time

PROPOSITION 28 THEOREM 23

If a pendulous body oscillating in a cycloid be resisted in the ratio of the moments of the time its resistance will be to the force of gravity as the excess of the arc described in the whole descent above the arc described in the subsequent ascent is to twice the length of the pendulum

Let BC represent the arc described in the descent Ca the arc described in the ascent and Aa the difference of the arcs and things remaining as they were constructed and demonstrated in Prop 25 the force with which the oscillating body is urged in any place D will be to the force of resistance as the arc CD to the arc CO which is half of that difference Aa Therefore the force with which the oscillating body is urged at the beginning or the highest point of the cycloid



that is the force of gravity will be to the resistance as the arc of the cycloid between that highest point and the lowest point C is to the arc CO that is

equal Hence the area $\frac{OR}{OQ}$ IEF−IGH is equal to the area Z by which the resi tance is expressed and therefore is to the area PINM by which the gravity is expressed as the resi tance is to the gravity Q E D

COR I Therefore the re i tance in the lowest place C is to the force of gravity as the area $\frac{OP}{\ }$ IEF is to the area PINM

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COR III Hence al o may be known the quare root of the resistance and at the beginning of the motion being equal to the velocity of the body o cillatin in the same cycloid without any

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PROPOSITION 30 THEOREM 24

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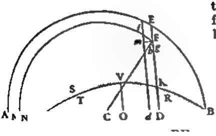
to the force of gravity Let t at force therefore be expressed by that length CD and the force of gravity



be the exponent of the resistance From the centre C with the interval

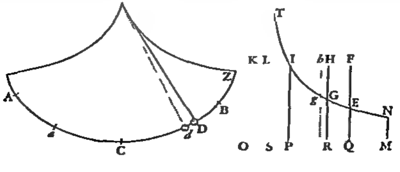
pears by Prop 5 B ok I Let therefore these velocities be expressed by tho e pe pend culars DF de and let DF be the veloc ty which it acquire in D by falling from B in the re isting medium An l if from the centre C with the in t rv l CF we describe the circle FfM meeting the right lines de and AB in f and M then M will be the place to which it ould thenceforward without further

PIHR and PIGR+IGH) as PIGR+IGH$-\frac{OR}{OQ}$IEF is to OPIK. Therefore if the area $\frac{OR}{OQ}$IEF$-$IGH be called Y, and RG*gr* the decrement of the area PIGR be given, the increment of the area Y will be as PIGR$-$Y.

Then if V represent the force arising from the gravity, proportional to the arc CD to be described, by which the body is acted upon in D, and R be put for the resistance, V$-$R will be the whole force with which the body is urged in D. Therefore the increment of the velocity is as V$-$R and the interval of time in which it is generated conjointly. But the velocity itself is directly as the contemporaneous increment of the space described and inversely as the same interval of time. Therefore, since the resistance is, by the supposition, as the square of the velocity, the increment of the resistance will (by Lem. 2) be as the velocity and the increment of the velocity conjointly, that is, as the moment of the space and V$-$R conjointly; and, therefore, if the moment of the space be given, as V$-$R; that is, if for the force V we put its expression PIGR, and the resistance R be expressed by any other area Z, as PIGR$-$Z.

Therefore the area PIGR uniformly decreasing by the subtraction of given moments, the area Y increases in proportion of PIGR$-$Y, and the area Z in proportion of PIGR$-$Z. And therefore if the areas Y and Z begin together, and at the beginning are equal, these, by the addition of equal moments, will continue to be equal; and in like manner decreasing by equal moments, will vanish together. And, conversely, if they together begin and vanish, they will have equal moments and be always equal. For, if the resistance Z be augmented, then the velocity together with the arc C*a*, described in the ascent of the body, will be diminished; and the point in which all the motion together with the resistance ceases coming nearer to the point C, then the resistance vanishes sooner than the area Y. And the contrary will happen when the resistance is diminished.

Now the area Z begins and ends where the resistance is nothing, that is, at the beginning of the motion where the arc CD is equal to the arc CB, and the right line RG falls upon the right line QE; and at the end of the motion where the arc CD is equal to the arc C*a*, and RG falls upon the right line ST. And the area Y or $\frac{OR}{OQ}$IEF$-$IGH begins and ends also where the resistance is nothing, and therefore where $\frac{OR}{OQ}$IEF and IGH are equal; that is (by the construction) where the right line RG falls successively upon the right lines QE and ST. Therefore those areas begin and vanish together, and are therefore always

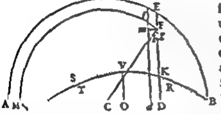


therefore OV is equal to $\frac{3}{4}Aa$ and therefore the resistance in O made to the oscillating body is to its gravity as $\frac{3}{4}Aa$ is to the length of the pendulum

And I take these conclusions to be accurate enough for practical uses. For since an ellipse or parabola BRVSa falls in with the figure BKVTa in the middle point V that figure if greater towards the part BRV or VSa is less towards the contrary part and is therefore nearly equal to it.

PROPOSITION 31 THEOREM 25

If the resistance made to an oscillating body in each of the proportional parts of the ... be a... nted or diminished in a given ratio the difference between ... bed in the subsequent ascent will be



on of the pendulum by the resist ... hole retardation and the retarding resistance proportional thereto In the foregoing Proposition the rectangle under the right line $\frac{1}{2}aB$ and the difference Aa of the arcs CB Ca was equal to the area BKTa And that area if the length aB remains is augmented or diminished in the ratio of the ordinates DK that is in the ratio of the resistance and is therefore as the length aB and the resistance conjointly And therefore the rectangle under Aa and $\frac{1}{2}aB$ is as aB and the resistance conjointly and therefore Aa is as the resistance Q.E.D

COR. I Hence if the resistance be as the velocity the difference of the arcs in the same medium will be as the whole arc described and conversely

COR. II If the resistance varies as the square of the velocity that difference will vary as the square of the whole arc and conversely

COR. III And generally if the resistance varies as the third or any other power of the velocity the difference will vary as the same power of the whole arc and conversely

COR. IV If the resistance varies partly as the first power of the velocity and partly as the square of the same the difference will vary partly as the first power and partly as the square of the whole arc and conversely So that the law and ratio of the resistance will be the same *for the velocity* as the law and ratio of that difference for the length of the arc

COR. V And therefore if a pendulum describe successively unequal arcs and we can find the ratio of the increment or decrement of this difference for the length of the arc described there will be had also the ratio of the increment or decrement of the resistance for a greater or less velocity

GENERAL SCHOLIUM

From these Propositions we may find the resistance of mediums by pendulums oscillating therein. I found the resistance of the air by the following experiments. I suspended a wooden globe or ball weighing 57 7/22 ounces troy its diameter 6⅞ London inches by a fine thread on a firm hook, so that the distance between the hook and the centre of oscillation of the globe was 10½

resistance ascend and df the velocity it would acquire in d Hence also if Fg represent the moment of the velocity which the body D in describing the least space Dd loses by the resistance of the medium and CN be taken equal to Cg then will N be the place to which the body if it met no further resistance would thenceforward ascend and MN will be the decrement of the ascent arising from the loss of that velocity Draw Fm perpendicular to df d the decrement Fg of the velocity DF ~ -
the increment fm of the same veloci[illegible]
erating force DK to the generating f [illegible] the similar tri angles Fmf Fhg FDC fm is to Fm [illegible] s CD to DF and by multiplication [illegible] DF Also Fh is to Fg as DF to CF [illegible] ing terms Fh or MN to Dd as DK [illegible] the MN CM will be equal to the [illegible] Dd DK At the movable point M suppose always a rectangular ordinate erected equal to the indeterminate CM which by a continual motion is multiplied by the whole length Aa and the trapezium described by that motion or its equal the rectangle Aa ½aB will be equal to the sum of all the MN CM and therefore to the sum of all the Dd DK that is to the area BKVTa

Q E D

COR Hence from the law of resistance and the difference Aa of the arcs Ca CB may be derived the proportion of the resistance to the gravity nearly

For if the resistance DK be uniform the figure BKTa will be a rectangle under Ba and DK and hence the rectangle under ½Ba and Aa will be equal to the rectangle under Ba and DK and DK will be equal to ½ Aa Therefore since DK represents the resistance and the length of the pendulum represents the gravity the resistance will be to the gravity [illegible]
pendul[illegible]

If th[illegible]
For if [illegible]
scribe [illegible] ny place D would be as the ordinate DF of the circle described on the diameter AB Therefore since Ba in the resisting medium and BA in the nonresisting one are described nearly in the same times and therefore the velocities in each of the [illegible] vel [illegible] the [illegible] the [illegible] diameter Ba [illegible] e the figure BKV[illegible] proportional [illegible] it O and an [illegible] OV will be [illegible] n and to its equal the rectangle Aa BO Therefore Aa BO is to OV BO as the area of this ellipse to OV BO that is Aa is to OV as the area of the semicircle is to the square of the radius or as 11 to 7 [illegible] of the pendulum as the resistance

[illegible] the resistance DK varies as the square of the velo[illegible] he figure BKVTa will be [illegible] its axis [illegible] There- [illegible] and [illegible] is equal to the rectangle ⅔Ba OV and

is to the length of the pendulum between the centre of suspension and the 121 inches Therefore ince V in the second case represents 1 ... will be to the weight of ... 4th as 0 041 48 is to

... the thread described in the 6th case was ... fore since the radius was 121 inches and the length of the pendulum ... the point of suspension and the centre of ... arc which the centre of the globe described was ... by reason ... scribed ...

equal to the versed sine of that arc ... the length of the pendulum 2?? and that arc 02 as the same arc to twice the length of the pendulum ... 278 ... Therefore the velocity of the pendulum is the

... which is in the squared ...

... with the ... 1 to 376 ... Since the weight of a globe ... with a velocity uniformly continued ...

of its motion

I also counted the oscillations in which the pendulum lost ¼ part of its motion In the following table the upper numbers denote the length of the arc described in the first descent expressed in inches and parts of an inch the middle numbers denote the length of the arc described in the last ascent and ...

First descent		4	8	16	32	64
Last ascent	1½	3	6	1	4	48
N... of oscillations	374	272	16 ½	83⅓	41⅔	22⅔

I afterwards suspended a leaden globe of 2 inches in diameter weighing 26¼ ounces troy by the same thread so that between the centre of the globe and

feet I marked on the thread a point 10 feet and 1 inch distant from the centre of suspension and even with that point I placed a ruler divided into inches by the help of which I observed the lengths of the arcs described by the pendulum Then I numbered the oscillations in which the globe would lose $\frac{1}{8}$ part of its motion If the pendulum was drawn aside from the perpendicular to the distance of 2 inches and then let go so that in its whole descent it described an a c of [illegible] inches and in the first whole oscillation compounded of the descent

inches If in the first descent it described an arc of 8 16 32 or 64 ... $\frac{1}{8}$ ... t of its motion in 69 $35\frac{1}{2}$ $18\frac{1}{2}$ $9\frac{2}{3}$ oscillations respectively Therefore ... t a cent

Divide those differences by the number of mean oscillation in which an arc of $3\frac{3}{4}$ scribed the difference of the arcs described in the descent and subsequent ascent will be $\frac{1}{656}$ $\frac{1}{242}$ $\frac{1}{69}$ $\frac{4}{71}$ $\frac{8}{37}$ $\frac{24}{29}$ parts of an inch respectively But these ... square of the arcs described r than in that ratio and there- sistance of the globe when it velocity nearly and when it moves slowly in a somewhat greater ratio

Now let V represent the greatest velocity in any oscillation and let A B and C be given quantities and let us suppose the difference of the arcs to be $AV+BV^{3/2}+CV^2$ Since the greatest velocities are in the cycloid as $\frac{1}{2}$ the arcs described in oscillating and in the circle as $\frac{1}{2}$ the chords of those arcs and ... n the cycloid than in the circle in the ratio the circle are greater than in the is plain that the differences of the arcs (which are a ... uare of the time conjointly) are nearly the same in both curves for in the cycloid those differences must be on the one hand augmented with the resistance in about the squared ratio of the arc to the chord because of the velocity augmented in the simple ratio of the same and on the other hand diminished with the square of the time in the same squared ratio Therefore to reduce these observations to the cycloid we ... me differences of the arcs as were observed in the circle and that i t 1 4 $\frac{1}{T}$= A +64B C= $7V^{3/2}$) the where length

re istance of the globe in ... the velocity is V is to its weight as ... istance

of the globe in one
ce 0 4475 If the
the pendulum
the oscillation
e diminished as
would be augmented and the
the square root of that ratio so that the difference 0 4475 of the arcs described
ll remain Then if the arc described

therefore the resis

greater than may arise from the resistance of the thread
of the resistances which are when the globes are equal as the squares of the velocities are also when the velocities are equal as the squares of the diameters of the globes

But the greatest of the globes I used in these experiments was not perfectly spherical and therefore in this calculation I have for brevity's sake neglected some little niceties being not very solicitous for an accurate calculus in an experiment that was not very accurate So that I could wish that these experiments were tried again with other globes of a larger size more in number and more accurately formed since the demonstration of a vacuum depends thereon If the globes be taken in a geometrical proportion whose diameters

the following trials I procured a wooden vessel 4 feet long 1 foot broad and 1 foot high This vessel being uncovered I filled with spring water and having

the point of suspension there was an interval of 10½ feet and I counted the oscillations in which a given part of the motion was lost The first of the following tables exhibits the number of oscillations in which ⅛ part of the whole motion was lost the second the number of oscillations in which there was lost ¼ part of the same

First descent	1	2	4	8	16	32	64
Last ascent	⅞	7/4	3½	7	14	28	56
No of oscillations	226	228	193	140	90½	53	30
First descent	1	2	4	8	16	32	64
Last ascent	¾	1½	3	6	12	24	48
No of oscillations	510	518	420	318	204	121	70

Selecting in the first table the 3d 5th and 7th observations and expressing the greatest velocities in these observations particularly by the numbers 1 4 16 respectively and generally by the quantity V as above there will come out in the 3d observation $\frac{\frac{1}{2}}{193}=A+B+C$ in the 5th observation $\frac{2}{90\frac{1}{2}}=4A+8B+16C$ in the 7th observation $\frac{8}{30}=16A+64B+256C$ These equations reduced give $A=0\ 001414$ $B=0\ 000297$ $C=0\ 000879$ And thence the resistance of the globe moving with the velocity V will be to its weight 26¼ ounces in the same ratio as $0\ 0009V+0\ 000208V^{3/2}+0\ 000659V^2$ to 121 inches the length of the pendulum And if we regard that part only of the resistance which is as the square of the velocity it will be to the weight of the globe as $0\ 000659V^2$ to 121 inches But this part of the resistance in the first experiment was to the weight of the wooden globe of $57\frac{7}{22}$ ounces as $0\ 002217V^2$ to 121 hence the resistance of the wooden globe is to the resistance of the leaden one (their velocities being equal) as $57\frac{7}{22}$ into 0 002217 to 26¼ into 0 000659 that is as 7⅓ to 1 The diameters of
to each other
of these equa
But we have not yet considered the resistance of the thread which was certainly very considerable and ought to be subtracted from the resistance of the pendulums here found I could not determine this accurately but I found it greater than ⅓ part of the whole resistance of the lesser pendulum hence I gathered that the resistances of the globes when the resistance of the thread is subtracted are nearly in the squared ratio of their diameters For the ratio of $7\frac{1}{2}-\frac{1}{3}$ to $1-\frac{1}{3}$ or 10½ to 1 is not very different from the squared ratio of the diameters $11\frac{13}{16}$ to 1

Since the resistance of the thread is of less moment in greater globes I tried the experiment also with a globe whose diameter was 18¾ inches The length of the pendulum between the point of suspension and the centre of oscillation was 122½ inches and between the point of suspension and the knot in the thread 109½ inches The arc described by the knot at the first descent of the pendulum was 32 inches The arc described by the same knot in the last ascent after five oscillations was 28 inches The sum of the arcs or the whole arc described in one mean oscillation was 60 inches the difference of the arcs 4 inches The $\frac{1}{10}$ part of this or the difference between the descent and ascent in one mean oscillation is ⅖ of an inch Then as the radius 109½ is to the radius 122½ so is the whole arc of 60 inches described by the knot in one mean oscil

the same velocity as about 850 to 1 that is, as the density of water is to the density of air nearly

In this calculation we ought also to have taken in that part of the resistance of the pendulum in the water which was as the square of the velocity but I found (which will perhaps seem strange) that the resistance in the water was augmented in more than a squared ratio of the velocity In searching after the cause I thought upon this that the vessel was too narrow for the magnitude of the pendulous globe and by its narrowness obstructed the motion of the water as it yielded to the oscillating globe For when I immersed a pendulous globe [illegible] one inch only the resistance was augmented nearly as the [illegible] of [illegible] her [illegible] air [illegible] nts

made by this contrivance resulted as [illegible]

Arc described [illegible] first descent	16	8	4	[illegible]	1	1/2	1/4
Arc described in last ascent	[illegible]	[illegible]	3	1 3/4	3/4	3/8	3/16
Difference of arc proportional to motion lost	4	[illegible]	1	[illegible]	[illegible]	1/8	1/16
Number of oscillations	3/8	6 1/2	1 [illegible]	1/5	34	53	62/5

In comparing the resistances of the mediums with each other I also caused iron pendulums to oscillate in quicksilver The length of the iron wire was about 3 feet and the diameter of the pendulous globe about 1/3 of an inch To the wire just above the quicksilver there was fixed another leaden globe of a bigness sufficient to continue the motion of the pendulum for some time Then a vessel that would hold about 3 pounds of quicksilver was filled by turns with quicksilver and common water so that by making the pendulum oscillate successively in these two different fluids, I might find the proportion of their resistances and the resistance of the quicksilver proved to be to the resistance of water as about 13 or 14 to 1 that is as the density of quicksilver to the density of water When I made use of a pendulous globe something bigger as of one whose diameter was about [illegible] or 2/3 of an inch the resistance of the quicksilver proved to be to the resistance of the water as about 12 or 10 to 1 But the former experiment is more to be relied on because in the latter the vessel [illegible]

[illegible] warm oil more than rain water and water more than spirit of wine But in liquors which are sensibly fluid enough as in air in salt and fresh water in spirit of wine of turpentine and salts in oil cleared of its feces by distillation

the point of suspension to a certain point marked in the thread being 126 inches and to the centre of oscillation 134⅜ inches

The arc described in the first descent by a point marked in the thread was inches	64	32	16	8	4	2	1	½	¼
The arc described in the last ascent was inches	48	24	12	6	3	1½	¾	⅜	3/16
The difference of the arcs proportional to the motion lost was inches	16	8	4	2	1	½	¼	⅛	1/16
The number of the oscillations in water			29/60	1⅕	3	7	11¼	12⅔	13⅓
The number of the oscillations in air	85½	287	535						

In the experiments of the 4th column there were equal motions lost in 535 oscillations made in the air and 1⅕ in water The oscillation ...
indeed a little swifter than those in th...
water were ...
be e...
osci... quantity of motion would be
lost ... resistance is increased and the sa...
diminished in the same squared ratio Th...
velocities there were equal motions lo...
in the water ...
resistance in ... ou of the whole resist
ances in the ... column

Now let $AV+CV^2$ represent the difference of the arcs described in the descent and subsequent ascent by the globe moving in air with the greatest velocity V and since the greatest velocity is in the case of the ... the
greatest velocity in the case of th... nce
of the arc... 1st
column a... the cases 1 and 8 for the vel
ocities a... 500 for the differences of the arcs and A+C will be $=85½$ and $8A+64C=4280$ or $A+8C=535$ and then by reducing these equations there will come out $7C=449½$ and $C=64\frac{3}{14}$ and $A=21\frac{2}{7}$ and therefore the resistance which is as $\frac{7}{11}AV+\frac{3}{4}CV^2$ will become ...
$+48\frac{9}{56}V^2$ Therefore in the case of th...
the whole re...
$13\frac{6}{11}+$ ...
in water

... as on to
... oscillating in the water had been im...
... ance would have been still greater so that the resistance of the pendulum oscillating in the water that is that part which is proportional to the square of the velocity and which only needs to be considered in swift bodies is to the resistance of the same whole pendulum oscillating in air with

B as — 77 to 1 and by subtraction again A to B as 5928 to 1 Therefore the resistance of the empty box in its internal parts will be above 5000 times less than the resistance on its external surface This reasoning depends upon the supposition that the greater resistance of the full box arises not from any other latent cause but only from the action of some subtile fluid upon the included metal

This experiment is related by memory the paper being lost in which I had described it so that I have been obliged to omit some fractional part which are slipped out of my memory and I have no leisure to try it again. The first time I made it the hook being weak the full box was retarded soon Th
ne

SECTION VII

THE MOTION OF FLUIDS AND THE RESISTANCE MADE TO PROJECTED BODIES

PROPOSITION 37 THEOREM 26

Suppose two similar systems of bodies consisting of an equal number of pa ticles and let the correspondent particles be similar and p opo t t
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prop
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I say that

p a nt particles by reason of the motions at their beginning will continue to be moved with like motions so long as they move without meeting one another for if they are acted on by no forces, they will go on uniformly in right lines by the first Law But if they agitate one another with some certain forces and those forces are inversely as the diameters of the correspondent particles and directly as the squares of the velocities then because the particles are in like situation and their forces are proportional the whole forces with which correspondent particles are agitated and which are compounded of each of the agitating forces (by Cor II of the Laws) will have like direction and have the same effect as

resolved into drops I doubt not that the rule already laid down may be accurate enough especially if the experiments be made with larger pendulous bodies and more swiftly moved

Lastly since it is the opinion of some that there is a certain ethereal medium extremely rare and subtile which freely pervades the pores of all bodies and from such a medium so pervading the pores of bodies some resistance must needs arise *in order to try whether the resistance which we experience in* bodies in motion be made upon their outward surfaces only or whether their internal parts meet with any considerable resistance upon their surfaces I thought of the following experiment I suspended a round deal box by a thread 11 feet long on a steel hook by means of a ring of the same metal so as to make a pendulum of the aforesaid length The hook had a sharp hollow edge on its upper part so that the upper arc of the ring pressing on the edge might move the more freely and the thread was fastened to the lower arc of the ring The pendulum being thus prepared I drew it aside from the perpendicular to the distance of about 6 feet and that in a plane perpendicular to the edge of the hook lest the ring while the pendulum oscillated should slide to and fro on the edge of the hook for the point of suspension in which the ring touches the hook ought to remain immovable I therefore accurately noted the place to which the pendulum was brought and letting it go I marked three other places to which it returned at the end of the 1st 2d and 3d oscillation This I often repeated that I might find those places as accurately as possible Then I filled the box with lead and other heavy metals that were near at hand But first I weighed the box when empty and that part of the thread that went round it and half the remaining part extended between the hook and the suspended box for the thread so extended always acts upon the pendulum when drawn aside from the perpendicular with half its weight To this weight I added the weight of the air contained in the box And this whole weight was about $\frac{1}{78}$ of the weight of the box when filled with the metals Then because the box when full of the metals by extending the thread with its weight increased the length of the pendulum I shortened the thread so as to make the length of the pendulum when oscillating the same as before Then drawing aside the pendulum to the place first marked and letting it go I reckoned about 77 oscillations before the box returned to the second mark and as many afterwards before it came to the third mark and as many after that before it came to the fourth mark From this I conclude that the whole resistance of the box when full had not a greater proportion to the resistance of the box when empty than 78 to 77 For if their resistances were equal the box when full by reason of its inertia which was 78 times greater than the inertia of the same when empty ought to have continued its oscillating motion so much the longer and therefore to have returned to those marks at the end of 78 oscillations But it returned to them at the end of 77 oscillations

Let therefore A represent the resistance of the box upon its external surface and B the resistance of the empty box on its internal surface and if the resistances to the internal parts of bodies equally swift be as the matter or the number of particles that are resisted then 78B will be the resistance made to the internal parts of the box when full and therefore the whole resistance A+B of the empty box will be to the whole resistance A+78B of the full box as 77 to 78 and by subtraction A+B to 77B as 77 to 1 and thence A+B to

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... bod... D will be to th... re... tance of the body F as the

resi... tances of the equal and equally swift bodies E and G in these mediums will co... t... n... lly approach to equality so that their difference will at last become less th... n any given Therefore since the resistances of the bodies D and F are t... ... ch other as the resistances of the bod... es E and G those will also in like ma... n... r ... pproach to the ratio of equality Therefore the bodies D and F when th... y m... ... with very great swiftness meet with resistances very nearly equal and therefore since the resi... tance of the body F is in a ... quared ratio of the velocity th... re... tance of the body D will be nearly in the same ratio

if they respected centres places alike among the particles and those whole forces will be to each other as the several forces which compose them that is inversely as the diameters of the correspondent particles and directly as the squares of the velocities and therefore will cause correspondent particles to continue to describe like figures These things will be so (by Cor I and VIII Prop 4 Book I) if those centres are at rest but if they are moved yet by reason of the similitude of the translations their situations among the particles of the system will remain similar so that the changes introduced into the figures described by the particles will still be similar So that the motions of correspondent and similar particles will continue similar till their first meeting with each other and thence will arise similar collisions and similar reflections which will again beget similar motions of the particles among themselves (by what was just now shown) till they mutually fall upon one another again and so on *ad infinitum* Q E D

COR I Hence if any two bodies which are similar and in like situations to the correspondent particles of the systems begin to move amongst them in like manner and in proportional times and their magnitudes and densities be to each other as the magnitudes and densities of the corresponding particles these bodies will continue to be moved in like manner and in proportional times for the case of the greater parts of both systems and of the particles is the very same

COR II And if all the similar and similarly situated parts of both systems be at rest among themselves and two of them which are greater than the rest and mutually correspondent in both systems begin to move in lines alike posited with any similar motion whatsoever they will excite similar motions in the rest of the parts of the systems and will continue to move among those parts in like manner and in proportional times and will therefore describe spaces proportional to their diameters

PROPOSITION 33 THEOREM 27

The same things being supposed I say that the greater parts of the systems are resisted in a ratio compounded of the squared ratio of their velocities and the squared ratio of their diameters and the simple ratio of the density of the parts of the systems

For the resistance arises partly from the centripetal or centrifugal forces with which the particles of the system act on each other partly from the collisions

parts that is (by the supposition) directly as the squares of the velocities and inversely as the distances of the corresponding particles and directly as the quantities of matter in the correspondent parts and therefore since the distances of the particles in one system are to the correspondent distances of the particles in the other as the diameter of one particle or part in the former system to the diameter of the correspondent particle or part in the other and since the quantities of matter are as the densities of the parts and the cubes of the diameters the resistances are to each other as the squares of the velocities and the squares of the diameters and the densities of the parts of the systems Q E D The resistances of the latter sort are as the number of correspondent

dec whod h

of the cylinder NAO and equal to the radiu AC we take bH equal to $\frac{BE^2}{CB}$

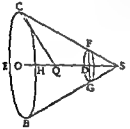
then bH will be to bE as the effect of the part I nw h w h

f h w w

its axis CA and latus rec —
in the paraboloid and i
cylinder Therefore the v
entire force of the same
the medium are at rest and I and globe move with equal velocities
the resi tance of the globe will be half the resi tance of the cylinder Q E D

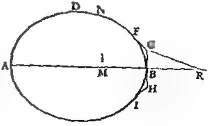
SCHOLIUM

By the same method other figures may be compared together as to their resi tance and those may be found which are most apt to continue their motions in resi ting mediums As if upon the circular base CEBH from the centre O with h w CG



so that QS may be equal to QC and S will be the vertex of the cone whose frustum is sought

Incidentally since the angle CSB



about its axis AB and the gr rati figure be touched by three right lines FG GH HI in the points F B and I so that GH shall be perpendicular to the axi in the point of contact B and FG HI may be
in the an les FI
of the figure AI

COR III Hence the resistance of a body moving very swiftly in an elastic fluid is almost the same as if the parts of the fluid were destitute of their centrifugal forces and did not fly from each other provided only that the elasticity of the fluid arise from the centrifugal forces of the particles and the velocity be so great as not to allow the particles time enough to act

COR IV Since the resistances of similar and eq... dium whose d...

from each other will
whether the particles
...fore impress on that matter an equal quantity of motion ... at a smaller or fewer and greater and (by the third Law of Motion) suffer ... and in return

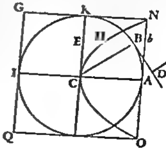
...resistance of projectiles moving ... parts or of parts ever so subtile For

... as from the expansion of the particles after the manner of wool or the boughs of trees or any other cause by which the particles are hindered from moving freely among ... of the medium ... the lesser fluidity

PROPOSITION 34 THEOREM 28

If in a rar... from eac... equal vel... will be but half as great as that of the cylinder ... of the cylinder the resistance of the globe

For since the action of the medium upon the body is the same (by Cor v of the Laws) whether the body move in a quiescent medium or whether the particles of the medium impinge with the same velocity upon the quiescent body let us consider the body as if it were quiescent and see with what force it would be impelled by the moving medium



Let therefore ABKI represent a spherical body described from the centre C with the semidiameter CA and let the particles of the medium impinge with a given velocity upon that spherical body in the directions of right lines parallel to AC and let FB be one of

... BD let fall the

square of the velocity

Cor III The resistance of the globe other things being equal varies as the square of the diameter

Cor IV The resistance of the globe other things being equal varies as the density of the medium

Cor V The resistance of the globe varies jointly as the square of the velocity as the square of the diameter and as the density of the medium

Cor VI The motion of the globe and its resistance may be thus represented Let AB be the time in which the globe may by its resistance uniformly con ... le motion Erect AD BC perpendicular to AB Let BC be ... AD AB ... to any ... ting the ... CBEG

All these things appear by Cor I and III Prop ...

Cor. VII Hence if the globe in the time T by the resistance R uniformly continued ... lose its whole motion M the same globe in the time t in a resisting medium wherein the resistance R decreases as the square of the velocity will lose out of its motion M the part $\frac{tM}{T+t}$ the part $\frac{TM}{T+t}$ remaining and will describe a space which is to the space described in the same time t with the uniform motion M as the logarithm of the number $\frac{T+t}{T}$ multiplied by the number 30?585092994 is to the number $\frac{t}{T}$ because the hyperbolic area BCFE is to the rectangle BCGE in that proportion

Scholium

I have exhibited in this Proposition the resistance and retardation of spherical projectiles in mediums that are not continued and shown that this resist ...

this force where the globe and particles of the medium are infinitely hard and void of any reflecting force is diminished one-half But in continued mediums

former solid provided that both move forwards in the direction of their axis AB and that the extremity B of each go foremost This Proposition I conceive may be of use in the building of ships

If the figure DNFG be such a curve that if from any point thereof as N the perpendicular NM be let fall on the axis AB and from the given point G there be drawn the right line GR parallel to a right line touching the figure in N and cutting the axis produced in R MN becomes to GR as GR^3 to $4BR \cdot GB^2$ the solid described by the revolution of this figure about its axis AB moving in the before mentioned rare medium from A towards B will be less resisted than any other circular solid whatsoever described of the same length and breadth

PROPOSITION 35 PROBLEM 7

If a rare medium consist of very small quiescent particles of equal magnitudes and freely disposed at equal distances from one another to find the resistance of a globe moving uniformly forwards in this medium

CASE 1 Let a cylinder described with the same diameter and altitude be conceived to go forwards with the same velocity in the direction of its axis

but half the resistance of the cylinder and since the globe is to the cylinder as 2 to 3 and since the cylinder by falling perpendicularly on the particles and *reflecting them with the utmost force communicates to them a velocity double* to its own it follows that the cylinder in moving forwards uniformly half the length of its axis will communicate a motion to the particles which is to the whole motion of the cylinder as the density of the medium to the density of the cylinder and that the globe in the time it describes one length of its diameter in moving uniformly forwards will communicate the same motion to the particles and in the time that it describes two-thirds of its diameter will communicate a motion to the particles which is to the whole motion of the globe as the density of the medium to the density of the globe And therefore the globe meets with a resistance which is to the force by which its whole motion may be either taken away or generated in the time in which it describes two-thirds of its diameter moving uniformly forwards as the density of the medium is to the density of the globe

CASE 2 Let us suppose that the particles of the medium incident on the globe or cylinder are not reflected and then the cylinder falling perpendicularly on the particles will communicate its own simple velocity to them and therefore meets a resistance but half so great as in the former case and the globe also meets with a resistance but half so great

CASE 3 Let us suppose the particles of the medium to fly back from the globe with *a force which is neither the greatest nor yet none at all* but with a certain mean force then the resistance of the globe will be in the same mean ratio between the resistance in the first case and the resistance in the second

Q E I

COR I Hence if the globe and the particles are infinitely hard and destitute of all elastic force and *therefore of all force of reflection the resistance of the* globe will be to the force by which its whole motion may be destroyed or

the ice as through a funnel Then if the water pass very near to the ice only without touching it or which is the same thing if by reason of the perfect smoothness of the surface of the ice the water though touching it glides over it with the utmost freedom and without the least resistance the water will run through the hole EF with the same velocity as before and the whole weight of the column of water ABNFEM will be taken up as before in forcing out the water and the bottom of the vessel will sustain the weight of the ice surround i h t column.

into water but the efflux of the water e as before It will not be less because descend it will not be greater because the ice now become water descend without hindering the descent of other water equal to its own descent The same force ought always to generate the same velocity in the effluent water

But the hole at the bottom of the vessel by reason of the oblique motions of the particles of the effluent water must be a little greater than before For now the particles of the water do not all of them pass through the hole perpendicularly but flowing down on all parts from the sides of the vessel and converging towards the hole pass through it with oblique motions and in tending downwards they meet in a stream whose diameter is a little smaller below the hole than at the hole itself its diameter being to the diameter of the hole as 5 to 6 or as $5\frac{1}{2}$ to $6\frac{1}{2}$ very nearly if I measured those diameters rightly I procured a thin flat plate having a hole pierced in the middle the diameter of the circular hole being five eighth parts of an inch And that the stream of running water might not be accelerated in falling and by that accel

plate not to the bottom but to the side of

stream

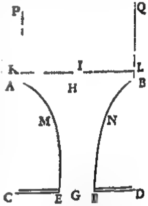
m of a vessel is equal to the ud velocity would run out in the same time through another circular hole whose diameter is to the diameter of the former as 1 to 2 And therefore the running water in passing through the hole itself has a velocity downwards nearly equal to that which a heavy body would acquire in falling through half the height of the stagnant water in the vessel But then after it has run out it is still accelerated by converging till it arrives at a distance from the hole that is nearly equal to its diameter and acquires a velocity greater than the other in about the ratio of $\sqrt{2}$ to 1 this velocity a heavy body would nearly acquire by falling freely through the whole height of the stagnant water in the vessel

as water, hot oil and quicksilver the globe as it passes through them does not immediately strike against all the particles of the fluid that generate the resistance made to it but presses only the particles that lie next to it which press the particles beyond which press other particles and so on and in these mediums the resistance is diminished one other half A globe in these extremely fluid mediums meets with a resistance that is to the force by which its whole motion may be destroyed or generated in the time wherein it can describe with that motion uniformly continued eight third parts of its diameter as the density of the medium is to the density of the globe This I shall endeavor to show in what follows

PROPOSITION 36 PROBLEM 8

To find the motion of water running out of a cylindrical vessel through a hole made at the bottom

Let ACDB be a cylindrical vessel AB the mouth of it CD the bottom parallel to the horizon EF a circular hole in the middle of the bottom G the centre of the hole and GH the axis of the cylinder perpendicular to the horizon And suppose a cylinder of ice APQB to be of the same breadth with the cavity of the vessel and to have the same axis and to descend continually with an uniform motion and that its parts as soon as they touch the surface AB dissolve into water and flow down by their weight into the vessel and in their fall compose the cataract or column of water ABNFEM passing through the hole EF and filling up the same exactly Let the uniform velocity of the descending ice and of the contiguous water in the circle AB be that which the water would acquire by falling through the space IH and let IH and HG lie in the same right line and through the point I let there be drawn the right line KL parallel to the horizon and meeting the ice on both the sides thereof in K and L Then the velocity of the water running out at the hole EF will be the same that it would acquire by falling from I through the space IG Therefore by Galileo's Theorems IG will be to IH as the square of the velocity of the water that runs out at the hole to the velocity of the water in the circle AB that is as the square of the ratio of the circle AB to the circle EF those circles being inversely as the velocities of the water which in the same time and in equal quantities passes through each of them and completely fills them both We are now considering the velocity with which the water tends to the plane of the horizon But the motion parallel to the same by which the parts of the falling water approach to each other is not here taken notice of since it is neither produced by gravity nor at all changes the motion perpendicular to the horizon which the gravity produces We suppose indeed that the parts of the water cohere a little that by their cohesion they may in falling approach to each other with motions parallel to the horizon in order to form one single cataract and to prevent their being divided into several but the motion parallel to the horizon arising from this cohesion does not come under our present consideration



CASE 1 Conceive now the whole cavity in the vessel which surrounds the falling water ABNFEM to be full of ice so that the water may pass through

– t th distance of 40 inches the

still issue forth with the
upward ascend with
— – water in the
ice of the air
dd acquire in
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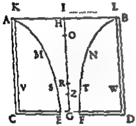
passes into a canal and spring u
the upper part of the canal And it may not only be inferred from reason a
but is manifest also from the well known experiments just mentioned that the velocity with which the water runs out is the very same that is assigned in this Proposition

Case 5 The velocity of the effluent water is the same whether the figure of the hole be circular or square or triangular or of any other figure whatever equal to the circular for the velocity of the effluent water does not depend upon the figure of the hole but arises from such depth of the hole as it may have below the plane KL

Case 6 If the lower part of the vessel ABDC be immersed into stagnant water and the height of the stagnant water above the bottom of the vessel be GR the velocity with which the water that is in the
– – – f th hole EF into the stagnant



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the stagnant water will be sustained in equ u i im by the weight of the stagnant water and therefore does not at all accelerate the motion of the descending water in the vessel This case will also become evident from experiment measuring the times in which the water will run out

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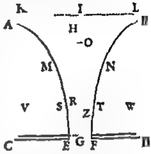
the weight which is employed in forcing out the water as the sum of the circles AB and EF is to twice the circle EF For let IO be a mean proportional between IH and IG and the water running out at the hole EF will in the time that a drop falling from I would describe the altitude IG become equal to a

Therefore in what follows let the diameter of the stream be represented by that lesser hole which we shall call EF And imagine another plane VW above the hole EF and parallel to the plane thereof to be placed at a distance equal to the diameter of the same hole and to be pierced through with a greater hole ST of such a magnitude that a stream which will exactly fill the lower hole EF may pass through it the diameter of this hole will therefore be to the diameter of the lower hole nearly as 25 to 21 By this means the water will run perpendicularly out at the lower hole and the quantity of the water running out will be according to the magnitude of this last hole very nearly the same as that which the solution of the Problem requires The space included between the two planes and the falling stream may be considered as the bottom of the vessel But to make the solution more simple and mathematical it is better to take the lower plane alone for the bottom of the vessel and to suppose that the water which flowed through the ice as through a funnel and ran out of the vessel through the hole EF made in the lower plane preserves its motion continually and that the ice continues at rest



Therefore in what follows let ST be the diameter of a circular hole described from the centre Z and let the stream run out of the vessel through that hole when the water in the vessel is all fluid And let EF be the diameter of the hole which the stream in falling through exactly fills up whether the water runs out of the vessel by that upper hole ST or flows through the middle of the ice in the vessel as through a funnel And let the diameter of the upper hole ST be to the diameter of the lower EF as about 25 to 21 and let the perpendicular distance between the planes of the holes be equal to the diameter of the lesser hole EF Then the velocity of the water downwards in running out of the vessel through the hole ST will be in that hole the same that a body may ac- ... by falling freely from half the height IZ and the velocity of both the ...

Case 2 If the hole EF be not in the middle of the bottom of ... in some other part thereof the water will still run out with the same velocity as before if the magnitude of the hole be the same For though a heavy body ... e line than ... it the same

... rough a hole in the side of the vessel I or if the hole be small so that the interval between the surfaces AB and KL may vanish as to sense and the stream of water horizontally issuing out may form a parabolic figure from the latus rectum of this parabola one may see that the velocity of the effluent water is that which a body may acquire by falling the height IG or HG of the stagnant water in the ... ht of the stagnant ... ole above a plane ... pringing out from ... ly 37 inches from ... ithout resistance

and suppose this cat
to be against
less perhaps
ginning of its
part at the very vertex of the ice
fall may tend to a concave figure And as the congealed water AMEC BNFD
lying round the cataract is convex in its internal surfaces AME BNF towards
PHQ will be convex towards the cataract
a cone whose base is that little circle

tains seems to be less than the weight of two-thirds of a cylinder
whose base is that little circle and its altitude HG For things standing as
here supposed imagine the half of a spheroid described whose base is that

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tion
becomes narrower Therefore since that angle is less than a right one this column in the lower parts thereof will lie within the hemispheroid In the upper parts also it will be acute or pointed because to make it otherwise the horizontal motion of the water must be at the vertex infinitely more swift than its motion towards the horizon And the less this circle PQ is the more acute will the vertex of this column be and the circle being diminished *in infinitum* the

altitude GH Now the little circle sustains a force of water equal to the weight of this column the weight of the ambient water being employed in causing its efflux out at the hole

COR. IX The weight of water which the little circle PQ sustains when it is very small is very nearly equal to the weight of a cylinder of water whose base is that little circle and its altitude ½GH for this weight is an arithmetical mean between the weights of the cone and the hemispheroid above mentioned. But if that little circle be not very small but on the contrary increased till it be equal to the hole EF it will sustain the weight of all the water lying perpendicularly above it that is the weight of a cylinder of water whose base is that little circle and its altitude GH

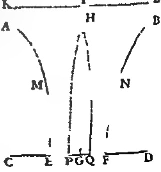
cylinder whose base is the circle EF and its altitude 2IG that is to a cylinder whose base is the circle AB and whose altitude is 2IO For the circle EF is to the circle AB as the square root of the ratio of the altitude IH to the altitude IG that is in the simple ratio of the mean proportional IO to the altitude IG Moreover in the time that a drop falling from I can describe the altitude IH the water that runs out will have become equal to a cylinder whose base is the circle AB and its altitude 2IH and in the time that a drop falling from I through H to G describes HG the difference of the altitudes the effluent water that is the water contained within the solid ABNFEM will be equal to the difference of the cylinders that is to a cylinder whose base is AB and its altitude 2HO And therefore all the water contained in the vessel ABDC is to the whole falling water contained in the said solid ABNFEM as HG is to 2HO that is as HO+OG to 2HO or IH+IO to 2IH But the weight of all the water in the solid ABNFEM is employed in forcing out the water and therefore the weight of all the water in the vessel is to that part of the weight that is employed in forcing out the water as IH+IO is to 2IH and therefore as the sum of the circles EF and AB is to twice the circle EF

COR IV And hence the weight of all the water in the vessel ABDC is to the other part of the weight which is sustained by the bottom of the vessel as the sum of the circles AB and EF is to the difference of the same circles

COR V And that part of the weight which the bottom of the vessel sustains is to the other part of the weight employed in forcing out the water as the difference of the circles AB and EF is to twice the lesser circle EF, or as the area of the bottom to twice the hole

COR VI That part of the weight which presses upon the bottom is to the whole weight of the water perpendicularly incumbent thereon as the circle AB is to the sum of the circles AB and EF or as the circle AB is to the excess of twice the circle AB above the area of the bottom For that part of the weight which presses upon the bottom is to the weight of the whole water in the vessel as the difference of the circles AB and EF is to the sum of the same circles (by Cor IV) and the weight of the whole water in the vessel is to the weight of the whole water perpendicularly incumbent on the bottom as the circle AB is to the difference of the circles AB and EF Therefore multiplying together corresponding terms of the two proportions that part of the weight which presses upon the bottom is to the weight of the whole water perpendicularly incumbent thereon as the circle AB to the sum of the circles AB and EF or the excess of twice the circle AB above the bottom

COR VII If in the middle of the hole EF there be placed the little circle PQ described about the centre G and parallel to the horizon the weight of water which that little circle sustains is greater than the weight of a third part of a cylinder of water whose base is that little circle and its height GH For let ABNFEM be the cataract or column of falling water whose axis is GH as above and let all the water whose fluidity is not requisite for the ready and quick descent of the water be supposed to be congealed as well round about the cataract as above the little circle And let PHQ be the column of water congealed above



C –

PQ and its altitude GH that i greater than a third part of a cylinder described with the same base and altitude Now that little circle sustains the weight of this column that is a weight greater than the weight of the cone or a third part of the cylinder

Cor viii The weight of water which the circle PQ when very small sustains seems to be less than the weight of two-thirds of a cylinder of water whose base is that little circle and its altitude HG For things standing as above supposed imagine the half of a spheroid described whose base is that little circle and its semiaxis or altitude HG This figure will be equal to two-thirds of that cylinder and will comprehend within it the column of congealed water PHQ the weight of which is sustained by that little circle For though

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becomes narrower Therefore since that angle is less than a right one this column in the low r p rt th m f ll

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u it e t tle sustains a force of water equal to the weight of this column the weight of the ambient water being employed in causing its efflux out at the hole

Cor ix The weight of wate –
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little circle and its altitude GH b o a cylinder of water whose base is that

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cylinder whose base is the circle EF an . . . whose base is the circle AB and wh . . . the circle AB as the square root of t . . . IG that is in the simple ratio of the me . . . o the altitud

. . . AB through H . . . IG the difference of the altitudes the effluent water that is the water contained within the solid ABNFEM will be equal to th . . . difference of the cylinders th . . .

. . . ABNFEM is employed in forcing out the water . . . the weight of all the water weight of all the water in the vessel is to that part of the weight that is employed in forcing out the water as IH+IG is to 2IH and therefore as the sum of . . . ircle EF

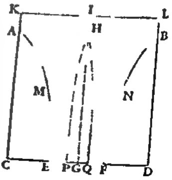
oth . . . the weight which is sustained by the bottom of the vessel . . . water in the vessel ABDC is to the sum of the circles AB and EF is to the difference of the same circles

COR V And that part of the weight which the bottom of the vessel sustains is to the other part of the weight employed in forcing out the water as the difference of the circles AB and EF is to twice the lesser circle EF or as the area of the bottom to twice the hole

COR VI That part of the weight which presses upon the bottom is to the whole weight of the water perpendicularly incumbent thereon as the circle AB is to the . . . twice th . . . AB is to the excess of which p . . . at part of the weight as the d . . . le water in the vessel Cor IV) . . . the same circles (by . . . the whole water in the vessel is to the weight of the whole water perpendicularly incumbent on the bottom as the circle AB is to the difference of the circles AB and EF Therefore multiplying together corresponding terms of the two proportion . . .

. . . AB above the bottom . . . or the excess of

COR VII If in the middle of the hole EF there be placed the little circle PQ described about the centre G and parallel to the horizon the weight of water which that little circle sustains is greater than the weight of a third part of a cylinder of water whose base is that little circle and its height GH For let ABNFEM be the cataract or column of falling water whose axis is GH as above and let all the water whose fluidity is not requisite for the ready and quick descent of the water be supposed to be congealed as well round about the cataract as above the little circle And let PHQ be the column of water congealed above



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velocity in
length But the resistance of the c
the direction of its length is the same with the resistance of the little circle (by Lem 4) and is therefore nearly equal to the force by which its motion may be generated while it describes four times its length

If the length of the cylinder be augmented or diminished its motion and
when four times its length will be augmented or

Lem 4) that resistance will also rem

If the density of the cylinder be augmented or diminished its motion and the force by which its motion may be generated or destroyed in the same time will be augmented or diminished in the same ratio *Therefore the resistance of* any cylinder whatsoever will be to the force by which its whole motion may be generated or destroyed in the time during which it moves four times its length as the density of the medium is to the density of the cylinder nearly Q.E.D

pressed to become continued it must be continued and
props
moved
e motion
of the body spent in generating a motion in the parts of the fluid and this creates the resistance But the pressure arising from the compression of the fluid be it ever so forcible if it be propagated in an instant generates no

stronger on the hinder parts of the body moved than on its fore parts and therefore cannot lessen the resistance described in this Proposition And if its propagation be infinitely swifter than the motion of the body pressed it will not be stronger on the fore parts than on the hinder parts. But that action will be infinitely swifter and propagated in an instant if the fluid be continued and nonelastic

Cor. 1 The resistances made to cylinders going uniformly forwards in the

LEMMA 4

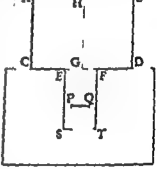
If a cylinder moves ... made ther ... therefore and mov ... dicular to

For ... circle ...

PROPOSITION 37 THEOREM 29

If a cylinder moves uniformly forwards in a ... fluid in th... transvers ... otion may be destroyed or generated ... that it moves four times its length as the density of the medium is to the density of the cylinder nearly

For let the vessel ABDC touch the surface of stagnant water with its bottom CD and let the water run out of this vessel into the stagnant water through the cylindric canal EFTS perpendicular to the horizon and let the little circle PQ be placed parallel to the horizon anywhere in the middle of the canal and produce CA to K so that AK may be to CK as the square of the ratio which the excess of the orifice of the canal EF above the little circle PQ bears to the circle AB Then it is manifest (by Case 5 Case 6 and Cor 1 Prop 36) that the velocity of the water passing through the annular space between the little circle and the sides of the vessel will be the very same as that which the water would acquire by falling and in its fall describing the altitude KC or IC



And (by Cor 10 Prop 36) if the breadth of the vessel be infinite so that the short line HI may vanish and the altitudes IG HG become equal the force of the water that flows down and presses upon the circle will be to the weight of a cylinder whose base is that little circle and the altitude $\frac{1}{2}IC$ as EF^2 is to $EF^2 - \frac{1}{2}PQ^2$ very nearly For the force of the water flowing downwards uniformly through the whole canal will be the same upon the little circle PQ in whatsoever part of the canal it be placed

Let now the orifices of the canal EF ST be closed and let the little circle ascend in the fluid compressed on every side an... water th... circle

... the velocity of the descending water with which it passes by the little circle in its ascent as the difference of the circles EF and PQ is to the circle EF or as $EF^2 - PQ^2$ to EF^2 Let that relative velocity be equal to the velocity with which it was shown above that the water would pass through the annular space if the circle were to remain unmoved that is to the velocity which the water would acquire by falling and

upon the as-
s of Motion)
of a
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is to EF ... he
velocity which the water acquires by falling ...
altitude IG as EF² − PQ² is to EF²

Let the breadth of the canal be increased *in infinitum* and the ratios between ... and between EF² and EF² − 16PQ will become at last ... ll now be the ... ll describing ... of a cylinder
the altitude ... from which ... circle and its altitude half ... and with ... times its ... velocity in ... by ... be ... and ... l or
diminished in the same ratio and therefore ... so
increased or diminished, may be destroyed or generated will continue the same because the time is increased or diminished in the same proportion and therefore that force remains still equal to the resistance of the cylinder because (by Lem. 4) that resistance will also remain the same

If the density of the cylinder be augmented or diminished its motion and the force by which its motion may be generated or destroyed in the same time will be augmented or diminished in the same ratio Therefore the resistance of any cylinder whatsoever will be to the force by which its whole motion may be generated or destroyed, in the time during which it moves four times its length, as the density of the medium is to the density of the cylinder nearly Q.E.D.

... compressed ... continued it must be continued and ... ops ... ved ... otion ... of the fluid and this ... compression of the ... nstant generates no motion in the parts of a continued fluid, produces no change at all of motion therein and therefore neither augments nor lessens the resistance This is certain, that the action of the fluid arising from the compression cannot be stronger on the hinder parts of the body moved than on its fore parts and ... described in ... Propos. ...

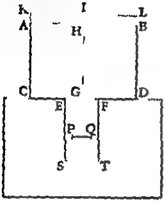
nonelastic.

COR. 1 The resistances, made to cylinders going uniformly forwards in the

direction of their lengths through continued infinite mediums are in a ratio compounded of the square of the ratio of the velocities and the square of th ratio of the diameters and the ratio of th

COR II If

der go forw ough an included quiescent medium it while coinciding with the axis of the canal its resistance



will be to the force by which its whole motion in the time in which it describes four t

t

t — PQ and the ratio of the density of the medium to the density of the cylinder

COR III The same thing supposed and that a length L is to four times the length of the cylinder in a ratio compounded of the ratio $EF^2 - \frac{1}{2}PQ^2$ to EF^2 and the square of the ratio of $EF^2 - PQ^2$ to EF^2 the resistance of the cylinder will be to the force by which its whole motion in the time during which it describes the length L may be destroyed or generated as the density of the medium is to the density of the cylinder

SCHOLIUM

In this Proposition we have investigated that resistance alone which arises from the magnitude of the transverse section of the cylinder neglecting that part of the same which may arise from the obliquity of the motions For as in Case 1 of Prop 36 the obliquity of the motions with which the parts of the water in the vessel converged on every side to the hole EF hindered the efflux of the water through the hole so in this Proposition the obliquity of the motions with which the parts of the water pressed by the antecedent extremity of the cylinder yield to the pressure and diverge on all sides retards their passage through the places that lie round that antecedent extremity towards the hinder parts of the cylinder and causes the fluid to be moved to a greater distance which increases the resistance and that in the same ratio almost in which it diminished the effl t of the vessel that is in the 1 of that Proposition

we

in

and b

to acquire the velocity with which it moves as HC to 4AB Let CF and DF be two other parabolic arcs described with the axis CD and a latus rectum four times the former and by the revolution of the figure about the axis EF let there be generated a solid whose middle part ABDC is the cylinder we are here speaking of and whose extreme part ABE and CDF contain the part of the fluid at rest

have the same ratio to the force with which the whole motion of the cylinder may be destroyed or generated in the time that it is describing the length 4AC with that motion uniformly continued as the density of the fluid has to the density of the cylinder nearly And (by Cor VII Prop 36) the resistance must be to this force in the ratio of 2 to 3 at the least

LEMMA 5

If a cylinder a sphere and a spheroid of equal breadths be placed successively in the middle of a cylindric canal so that their axes may coincide with the axis of the canal these bodies will equally hinder the passage of the water through the canal

For the spaces lying between the sides of the canal and the cylinder sphere and spheroid, through which the water passes are equal and the water will pass equally through equal spaces.

This is true upon the supposition that all the water above the cylinder sphere or spheroid, whose fluidity is not necessary to make the passage of the water the quickest possible is congealed as was explained above in Cor VII Prop 36

LEMMA 6

The same supposition remaining the fore mentioned bodies are equally acted on by the water flowing through the canal

This appears by Lem 5 and the third Law For the water and the bodies act upon each other mutually and equally

LEMMA 7

If the water be at rest in the canal and these bodies move with equal velocity and opposite directions through the canal their resistances will be equal among themselves

This appears from the last Lemma for the relative motions remain the same among themselves.

SCHOLIUM

The case is the same for all convex and round bodies whose axes coincide with the axis of the canal Some difference may arise from a greater or less friction but in these Lemmas we suppose the bodies to be perfectly smooth and the medium to be void of all tenacity and friction and that those parts of the fluid which by their oblique and superfluous motions may disturb hinder

and retard the flux of the water through the canal are at rest among themselves being fixed like water by frost and adhering to the force and hinder parts of the bodies in the manner explained in the Scholium of the last Proposition for in what follows we consider the very least resistance that round bodies described with the greatest given transverse sections can possibly meet with

Bodies swimming upon fluids when they move straight forwards cause the fluid to ascend at their fore parts and subside at their hinder parts especially

u us if they are obtuse behind and before condense the fluid a little more at their fore parts and relax the same at their hinder parts and therefore meet also with a little more resistance than if they were acute at the head and tail But in these Lemmas and Propositions we are not treating of elastic but non elastic fluids not of bodies floating on the surface of the fluid but deeply immersed therein

known we may

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PROPOSITION 38 THEOREM 30

If a globe move uniformly forwards in a compressed infinite and nonelastic fluid its resistance is to the force by which its whole motion may be destroyed or generated in the time that it describes eight third parts of its diameter as the density of the fluid is to the density of the globe very nearly

For the globe is to its circumscribed cylinder as 2 to 3 and therefore the force which can destroy all the motion of the cylinder while the same cylinder is describing the length of four of its diameters will destroy all the motion of the globe while the globe is describing two-thirds of this length that is eight third parts of its own diameter Now the resistance of the cylinder is to this force very nearly as the density of the fluid is to the density of the cylinder or globe (by Prop 37) and the resistance of the globe is equal to the resistance of the cylinder (by Lems 5 6 7) Q E D

COR I The resistances of globes in infinite compressed mediums are in a ratio compounded of the squared ratio of the velocity and the squared ratio of the diameter and the ratio of the density of the mediums

COR II The greatest velocity with which a globe can descend by its comparative weight through a resisting fluid is the same as that which it may acquire by falling with the same weight and without any resistance and in its fall describing a space that is to four third parts of its diameter as the density of the globe is to the density of the fluid For the globe in the time of its fall moving with the velocity acquired in falling will describe a space that will be to eight third parts of its diameter as the density of the globe is to the density of the fluid and the force of its weight which generates this motion will be to the force that can generate the same motion in the time that the globe describes eight third parts of its diameter with the same velocity as the density of the fluid is to the density of the globe and therefore (by this Proposition) the force of weight will be equal to the force of resistance and therefore cannot accelerate the globe

COR III If there be given both the density of the globe and its velocity at the beginning of the motion and the density of the compressed quiescent fluid

ι ... l he moves there is given at any time both the velocity of the
escribed by it (by Cor VII Prop 35)
ed quiescent fluid of the same density
it can describe the length of two of its

PROPO ITION 39 THEOREM 31

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SCHOLIUM

In the last two Propositions we suppose (as was done before in Lem 5) that

PROPOSITION 40 PROBLEM 9

To find by experiment the resistance of a globe moving through a perfectly fluid compressed medium

which the globe meets with when descending with that velocity will be equal to its weight B and the resistance it meets with in any other velocity will be to the weight B as the square of the ratio of that velocity to the greatest velocity H by Cor 1 Prop 38

This is the resistance that arises from the inactivity of the matter of the

arithm 4342944819 $\frac{}{C}$ and let L be the logarithm of th number $\frac{}{}$ and

the velocity acquired in falling will be $\frac{N-1}{N+1}H$ and the height described will be $\frac{2PF}{G} - 1.3862943611F + 4.605170186LF$. If the fluid be of a sufficient depth, we may neglect the term $4.605170186LF$; and $\frac{2PF}{G} - 1.3862943611F$ will be the altitude described nearly. These things appear by Prop. 9, Book II, and its Corollaries, and are true upon this suppo[illegible] that the globe meets with no other [illegible] really [illegible] inactivity of matter. Now if it from t[illegible] the descent will be slower, and resistance [illegible] that retardation will be known the amount of this new

That the velocity and descent of a body falling in a fluid might more easily be known, I have composed the following table, the first column of which

The T[illegible]m[illegible] P	V[illegible]l[illegible]ty of [illegible] body f[illegible]ll[illegible]g in th[illegible] fl[illegible]d	Th[illegible] spa[illegible]e de[illegible]crib[illegible]d [illegible] f[illegible]ll[illegible]g [illegible] the fl[illegible]id	Th[illegible] spa[illegible] describ[illegible]d with th[illegible] greatest m[illegible]t[illegible]n	The spa[illegible] descri[illegible]ed by fall[illegible]g [illegible] a vacu[illegible]m
0.001G	99999 29/30	0.000001F	0.002F	0.000001F
0.01G	999967	0.0001F	0.0[illegible]F	0.0001F
0.1G	9966799	0.0099834F	0.[illegible]F	0.01F
0.2G	1973[illegible]532	0.0397361F	0.4F	0.04F
0.3G	29131261	0.0886815F	0.6F	0.09F
0.4G	37994896	0.1559070F	0.8F	0.16F
0.5G	46211716	0.2402290F	1.0F	0.25F
0.6G	53704957	0.3402706F	1.2F	0.36F
0.7G	60436778	0.4545405F	1.4F	0.49F
0.8G	66403677	0.5815071F	1.6F	0.64F
0.9G	716[illegible]978[illegible]	0.7196609F	1.8F	0.81F
1G	76159416	0.8675617F	2F	1F
2G	964027[illegible]8	[illegible]6500055F	4F	4F
3G	99505475	4.6186[illegible]70F	6F	9F
4G	99932930	6.6143[illegible]65F	8F	16F
5G	99990920	8.6137964F	10F	25F
6G	99998771	10.6137179F	12F	36F
7G	99999834	12.6137073F	14F	49F
8G	99999980	14.6137059F	16F	64F
9G	99999997	16.61370[illegible]7F	18F	81F
10G	99999999 3/5	18.61370[illegible]6F	20F	100F

denotes the times of descent, the second shows the velocity [illegible] falling, the greatest velocity [illegible] described by falling in tho[illegible]e ti[illegible] in the time G with the greate[illegible] [illegible] the fourth gives the spaces described with the greatest velocity in the same times. The numbers in the fourth column are $\frac{2P}{G}$, and by subtracting the number $1.3862944 - 4.6051702L$ are found the numbers in th[illegible] by the space F [illegible] to all these cor[illegible] in a vacuum w[illegible]

SCHOLIUM

... to investigate the resistances of fluids from experiments I procured ... was 9 inches ... and having provided globes ... ted the times of the descents of these globes the height through which they descended being 11 inches. A solid cubic foot of English measure contains 76 pounds troy weight of rain water and a solid inch contains 19/36 ounces troy weight or 253⅓ grains and a globe of water of one inch in diameter contains 132.645 grains in air or 132.8 grains in a vacuum and any other globe will be as the excess of its weight in a vacuum above its weight in water

EXPER 1 A globe whose weight was 156¼ grains in air and ... grains in water described the whole height of 112 inches in 4 seconds And upon repeating the experiment the globe spent again the very same time of 4 seconds in falling ...

... 156 13/16 grains and the excess of this ... Hence the diameter ... will be as that excess to the weight of the globe in a vacuum so is the density of the water to ...

the space 1.3802944 F or 3.0 6 inches and there will remain a space of 113.0569 inches which the globe falling through water in a very wide vessel will describe in 4 seconds But this space by reason of the narrowness of the wooden vessel before mentioned ought to be diminished in a ratio compounded of the square root of the ratio of the orifice of the vessel to the excess of this orifice above half a great circle of the globe and of the simple ratio of the same orifice to its excess above a great circle of the globe that is in a ratio of 1 to 0.9914 This done we have a space of 112.08 inches which a globe falling through the water in this wooden vessel in 4 seconds of time ought nearly to describe by this theory but it described 112 inches by the experiment

EXPER. ■ Three equal globes whose weights were severally 76⅓ grains in air and 5 1/16 grains in water were let fall successively and every one fell through the water in 15 seconds of time describing in its fall a height of 112 inches

By comp...

... in one second without resistance 1... 808 inches, and the time G

the velocity acquired in falling will be $\frac{N-1}{N+1}H$ and the height described will be $\frac{2PF}{G}$ − 1 3862943611F+4 605170186LF If the fluid be of a sufficient depth we may neglect the term 4 605170186LF and $\frac{2PF}{G}$ − 1 3862943611F will be the altitude described nearly These things appear by Prop 9 Book II and its Corollaries and are true upon this suppo t on other re t n k really) from t resista n own the amount of this new

That the velocity and descent of a body falling in a fluid might more easily be known I have composed the following table the first column of which

The Times P	*Velocities of the body falling in the fluid*	*The spaces described in falling in the fluid*	*The spaces described with the greatest motion*	*The spaces described by falling in vacuum*
0 001G	99999 29/30	0 000001F	0 002F	0 000001F
0 01G	999967	0 0001F	0 02F	0 0001F
0 1G	9966799	0 0099834F	0 2F	0 01F
0 2G	19737532	0 0397361F	0 4F	0 04F
0 3G	29131261	0 0886815F	0 6F	0 09F
0 4G	37994896	0 1559070F	0 8F	0 16F
0 5G	46211716	0 2402290F	1 0F	0 25F
0 6G	53704957	0 3402706F	1 2F	0 36F
0 7G	60436778	0 4545405F	1 4F	0 49F
0 8G	66403677	0 5815071F	1 6F	0 64F
0 9G	71629787	0 7196609F	1 8F	0 81F
1G	76159416	0 8675617F	2F	1F
2G	96402758	2 6500055F	4F	4F
3G	99505475	4 6186570F	6F	9F
4G	99932930	6 6143765F	8F	16F
5G	99990920	8 6137964F	10F	25F
6G	99998771	10 6137179F	12F	36F
7G	99999834	12 6137073F	14F	49F
8G	99999980	14 6137059F	16F	64F
9G	99999997	16 6137057F	18F	81F
10G	99999999 3/5	18 6137056F	20F	100F

denotes the times of descent the second shows the velocities acquired in falling the greatest velocity being 100 000 000 the third exhibits the spaces described by falling in those times 2F being the space which the body describes in the time G with the greatest velocity and the fourth gives the spaces described with the greatest velocity in the same times The numbers in the fourth column are $\frac{2P}{G}$ and by subtracting the number 1 3862944 − 4 6051702L are found the numbers in the third column and these numbers must be multiplied by the space F to obtain the spaces described in falling A fifth column is added to all these containing the spaces described in the same times by a body falling in a vacuum with the force of B its comparative weight

— th weight of the globe in a vacuum is

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inche
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By the theory they ought to have fallen in the t u o o ~ ~
nea ly
~ — nd 3 7/ grains in
31 32

By the theory they ought to have fallen in the time of 25 oscillations nearly

centre that ide hicl chanced to be the heavier descending first and producin an o c llating motion No by oscillatin thus the globe communicates a greater motion to the water than if it de cended without any oscillations and by this communication loses part of its own motion with which it should de ce d and therefo e as th s oscillation is greater or less it will be more or
~ h if h h

near its surface and I let fall the globe in such a manner that as near as possible the heavier side might be lo est at the beginning of the descent By this

... locity it is capable of ... describe ... onds the space 115 678 inches Subtract the space 1 3862944 or 1 609 inches and there remains the space 114 069 inches which therefore the falling globe ought to describe in the same time if the vessel were very wide But because our

inches The difference is ...

EXPER 3 Three equal globes whose weights were severally 121 grains in air and 1 grain in water were successively let fall, and they fell through the water in the times 46 seconds 47 seconds and 50 seconds describing a height of 112 inches

By the theory these globes ought to have fallen in about 40 second Now whether their falling more slowly were occasioned from the consideration that in slow motions the resistance arising from the force of inactivity does really bear a less proportion to the resistance arising from other causes or whether it is to be attributed to little bubbles that might chance to stick to the globes or to the rarefaction of the wax by the warmth of the weather or of the hand that let them fall or lastly whether it proceeded from some insensible errors in weighing the globes in the water I am not certain Therefore the weight of the globe in water should be of several grains that the experiment may be certain and to be depended on

EXPER 4 I began the foregoing Experiments to investigate the resistance of fluids before I was acquainted with the theory laid down in the Propositions immediately preceding Afterwards in order to examine the theory after it was discovered I procured a wooden vessel whose breadth on the inside was $8\frac{2}{3}$ inches and its depth $15\frac{1}{3}$ feet Then I made four globes of wax with lead included each of which weighed $139\frac{1}{4}$ grains in air and $7\frac{1}{8}$ grains in water These I let fall measuring the times of their falling in the water with a pen ...

warmth rarefies the wax and by rarefying it diminishes ... in the water and wax when rarefied is not instantly reduced by cold to its former density Before they were let fall they were totally immersed under water lest by the weight of any part of them that might chance to be above the water their descent should be accelerated in its beginning Then when after their immersion they were perfectly at rest they were let go with the ... se from the hand that let ... of $47\frac{1}{2}$ $48\frac{1}{2}$ 50 and 51 ... es But the weather was ... hed and therefore I repeated the experiment another day and then the globes fell in the times of 49 $49\frac{1}{2}$ 50 and 53 and at a third trial in the times of $49\frac{1}{2}$ 50 51 and 53 oscillations And by making the experiment several times over I found that the globes fell mostly in the times of $49\frac{1}{2}$ and 50 oscillations When they fell slower I suspect them to have been retarded by striking against the sides of the vessel

of air and in their fall they described a height of 220 English feet. A wooden table was suspended upon iron hinges on one side and the other side of the table was supported by a wooden pin. The two globes lying upon this table ... fell together by pulling out the pin by means of an iron wire reaching ... moved the table which ... urning round upon the ... t the same instant with the same pull of the iron wire that took out the pin a pendulum oscillating to seconds was let go and began to oscillate The diameters and weights of the globes and their times of falling are exhibited in the accompanying table

The globes filled with mercury			*The globes full of air*		
Weights	*Diameters*	*Times in falling*	*Weights*	*Diameters*	*Times in falling*
grains	*inches*	*seconds*	*grains*	*inches*	*seconds*
908	0 8	4	510	5 1	8½
983	0 8	4−	64	5 [illegible]	8
866	0 8	4	599	5 1	8
4	0	4+	51	5 0	8¼
808	0 [illegible]	4	483	5 0	8½
84	0	4+	641	5	8

But the times observed must be corrected for the globes of mercury (by Galileo's theory) in 4 seconds of time will describe 25 English feet and 220 feet in onl 3 seconds 42 thirds So that the wooden table when the pin was taken out did not turn upon its hinges so quickly as it ought to have done and the slowness of that revolution hindered the descent of the globes at the beginnin For the globes lay about the middle of the table and indeed were rathe nearer to the axis upon which it turned than to the pin. And hence the

whole weight of $50\frac{?}{10}$ grains will in one second of time describe 193⅓ inches as above and with the weight 483 grains will describe 185 905 inches and with that weight 483 grains in a vacuum will describe the space F or 14 feet 5½ inches in the time of 5 thirds and 58 fourths and acquire the greatest velocity it is capable of descending with in the air With this velocity the globe in

means the oscillations became much less than before and the times in which the globes fell were not so unequal as in the following Experiments

EXPER 8 Four globes weighing 139 grains in air and $6\frac{1}{2}$ in water were let fall several times and fell mostly in the time of 51 oscillations never in more than 52 or in fewer than 50 describing a height of 182 inches

By the theory they ought to fall in about the time of 52 oscillations

EXPER 9 Four globes weighing $273\frac{1}{4}$ grains in air and $140\frac{3}{4}$ in water being several times let fall fell in never fewer than 12 and never more than 13 oscillations describing a height of 182 inches

These globes by the theory ought to have fallen in the time of $11\frac{1}{3}$ oscillations nearly

EXPER 10 Four globes weighing 384 grains in air and $119\frac{1}{2}$ in water being let fall several times fell in the times of $17\frac{3}{4}$ 18 $18\frac{1}{2}$ and 19 oscillations describing a height of $181\frac{1}{2}$ inches And when they fell in the time of 19 oscillations I sometimes heard them hit against the sides of the vessel before they reached the bottom

By the theory they ought to have fallen in the time of $15\frac{5}{9}$ oscillations nearly

EXPER 11 Three equal globes weighing 48 grains in air and $3\frac{29}{32}$ in water, being several times let fall fell in the times of $43\frac{1}{2}$ 44 $44\frac{1}{2}$ 45 and 46 oscillations and mostly in 44 and 45 describing a height of $182\frac{1}{2}$ inches nearly

By the theory they ought to have fallen in the time of $46\frac{5}{9}$ oscillations nearly

EXPER 12 Three equal globes weighing 141 grains in air and $4\frac{3}{8}$ in water being let fall several times fell in the times of 61 62 63 64 and 65 oscillations describing a space of 182 inches

And by the theory they ought to have fallen in $64\frac{1}{2}$ oscillations nearly

From these Experiments it is manifest that when the globes fell slowly as in the second fourth fifth eighth eleventh and twelfth Experiments the times of falling are rightly exhibited by the theory but when the globes fell more swiftly as in the sixth ninth and tenth Experiments the resistance was somewhat greater than the square of the velocity For the globes in falling oscillate a little and this oscillation in those globes that are light and fall slowly soon ceases by the weakness of the motion but in greater and heavier globes the motion being strong it continues longer and is not to be checked by the ambient water till after several oscillations Besides the more swiftly the globes move the less are they pressed by the fluid at their hinder parts and if the velocity be continually increased they will at last leave an empty space behind them unless the compression of the fluid be in [illegible] sed at the same time For [illegible] Props 32 and 33) as the [illegible] istance in the same squa [illegible] But because this is not done the globes that move swiftly are not so much pressed at their hinder parts as the others and by the defect of this pressure it comes to pass that their resistance is a little greater than the square of their velocity

So that the theory agrees with the experiments on bodies falling in water It remains that we examine the observations of bodies falling in air

EXPER 13 From the top of St Paul s Church in London in June 1710 there were let fall together two glass globes one full of quicksilver the other

The globes filled with mercury			The globes filled with air		
W [illegible]	Diameters	Times falling	Weights	Diameter	Times falling
grains	inches	seconds	grains	inches	seconds
908	0 8	4	510	5 1	8½
983	0 8	4−	64	5 2	[illegible]
866	0 8	4	599	5 1	8
47	0 75	4+	515	5 0	8¼
808	0 75	4	483	5 0	8½
84	0 75	4+	641	5	8

But the times observed must be corrected for the globes of mercury (by Galileo s theory) in 4 seconds of time will describe 257 English feet and 220 feet in only 3 seconds 42 thirds So that the wooden table when the pin was taken out did not turn upon its hinges o quickly as it ought to have done

reason of the largeness of their diameters lay longer upon the revolving table than the others This being done the times in which the six larger globes fell will come forth 8 seconds 12 thirds 7 seconds 42 thirds " seconds 42 thirds 7 seconds 5 thirds 8 seconds 12 thirds and 7 seconds 42 thirds

Therefore the fifth in order among the globes that were full of air being 5

/ weight ins will in one second of time describe 193⅓ inches as above and with the weight 483 grains will describe 185 90 n hes

that we

inches

ity it i

means the oscillations became much less than before and the times in which the globes fell were not so unequal as in the following Experiments

EXPER 8 Four globes weighing 139 grains in air and $6\frac{1}{2}$ in water were let fall several times and fell mostly in the time of 51 oscillations never in more than 52 or in fewer than 50 describing a height of 182 inches

By the theory they ought to fall in about the time of 52 oscillations

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These globes by the theory ought to have fallen in the time of $11\frac{1}{3}$ oscillations nearly

EXPER 10 Four globes weighing 384 grains in air and $119\frac{1}{2}$ in water being let fall several times fell in the times of $17\frac{3}{4}$ 18 $18\frac{1}{2}$ and 19 oscillations describing a height of $181\frac{1}{2}$ inches And when they fell in the time of 19 oscillations I sometimes heard them hit against the sides of the vessel before they reached the bottom

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By the theory they ought to have fallen in the time of $46\frac{5}{9}$ oscillations nearly

EXPER 12 Three equal globes weighing 141 grains in air and $4\frac{3}{8}$ in water being let fall several times fell in the times of 61 62 63 64 and 65 oscillations describing a space of 182 inches

And by the theory they ought to have fallen in $64\frac{1}{2}$ oscillations nearly

From these Experiments it is manifest that when the globes fell slowly as in the second fourth fifth eighth eleventh and twelfth Experiments the times of falling are rightly exhibited by the theory but when the globes fell more swiftly as in the sixth ninth and tenth Experiments the resistance was somewhat greater than the square of the velocity For the globes in falling oscillate a little and this oscillation in those globes that are light and fall slowly soon ceases by the weakness of the motion but in greater and heavier globes the motion being strong it continues longer and is not to be checked by the ambient water till after several oscillations Besides the more swiftly the globes move the less are they pressed by the fluid at their hinder parts and if the velocity be continually increased they will at last leave an empty space behind them unless the compression of the fluid be increased at the same time For the compression of the fluid ought to be increased (by Props 32 and 33) as the square of the velocity in order to maintain the resistance in the

So that the theory agrees with the experiments on bodies falling in water It remains that we examine the observations of bodies falling in air

EXPER 13 From the top of St Paul s Church in London in June 1710 there were let fall together two glass globes one full of quicksilver the other

D t the

times of their fal ng
— d som times by a whole second The second and fo rth ... ... had h sec nd time The fifth nd their l wound eriments

The weight of the bladder	The diameters	The times of falling from height of f et	The spaces which by the theory ght t ha been described the time		The difference between the theory and the eriments	
grains	inch s	seco ds	feet	inches	f t	inches
1_S	5 _S	19	71	11	− 0	1
156	5 19	17		0½	+ 0	0½
137½	5 3	18	7	7	+ 0	7
97 ½	5 6		7	4	+ 5	4
99 s	5	21⅛	82	0	+10	0

l

magnitudes

In the Scholium subjoined to the sixth Section we showed by experiments of pendulums that the resistances of equal and equally swift globes moving in air water and quicksilver are as the densities of the fluids We here prove the same more accurately by experiments of bodies falling in air and water

u pended makes the whole resistance of a pendulum greater than the resist ance deduced from the experiments of falling bodies For by the experiments

would lose only a part of its motion equal to $\frac{1}{1586}$ supposin the density of water to be to the density of air as 860 to 1 Therefore the resistances were

i these mediums will be rightly enou h exhibited by the experiments of pen dulums as w ll as by the experiments of falling bodies And from all this it may be con luded that the resistance of bodies moving in any fluids whatso-

8 seconds 12 thirds of time will describe 245 feet and 5⅓ inches Subtract 1 3863 F or 20 feet and ½ an inch and there remain 225 feet 5 inches This space therefore the falling globe ought by the theory to describe in 8 seconds 12 thirds But by the experiment it described a space of 220 feet The difference is inappreciable

By like calculations applied to the other globes full of air I composed the following table

The weight of the globes	The diameter	The times of falling from a height of 220 feet		The spaces which they would describe by the theory		The excesses	
grains	inches	seconds	thirds	feet	inches	feet	inches
510	5 1	8	12	226	11	6	11
642	5 2	7	42	230	9	10	9
599	5 1	7	42	227	10	7	0
515	5	7	57	224	5	4	5
483	5	8	12	225	5	5	5
641	5 2	7	42	230	7	10	7

EXPER 14 In the year 1719 in the month of July Dr Desaguliers made some experiments of this kind again by forming hogs bladders into spherical orbs which was done by means of a concave wooden sphere which the bladders being wetted ...

they were c

when dry ...

cupola of the same church namely from a height of 272 feet and at the same moment of time there was let fall a leaden globe whose weight was about 2 pounds troy weight And in the meantime some persons standing in the upper part of the church where the globes were let fall observed the whole times of falling and others standing on the ground observed the differences of the times between the fall of the leaden weight and the fall of the bladder The times were measured by pendulums oscillating to half-seconds And one of those that stood upon the ground had a machine vibrating four times in one second and another had another machine accurately made with a pendulum vibrating four times in a second also One of those also who stood at the top of

were so contrived that

Now the leaden globe

tion of this time to the difference of time above spoken of was obtained the whole time in which the bladder was falling The times which the five bladders spent in falling after the leaden globe had reached the ground were the first time 14¾ seconds 12¾ seconds 14⅝ seconds 17¾ seconds and 16⅞ seconds and the second time 14½ seconds 14¼ seconds 14 seconds 19 seconds and 16¾ seconds Add to these 4¼ seconds the time in which the leaden globe was falling and the whole times in which the five bladders fell were the first time 19 seconds 17 seconds 18⅞ seconds 22 seconds and 21⅛ seconds and the second time 18¾ seconds 18½ seconds 18¼ seconds 23¼ seconds and 21 seconds The times observed at the top of the church were the first time 19⅜ seconds 17¼ seconds 18¾ seconds 22⅛ seconds and 21⅝ seconds and the second time

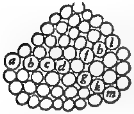
SECTION VIII

The motion propagated through fluids

Propo ition 41 Theorem 32

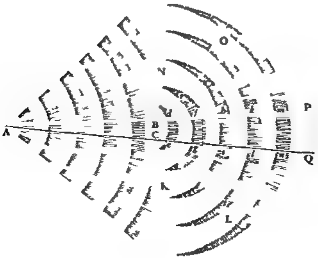
A p e ure is not propagated thro gh a fluid in rectilinear directions except where the particles of the fluid lie in a right line

If the particles a b c d e lie in a right line the pre ure may be indeed



ticles h and k lyin beyond them but the particles that upport them are al o pre ed by them and those particles cannot su tain that pre ure with out being upported by and pres ing upon those particles that lie still farther as l and m and so on *in infinitum* Therefore the pre ure as soon as it is propagated to particles that lie out of right line begins to deflect towards one hand and the other and will be propagated obliquely *in infinitum* and after it has begun to be propagated obliquely if it reaches more di tant particles lying out of the right line it will deflect again on each hand and this it will do as often as it lights on particles that do not lie exactly in a right line Q E D

Cor If any part of a pressure propagated through a fluid from a given point be intercepted by any obstacle the remaining part which is not inter cepted will deflect into the paces behind the obstacle This may be demon



ever though of the most extreme fluidity are other things being equal as the densities of the fluids

These things being thus established we may now determine what part of its motion any globe projected in any fluid whatsoever would nearly lose in a given time Let D be the diameter of the globe and V its velocity at the beginning of its motion and T the time in which a globe with the velocity V can describe in a vacuum a space that is to the space $\frac{8}{3}$D as the density of the globe to the density of the fluid and the globe projected in that fluid will in any other time t lose the part $\frac{tV}{T+t}$ the part $\frac{TV}{T+t}$ remaining and will describe a space which will be to that described in the same time in a vacuum with the uniform velocity V as the logarithm of the number $\frac{T+t}{T}$ multiplied by the number 2 302585093 is to the number $\frac{t}{T}$ by Cor VII Prop 35 In slow motions the resistance may be a little less because the figure of a globe is more adapted to motion than the figure of a cylinder described with the same diameter In swift motions the resistance may be a little greater because the elasticity and compression of the fluid do not increase as the square of the velocity But these little niceties I take no notice of

And though air water quicksilver and the like fluids by the division of their parts *in infinitum* should be subtilized and become mediums infinitely

d fluid

the same its

here spoken of

force diminish this

which the bodies move

es through which the

globes of the planets and comets are continually passing towards all parts with the utmost freedom and without the least sensible diminution of their motion must be utterly void of any corporeal fluid excepting perhaps some extremely rare vapors and the rays of light

Projectiles excite a motion in fluids as they pass through them and this motion arises from the excess of the pressure of the fluid at the fore parts of the projectile above the pressure of the same at the hinder parts and cannot be less in mediums infinitely fluid than it is in air water and quicksilver in proportion to the density of matter in each Now this excess of pressure does in proportion to its quantity not only excite a motion in the fluid but al

the resistance

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be less in th

it is in air

than in the unmoved parts of the fluid KL, NO it will run down from off the tops of those ridges *e g i l* &c *d f h k* &c this way and that way towards KL and NO and because the water is more depressed in the hollows of the [illegible]

filled by the dilated waves *rfor shs tkl rrnr* &c. Q E D That these things are so anyone may find by making the experiment in still water

CASE 2 Let us suppose that *d fg hi kl mn* represent pulses successively propagated from the point A through an elastic medium Conceive the pulses to be propagated by successive condensations and rarefactions of the medium so that the densest part of every pulse may occupy a spherical surface described [illegible]

relaxation of the denser parts towards the antecedent rare intervals and since the pulses will relax themselves on each hand towards the quiescent parts of the medium KL, NO with very near the same celerity therefore the pulses will dilate themselves on [illegible]

the

and

by experience also in sounds which are heard through a mountain interposed and, if they come into a chamber through the window dilate themselves into all the parts of the room and are heard in every corner and not as reflected from the opposite walls, but directly propagated from the window as far as our sense can judge

CASE 3 Let us suppose lastly that a motion of any kind is propagated from A through the hole BC Then since the cause of this propagation is that the parts of the medium that are near the centre A disturb and agitate those which lie farther from it and since the parts which are urged are fluid and therefore recede every way towards those spaces where they are less pressed they will by consequence recede towards all the parts of the quiescent medium as well to the parts on each hand, as KL and NO as to those right before as PQ and by this means all the motion, as soon as it has passed through the hole BC will begin to dilate itself and from thence as from its principle and centre will be propagated directly every way

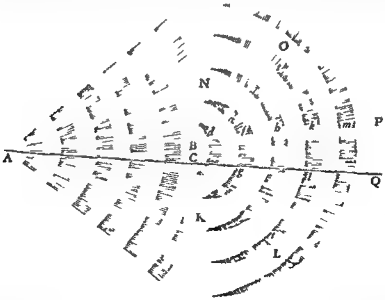
Q E D

the cone
en while
beyond
frustum urges the next frustum $fgih$ on the surface fg and that frustum urges a third frustum and so *in infinitum* it is manifest (by the third Law) that the first frustum $defg$ is by the reaction of the second frustum $fghi$ as much urged and pressed on the surface fg as it urges and presses that second frustum Therefore the frustum $degf$ is compressed on both sides that is between the cone Ade and the frustum $fhig$ and therefore (by Case 6 Prop 19) cannot preserve its figure unless it be compressed with the same force on all sides Therefore with the same force with which it is pressed on the surfaces de fg it will endeavor to break forth at the sides df eg and there (being not in the least tenacious or hard but perfectly fluid) it will run out expanding itself unless there be an ambient fluid opposing that endeavor Therefore by the effort it makes to run out it will press the ambient fluid at its sides df eg with the same force that it does the frustum $fghi$ and therefore the pressure will be propagated as much from the sides df eg into the spaces NO KL this way and that way as it is propagated from the surface fg towards PQ Q E D

PROPOSITION 42 THEOREM 33

All motion propagated through a fluid diverges from a rectilinear progress into the unmoved spaces

CASE 1 Let a motion be propagated from the point A through the hole BC and if it be possible let it proceed in the conic space BCQP according to right lines diverging from the point A And let us first suppose this motion to be



that of waves in the surface of standing water and let de fg hi kl &c be the tops of the several waves divided from each other by as many intermediate valleys or hollows Then because the water in the ridges of the waves is higher

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P OPOSITION 44 THEOREM

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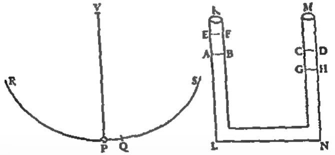
of the water ari ing from its attri u
AB CD represent the mean height of the water in both legs and when the water in the leg KL ascends to the height EF the water will descend in the



leg MN to the height GH Let P be a pendulous body VP the thread V the point of suspension RPQS the cycloid which the pendulum de cribes P its lowest point PQ an arc equal to the height AE The force with which the motion of the water is accelerated and retarded alternately is the excess of the
k h n h other and therefore

Cor Prop 51 Book 1) to its whole weight as its di tance PQ from the lowest place P to the length PR of the cyclo d Therefore the motive forces of the water and pendulum describ ng the equal spaces AE PQ are as the weights to be moved and therefore if the water and pendulum are quiescent at first those forces will move them in equal times and will cause them to go and return together with a reciprocal motion Q.E.D.

COR 1 Therefore the reciprocations of the water in ascending and descend

PROPOSITION 43 THEOREM 34

Every tremulous body in an elastic medium propagates the motion of the pulses on every side straight forwards but in a nonelastic medium excites a circular motion

CASE 1 The parts of the tremulous body alternately going and returning do in going urge and drive before them those parts of the medium that lie nearest and by that impulse compress and condense them and in returning suffer those compressed parts to recede again and expand themselves Therefore the parts of the medium that lie nearest to the tremulous body move to and fro by turns in like manner as the parts of the tremulous body itself do and for the same cause that the parts of this body agitate these parts of the medium these parts being agitated by like tremors will in their turn agitate others next to themselves and these others agitated in like manner will agitate those that lie beyond them and so on *in infinitum* And in the same manner as the first parts of the medium were condensed in going and relaxed in returning so will the other parts be condensed every time they go and expand themselves every time they return And therefore they will not be all going and all returning at the same instant (for in that case they would always maintain determined distances from each other and there could be no alternate condensation and rarefaction) but since in the places where they are condensed they approach to and in the places where they are rarefied recede from each other therefore some of them will be going while others are returning and so on *in infinitum* The parts so going and in their going condensed are pulses by reason of the progressive motion with which they strike obstacles in their way and therefore the successive pulses produced by a tremulous body will be propagated in rectilinear directions and that at nearly equal distances from each other because of the equal intervals of time in which the body by its several tremors produces the several pulses And though the parts of the tremulous body go and return in some certain and determinate direction yet the pulses propagated from thence through the medium will dilate themselves towards the sides by the foregoing Proposition and will be propagated on all sides from that tremulous body as from a common centre in surfaces nearly spherical and concentric as in waves excited by shaking a finger in water which proceed not only forwards and backwards agreeably to the motion of the finger but spread themselves in the manner of concentric circles all round the finger and are propagated on every side For the gravity of the water supplies the place of elastic force

CASE 2 If the medium be not elastic then because its parts cannot be condensed by the pressure arising from the vibrating parts of the tremulous body the motion will be propagated in an instant towards the parts where the medium yields most easily that is to the parts which the tremulous body would otherwise leave vacuous behind it The case is the same with that of a body projected in any medium whatever A medium yielding to projectiles does not recede *in infinitum* but with a circular motion comes round to the spaces which the body leaves behind it Therefore as often as a tremulous body tends to any part the medium yielding to it comes round in a circle to the parts which the body leaves and as often as the body returns to the first place the medium will be driven from the place it came round to and return to its original place And though the tremulous body be not firm and hard but

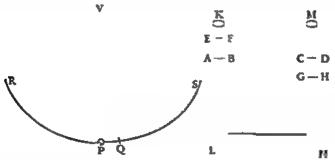
every way flexible yet if it continue of a given magnitude since it cannot impel the medium by its tremors anywhere without yielding to it omewhere

— — — f h body wh re t pressed all

PROPOSITION 44 THEOREM 35

If water ascend and descend alternately in the erected legs KL, MN of a canal or pipe and a pendulum be constructed whose length between the point of suspension and the cent e of oscillat on is equal to half the length of the water in the canal I say that the water will ascend and descend in the same times in which the pendulum oscillates

I measure the length of the water along the axes of the canal and its legs and make it equal to the sum of those axes and take no notice of the resistance of the water arising from its attrition by the sides of the canal Let therefore AB CD represent the mean height of the water in both legs and when the water in the leg KL ascends to the height EF the water will descend in the



leg MN to the h ht (H I P b n d b m

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motion of the water is accelerated and retarded alternately in the excess of the weight of the water in one leg above the weight in the other and therefore

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hole weight as its distance PQ from the lowest place P to the length PR of the cycloid Therefore the motive forces of the water and pendulum des nh

to be m

those fo

return t

procal motion

Q E D

COR. I Therefore the reciprocations of the water in ascending and descend

ing are all performed in equal times whether the motion be more or less intense or remiss

COR II If the length of the whole water in the canal be of $6\frac{1}{9}$ feet of French measure the water will descend in one second of time and will ascend in another second and so on by turns *in infinitum* for a pendulum of $3\frac{1}{18}$ such feet in length will oscillate in one second of time

COR III But if the length of the water be increased or diminished the time of the reciprocation will be increased or diminished as the square root of the length

PROPOSITION 45 THEOREM 36

The velocity of waves varies as the square root of the breadths

This follows from the construction of the following Proposition

PROPOSITION 46 PROBLEM 10

To find the velocity of waves

Let a pendulum be constructed whose length between the point of suspension and the centre of oscillation is equal to the breadth of the waves and in the time that the pendulum will perform one single oscillation the waves will advance forwards nearly a space equal to their breadth

That which I call the breadth of waves is the transverse measure lying between the deepest part of the hollows or the tops of the ridges Let ABCDEF represent the surface of stagnant water ascending and descending in successive

waves and let A C E &c be the tops of the waves and let B D F &c be the intermediate hollows Because the motion of the waves is carried on by the successive ascent and descent of the water so that the parts thereof as A C E &c which are highest at one time become lowest immediately after and because the motive force by which the highest parts descend and the lowest ascend is the weight of the elevated water that alternate ascent and descent will be analogous to the reciprocal motion of the water in the canal and will observe the same laws as to the times of ascent and descent and therefore (by Prop 44) if the distances between the highest places of the waves A C E and the lowest B D F be equal to twice the length of any pendulum the highest parts A C E will become the lowest in the time of one oscillation and in the time of another oscillation will ascend again Therefore with the passage of each wave the time of two oscillations will occur that is the wave will describe its breadth in the time that pendulum will oscillate twice but a pendulum of four times that length and therefore equal to the breadth of the waves will just oscillate once in that time Q E I

COR I Therefore waves whose breadth is equal to $3\frac{1}{18}$ French feet will advance through a space equal to their breadth in one second of time and therefore in one minute will go over a space of $183\frac{1}{3}$ feet and in an hour a space of 11 000 feet nearly

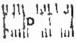
COR II And the velocity of greater or less waves will be augmented or diminished as the square root of their breadth

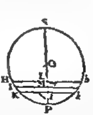
These things are true upon the supposition that the parts of water ascend or descend in a straight line but in truth that ascent and descent is rather performed in a circle and therefore I give the time defined by this Proposition as only approximate.

Proposition 47 Theorem 37

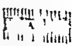
If pulses are propagated through a fluid, the several particles of the fluid going and returning with the shortest reciprocal motion are always accelerated or retarded according to the law of the oscillating pendulum

AB BC CD &c. represent equal distances of successive pulses ABC ... lses propagated from ... medium situate in the ... f Gg equal spaces of extreme shortness through which those points ... turn with a reciprocal motion in each vibration ϵ ϕ γ any intermediate places of the same point. EF FG physical short lines or linear parts of the medium lying between those points and successively transferred into the places $\epsilon\phi$ $\phi\gamma$ and ef fg Let there be drawn the right line PS equal to the right line Ee Bisect the same in O and from the centre O with the radius OP describe the circle SIPi Let the whole time of one vibration, with its proportional parts, be represented by the whole circumference of this circle and its parts in such sort that when any time PH or PHSh is completed, if there be let fall to PS the perpendicular HL or hl and there be taken Eϵ equal to PL or Pl the physical point E may be found in ϵ A point, as E, moving according to this law with a reciprocal motion, in its going from E through ϵ to e and returning again through ϵ to E, will perform its several vibrations with the same degrees of acceleration and retardation with those of an oscillating pendulum. We are now to prove that the several physical points of the medium will be agitated with such a kind of motion Let us suppose then that a medium hath such a motion excited in it from any cause whatsoever and consider what will follow from thence





In the circumference PHSh let there be taken the equal arcs HI IK or hi ik having the same ratio to the whole circumference as the equal right lines EF FG have to BC the whole interval of the pulses Let fall the perpendiculars IM KN or im kn then because the points E, F G are successively agitated with like motions, and perform their entire vibrations composed of their going and return while the pulse is transferred from B to C if PH or PHSh be the time elapsed since the beginning of the motion of the point E, then will PI or PHSi be the time elapsed since the beginning of the motion of the point F and PK or PHSk the time elapsed since the beginning of the motion of the point G and therefore Eϵ Fϕ Gγ will be respectively equal to PL, PM PN while the points



are going and to Pl Pm Pn when the points are returning Therefore $\epsilon\gamma$ o $EG+G\gamma-E\epsilon$ will when the points are going be eq 1 to EG

l

$LG+ln$ or $EG+LN$ is to EG Therefore since LN is to KH as IM to the radiu OP and KH to EG as the circumference PHShP to BC that is if we put V for the radius of a circle whose circumference is equal to BC the interval of the pulses as OP is to V and multiplying together corresponding terms of the proportions we obtain LN to EG as IM to V the expan ion of the part EG or of the physical point F in the place $\epsilon\gamma$ is to the mean expansion of the same part in its first place EG as $V-IM$ is to V in going and as $V+im$ is to V in its return Hence the elastic force of the point F in the place $\epsilon\gamma$ is to its mean elastic force in the place EG as $\frac{1}{V-IM}$ is to $\frac{1}{V}$ in its going and as $\frac{1}{V+im}$ is to $\frac{1}{V}$ in its return And by the same reasoning the elastic forces of the physical points E and G in going are as $\frac{1}{V-HL}$ and $\frac{1}{V-KN}$ is to $\frac{1}{V}$ and the difference of the forces is to the mean elastic force of the medium as $\frac{HL-KN}{VV-V\ HL-V\ KN+HL\ KN}$ is to $\frac{1}{V}$ that is as $\frac{HL-KN}{VV}$ is to $\frac{1}{V}$ or as $HL-KN$ is to V if we suppose (by reason of the very short extent of the vibrations) HL and KN to be indefinitely less than the quantity V Therefore since the quantity V is given the difference of the forces is as $HL-KN$ that is (because $HL-KN$ is proportional to HK and OM to OI or OP and because HK and OP are given) as OM that is if Ff be bisected in Ω as $\Omega\phi$ And for the same reason the difference of the elastic forces of the physical points ϵ and γ in the return of the physical short line $\epsilon\gamma$ is as $\Omega\phi$ But that difference (that is the exce s of the elastic force of the point ϵ above the elastic force of the point γ) is the very force by which the intervening physical short line $\epsilon\gamma$ of the medium is accelerated in going and retarded in returning and therefore the accelerative force of the physical sho t line $\epsilon\gamma$ is as its distance f om Ω

y Prop 38 Book I) the r part of the medium $\epsilon\gamma$ s according to the law all the l n of

C
with
in their progress For
place is at rest neith
either from the impulse of the tremulous body or of the pul es propagated from that body As soon therefore as the pulses cease to be propagated from the tremulous body it will return to a state of rest and move no more

PROPOSITION 48 THEOREM 38

ompounded of
root of the

ne medium is
corre-
rectly
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accurate However
tense the error will not be sensible and therefore this may be
ally exact Now the motive elastic forces are as the con
generated in the same time in equal
corresponding parts of correspond
through spaces proportional to their
es that are as those spaces and there-
oing and returning advance forwards
ays succeeding into the places of the
will by reason of the equality of the
with equal velocity

CASE 2 If the distances of the pulses or their lengths are greater in one medium than in another let us suppose that the correspondent parts describe going and returning each time proportional to the breadths of the and therefore if

time of one going and returning is in a ratio compounded of the matter and the square root of the space and therefore is as the space But the pulses advance a space equal to their breadths in the times of going once and returning once that is they go over spaces proportional to the times and therefore are equally swift

CASE 3 And therefore in mediums of equal density and elastic force all the pulses are equally swift Now if the density or the elastic force of the medium are augmented then because the motive force is increased in the ratio of the elastic force and the matter to be moved is increased in the ratio of the density the time which is necessary for producing the same motion as before will be increased as the square root of the ratio of the density and will be diminished And therefore the velocity of

elastic force
Q.E.D.

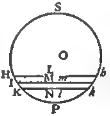
This Proposition will be made clearer from the construction of the following Problem

PROPOSITION 49 PROBLEM 11

The density and elastic force of a medium being given to find the velocity of the pulses

Suppose the medium to be pressed by an incumbent weight after the manner of our air and let A be the height of an homogeneous medium whose weight is equal to the incumbent weight and whose density is the same with the density of the compressed medium in which the pulses are propagated Suppose a pendulum to be constructed whose length between the point of suspension and the centre of oscillation is A and in the time in which that pendulum will perform one entire oscillation composed of its going and returning the pulse will be propagated right onwards through a space equal to the circumference of a circle described with the radius A

For letting those things stand which were constructed in Prop 47 if any physical line as EF describing the space PS in each vibration be acted on in the extremities P and S of every going and return that it makes by an elastic force that is equal to its weight it will perform its several vibrations in the time in which the same might oscillate in a cycloid whose whole perimeter is equal to the length PS and that because equal forces will impel equal corpuscles through equal spaces in the same or equal times Therefore since the times of the oscillations are as the square root of the lengths of the pendulums and the length of the pendulum is equal to half the arc of the whole cycloid the time of one vibration would be to the time of the oscillation of a pendulum whose length is A as the square root of the length $\frac{1}{2}$PS or PO to the length A But the elastic force with which the physical short line EG is urged when it is found in its extreme places P S was (in the demonstration of Prop 47) to its whole elastic force as HL−KN is to V that is (since the point K now falls upon P) as HK to V and all that force



or which is the same thing the incumbent weight by which the short line EG is compressed is to the weight of the short line as the altitude of the incumbent weight is to EG the length of the short line and therefore taking the product of corresponding terms the force with which the short line EG is urged in the places P and S is to the weight of that short line as HK A is to V EG or as PO A is to VV because HK was to EG as PO to V Therefore since the times in which equal bodies are impelled through equal spaces are inversely as the square root of the forces the time of one vibration produced by the action of that elastic force will be to the time of a vibration produced by the impulse of the weight as the square root of the ratio of VV to PO A and therefore to the time of the oscillation of a pendulum whose length is A as the square root of the ratio of VV to PO A and as the square root of the ratio of PO to A conjointly that is in the entire ratio of V to A But in the time of one vibration composed of the going

and returning of the pendulum the pulse will be propagated right onwards through a space equal to its breadth BC Therefore the time in which a pulse runs over the space BC is to the time of one oscillation composed of the going and returning of the pendulum as \ is to \ that is as BC is to the circumference of a circle whose radius is A. But the time in which the pulse will run over the space BC is to the time in which it will run over a length equal to that circumference in the same ratio and therefore in the time of such an oscillation the pulse will run over a length equal to that circumference Q E D

... of the pulses is equal to that which heavy bodies acquire ... describing half ... osing it to move with the velocity acquired ... will be equal to the whole altitude A and therefore in the time of one oscillation composed of one goin and return will go over a pace equal to the circumference of a circle described with the radius A for the time of the fall is to the time of oscillation as the radius of a circle to its circumference

... A is directly as the elastic force of the ...

PROPOSITION 50 PROBLEM 12

To find the distances of the pulses

Let the number of the vibrations of the body by whose tremor the pulses are produced be found to any given time By that number divide the space which a pulse can go over in the same time and the part found will be the breadth of one pulse Q E I

SCHOLIUM

The last Propositions respect the motions of light and sounds for since light is propagated in right lines it is certain that it cannot consist in action alone ... from tremulous bodies ... Prop ... ud and ... drums for quick and short tremors are less easily excited But it is ... that any sounds falling upon strings in unison with the sonorous bodies excite tremors in those strings This is also confirmed from the velocity of sounds for since the specific gravities of rain water and quicksilver are to one another as about 1 to 13⅔ and when the mercury in the barometer is at the height of 30 inches of our measure the specific gravities of the air and of rain water are to one ...

radius is 29 25 feet is 186 68 feet in circumference And since a pendulum 39⅕ inches in length completes one oscillation composed of its going and return in

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omputation we have made no allowance for the crassitude of the solid particles of the air by which the sound is propagated instantaneously Because the weight of air is to the weight of water as 1 to 870 and because salts are almost twice as dense as water if the particles of air are supposed to be of about the same density as those of water or salt and the rarity of the air arises from the intervals of the particles the diameter of one particle of air will be to the interval between the centres of the particles as 1 to about 9 or 10 and to the interval between the particles themselves as 1 to 8 or 9 Therefore to 979 feet which according to the above calculation a sound in one second of time
crassitude of
1088 feet in o

Moreover the vapors floating in the air being of another spring and ent tone will hardly if at all
the
will
root
true
a one part of vapor th atmosphere consist of ten

the sound will pass through 1142 feet in one second of time increased in that ratio By this These things will be found true in spring and autumn when the air is rarefied by the gentle warmth of those seasons and by that means its elastic force becomes somewhat more intense But in winter when the air is condensed by the cold and its elastic force is somewhat remitted the motion of sounds will be slower as the square root of the density and on the other hand the summer

sec
mo

The velocity of sounds being known the
also For M
pipe abou
string that
pulses in a space of 1070 Paris feet which a sound runs over in a second of time and therefore one pulse fills up a space of about $10\frac{7}{10}$ Paris feet that is about twice the length of the pipe From this it is probable that the breadths of the pulses in all sounds made in open pipes are equal to twice the length of the pipes Therefore there are near 100

Moreover from the Corollary of Prop 47 appears the reason why the sounds immediately cease with the motion of the sonorous body and why they are heard no longer when we are at a great distance from the sonorous bodies than when we are very near them And besides from the foregoing principles it plainly appears how it comes to pass that sounds are so mightily increased in speaking trumpets for all reciprocal motion tends to be increased by the gen

erating cause at each return. And in tubes hindering th dilatation of the sounds, the motion decays more lowly and recurs more forcibly and therefore is the more increased by the new motion impressed at each return And these are the principal phenomena of sounds.

SECTION IX

THE CIRCULAR MOTION OF FLUIDS

HYPOTHESIS

The resistance ari ng from the want of l bricity in the parts of a fuid is o.her things bei g eq al propo tional to the velocity with which the parts of the fuid are separa.ed from one a o.her

PROPOSITION 51 THEOREM 39

If a solid cyl nder infinitely long in an un form and infinite fuid revolves with an uniform motio about an axis given in posit on and the f id be forced round by only this imp lse of the cyl nder and every part of the fuid continues uniformly i its mo ion I say tha. the period c times of the parts of th fuid are as their distances f om the axis of the cyl nder

their tran.lations from each other and as the contiguous surfaces upon which the impressions are made If the impression made upon any orb be greater or less on its concave than on its convex side the stronger impression will prevail and will either accelerate or retard th motion of the orb according as it agrees with or is contrary to the mot on of the same Therefore that every orb may continue uniformly in its motion, the impre ion made on both sides mu.t be equal and their directions contrary Therefore since the impressions are as the contiguous surfaces and as their tran.lations from one another the tran.lations will be inversely as the surfaces, that is inversely as the distances of th ...

mo.ions abou t.
directly as the tr
ra.ios together i
erected the lines
of the in.nite ri

of SA SB SC SD SE &c and through the extremities of those perpendiculars there be supposed to pass an hyperbolic curve the sums of the differences that is the whole angular motions will be as the correspondent sums of the lines *Aa* *Bb* *Cc* *Dd* *Ee* that is (if to constitute a medium uniformly fluid the number of the orbs be increased and their breadth diminished *in infinitum*) as the hyperbolic areas *Aa*Q *Bb*Q *Cc*Q *Dd*Q *Ee*Q &c analogous to the sums and the times inversely proportional to the angular motions will be also inversely proportional to those areas Therefore the periodic time of any particle as D is inversely as the area *Dd*Q that is (as appears from the known methods of quadratures of curves) directly as the distance SD Q E D

COR I Hence the angular motions of the particles of the fluid are inversely as their distances from the axis of the cylinder and the absolute velocities are equal

COR II If a fluid be contained in a cylindric vessel of an infinite length and contain another cylinder within and both the cylinders revolve about one common axis and the times of their revolutions be as their semidiameters and every part of the fluid continues in its motion the periodic times of the several parts will be as the distances from the axis of the cylinders

COR III If there be added or taken away any common quantity of angular motion from the cylinder and fluid moving in this manner yet because this new motion will not alter the mutual attrition of the parts of the fluid the motion of the parts among themselves will not be changed for the translations of the parts from one another depend upon the attrition Any part will continue in that motion which by the attrition made on both sides with contrary directions is no more accelerated than it is retarded

COR VI Therefore if there be taken away from this whole system of the cylinders and the fluid all the angular motion of the outward cylinder we shall have the motion of the fluid in a quiescent cylinder

COR V Therefore if the fluid and outward cylinder are at rest and the inward cylinder revolve uniformly there will be communicated a circular motion to the fluid which will be propagated by degrees through the whole fluid and will go on continually increasing till such time as the several parts of the fluid acquire the motion determined in Cor IV

COR VI And because the fluid endeavors to propagate its motion still farther its impulse will carry the outmost cylinder also about with it unless the cylinder be forcibly held back and accelerate its motion till the periodic times of both cylinders become equal with each other But if the outward cylinder be forcibly held fast it will make an effort to retard the motion of the fluid and unless the inward cylinder preserve that motion by means of some external force impressed thereon it will make it cease by degrees

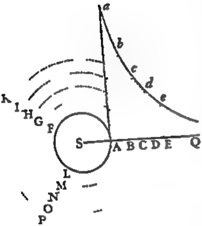
All these things will be found true by making the experiment in deep standing water

PROPOSITION 52 THEOREM 40

If a solid sphere in an uniform and infinite fluid revolves about an axis given in position with an uniform motion and the fluid be forced round by only this impulse of the sphere and every part of the fluid continues uniformly in its motion I say that the periodic times of the parts of the fluid are as the squares of their distances from the centre of the sphere

CASE 1 Let AFL be a sphere turning uniformly about the axis S and let the concentric circles BGM CHN DIO EKP &c divide the fluid into innumer

thickness Suppose those orbs to be solid and ... contiguous orbs ... nslations from ... made ... n upon ... her ac ... of the ... with a orb according as it ... conspiring or contrary motion to that of the orb Therefore that every orb may continue uniformly in its motion it is necessary that the impressions made upon both sides of the orb should be equal and have contrary directions Therefore since the impressions are as the contiguous surfaces and as their translations from one another the translations will be inversely as the surfaces that is inversely as the squares of the ...



... translations applied to the distances or directly as the translations and inversely as the distances that is by com... those ratios inversely as the cubes of the distances Therefore if ... DCDEQ there be erected

... DIO is inversely as the area DdQ that is (by the ... ture) directly as the square of the distance SD Which was first to be demonstrated

... 2 From the centre of the sphere let there be drawn a great number of ...

... trary directions by the attrition of the interior and exterior annuli unless the motion be communicated according to the law which we demonstrated in ... of

of SA SB SC SD SE &c and through the extremities of those perpendiculars there be supposed to pass an hyperbolic curve the sums of the differences that is the whole angular motions will be as the correspondent sums of the lines A*a* B*b* C*c* D*d* E*e* that is (if to constitute a medium uniformly fluid the number of the orbs be increased and their breadth diminished *in infinitum*) as the hyperbolic areas A*a*Q B*b*Q C*c*Q D*d*Q E*e*Q &c analogous to the sums and the times inversely proportional to the angular motions will be also inversely proportional to those areas Therefore the periodic time of any particle as D is inversely as the area D*d*Q that is (as appears from the known methods of quadratures of curves) directly as the distance SD Q E D

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COR II If a fluid be contained in a cylindric vessel of an infinite length and contain another cylinder within and both the cylinders revolve about one common axis and the times of their revolutions be as their semidiameters and every part of the fluid continues in its motion the periodic times of the several parts will be as the distances from the axis of the cylinders

COR III If there be added or taken away any common quantity of angular motion from the cylinder and fluid moving in this manner yet because this new motion will not alter the mutual attrition of the parts of the fluid the motion of the parts among themselves will not be changed for the translations of the parts from one another depend upon the attrition Any part will continue in that motion which by the attrition made on both sides with contrary directions is no more accelerated than it is retarded

COR IV Therefore if there be taken away from this whole system of the cylinders and the fluid all the angular motion of the outward cylinder we shall have the motion of the fluid in a quiescent cylinder

COR V Therefore if the fluid and outward cylinder are at rest and the in

acquire the motion determined in Cor IV

COR VI And because the fluid endeavors to propagate its motion still farther its impulse will carry the outmost cylinder also about with it unless the cylinder be forcibly held back and accelerate its motion till the periodic times of both cylinders become equal with each other But if the outward cylinder be forcibly held fast it will make an effort to retard the motion of the fluid and unless the inward cylinder preserve that motion by means of some external force impressed thereon it will make it cease by degrees

All these things will be found true by making the experiment in deep standing water

PROPOSITION 52 THEOREM 40

If a solid sphere in an uniform and infinite fluid revolves about an axis given in position with an uniform motion and the fluid be forced round by only this impulse

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COR. v If another globe no u u —
ce from its centre and in the meant.me by some force revolve con. tan y
— of l is globe will drive the fluid
new and small vortex will
r and in the meantime its
degrees be propagated in
or the same rea.on that the
— of h other
w
id
ne
ch
verything be left to
globes will langui.h
d the vortices at last

COR. VI. If several globes in given places .u uu ..tantly revolve with determined velocities about axes given in position there would arise from them as many vortices going on *in infinitum* For upon the same account that any one globe propagates its motion *in i finitum* each globe apart will propagate it motion *in infinit m* also so that every part of the infinite fluid will be agitated with a motion resulting from the actions of all the globes. Therefore the vortices will not be confined by any certain limit but by degrees run into
f h rt ces on each other the globes will be
Corollary
lves unless
v impressed
upon th globes to continue these motions hould cease the matter (for the rea son as.umed in Cor III and IV) will gradually stop and cease to move in vortices
od — oh real vessel and by the un

th ir motions without acceleration or retardation till their periodical times are

—

because the mutual attrition of the part of the fluid is not changed by this mot n the motions of the parts among themselves will not be changed for the tran lat ons of the parts amon themselves depend upon this attrition. Any part will continue in that mot on in which its attrit on on one side retards it just as mu h as its attrit n on the other side accelerates it

this law, no such is and therefore cannot be any obstacle to the motions continuing according to that law If annuli at equal distances from the centre revolve either more swiftly o m s than near the ecliptic

their mutual attrition

going on according

that is the periodi u be as the squares of their distances from the centre of the globe Which was to be demonstrated in the second place

CASE 3 Let now every annulus be divided by transverse sections into innumerable particles constituting a substan

and been

only ser

the same

asperity ount of the e sections or else they wil ange the same equally Therefore the propo tion of remaining the same

that is th

the circul ence arising is greater at the ecliptic than at the poles there must be some cause operating to retain the several particles in their circles otherwise the matter that is at the always recede from the centre

side of the vortex and from

continual circulation

COR I Hence the angular motions of the parts of the fluid about the axis of the globe are inversely as the squares of the distances from the centre of the globe and the absolute velocities are inversely as the same squares applied to the distances from the axis

COR II If a globe revolve with

at of a vortex and that by degrees be propagated onwards *in infinitum* and this motion will be increa ed continually in every part of the fluid till the periodical times of the several parts become as the squares of the distances from the centre of the globe

COR III Because the inward pa t of

the same quantity of motion to tho e that lie still beyond them and by this action preserve the quantity of their motion continually unchanged it

ays tran ferring the motion it receives from the matter nearer the centre to that matter which lies nearer the circumference

COR IV Therefore in order to continue a vortex in the same state of motion

all the more because of their greater swiftness for they then describe arcs of less curvity and the tendency to recede from the centre is as much diminished ... gmented by the increase of the

...

... paces they are a ... a mass in a rigid ... f this Proposition

I have endeavored in this Proposition to investigate the properties of vortices that I might find whether the celestial phenomena can be explained by ... is this, that the periodic times of the planet revolv ...

... the vortices must re... iodic times of the parts of the vortex to be as the square of the ... e centre of motion and this ratio cannot be diminished and reduced to the $^3/_2$th power unless either ... be more fluid the farther it is from the centre or the

...

PROPOSITION 53 THEOREM 41

...

For if any small part of the vortex, whose particles or physical points con...

...

nto a fluid this will move according to the same law as before except so far as it particles, now become fluid may be moved among themselves Neglect therefore the motion of the particles among themselves as not at all concerning

COR IX Therefore if the vessel be quiescent and the motion of the globe be given then
the axis of
sum of the
be to the time of the revolution of the globe as the square of the semidiameter of the vessel to the square of the semidiameter of the globe and the periodic times of the parts of the fluid in respect of this plane will be as the squares of their distances from the centre of the globe

COR X Therefore if the vessel move about the same axis with the globe or with a given velocity about a different one the motion of the fluid will be given For if from the whole system we take away the angular motion of the vessel all the motions will remain the same among themselves as before by Cor VIII and those motions will be given by Cor IX

COR XI If the vessel and the fluid are quiescent and the globe revolves with an uniform motion that motion will be propagated by degrees through the whole fluid to the vessel and the vessel will be carried round by it unless forcibly held back and the fluid and the vessel will be continually accelerated till their periodic times become equal to the periodic times of the globe If the vessel be either restrained by some force or revolve with any constant and uniform motion the medium will come little by little to the state of motion defined in Cor VIII IX X nor will it ever continue in any other state But if then the forces by which the globe and vessel revolve with certain motions should cease and the whole system be left to act according to the mechanical laws the vessel and globe by means of the intervening fluid will act upon each other and will continue to propagate their motions through the fluid to each other till their periodic times become equal among themselves and the whole system revolves together like one solid body

SCHOLIUM

In all these reasonings I suppose the fluid to consist of matter of uniform density and fluidity I mean that the fluid is such that a globe placed anywhere therein may propagate with the same motion of its own at distances from itself continually equal similar and equal motions in the fluid in the same interval of time The matter by its circular motion endeavors to recede from the axis of the vortex and therefore presses all the matter that lies beyond

the parts of the fluid are

the fluidity will be less in that place because there are fewer surfaces where the parts can be separated from each other In these cases I suppose the defect of the fluidity to be supplied by the smoothness or softness of the parts or some other condition otherwise the matter where it is less fluid will cohere more and be more sluggish and therefore will receive the motion more slowly and propagate it farther than agrees with the ratio above assigned If the vessel be not spherical the particles will move in lines not circular but answering to the figure of the vessel and the periodic times will be nearly as the squares of the mean distances from the centre In the parts between the centre and the circumference the motions will be slower where the spaces are wide and swifter where narrow nevertheless the particles will not tend to the circumference at

all the more because of their great swiftness for they then describe arcs of

in narrow spaces, they are again accelerated and so each particle is retarded and accelerated by turns forever These things will come to pass in a rigid vessel for the state of vortices in an infinite fluid is known by Cor. VI of this Proposition.

I have endeavored in this Proposition to investigate the properties of vortices, that I might find whether the celestial phenomena can be explained by them for the phenomenon is this that the periodic times of the planets revolving about Jupiter are as the $\frac{3}{2}$th power of their distances from Jupiter centre and the same rule obtains also among the planets that revolve about the sun. And these rules obtain also with the greatest accuracy as far as has been yet discovered by astronomical observation. Therefore if those planets are carried round in vortices revolving about Jupiter and the sun, the vortices must revolve according to that law But here we found the periodic times of the parts of the vortex to be as the square of the distances from the centre of motion and this ratio cannot be diminished and reduced to the $\frac{3}{2}$th power unless either the matter of the vortex be more fluid the farther it is from the centre or the resistance arising from

the velocity

be augmented

creases. But

and less fluid

wards the centre And though, for the sake of demonstration, I proposed, at the beginning of this Section, an Hypothesis that the resistance is proportional to the velocity nevertheless, it is in truth probable that the resistance is in a less ratio than that of the velocity which granted, the periodic times of the parts of the vortex will be in a greater ratio than the square of the distances from its centre If as some think, the vortices move more swiftly near the centre then slower to a certain limit then again swifter near the circumference certainly neither the $\frac{3}{2}$th power nor any other certain and determinate power can obtain in them. Let philosophers then see how that phenomenon of the $\frac{3}{2}$th power can be accounted for by vortices.

PROPOSITION 53 THEOREM 41

Bodies carried about in a vortex and returning in the same orbit are of the same density with the vortex and are moved according to the same law with the parts of the vortex as to velocity and direction of motion.

For if any small part of the vortex, whose particles or physical points continue a given situation among themselves be supposed to be congealed this part will move according to the same law as before since no change is made either in its density inertia, or figure And again if a congealed or solid part of the vortex be of the same density with the rest of the vortex, and be resolved into a fluid, this will move according to the same law as before except so far as its particles, now become fluid, may be moved among themselves. Neglect therefore the motion of the particles among themselves as not at all concerning

the progressive motion of the

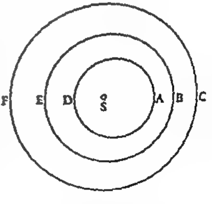
solid if it be of ... matter of the vortex will move with the same motion as the parts thereof being relatively at rest in the matter that surrounds it If it be more dense it will endeavor more than before to recede from the centre and therefore overcoming that ... being as it were ... from the cen... the same orb... it will approach to the centre ... d in the same orbit unle... shown in that case ... parts of th...

... that carries it

COR II ... vortex be of an uniform density the same body may revolve at any distance from the centre of the vortex

SCHOLIUM

Hence it is manifest that the planets are not carried round in ... vortices for according to the Co... the sun ... drawn ... times But the parts of a vortex ... with such a motion For let AD BE CF represent three orbits described about the sun S of which ... concentric to the sun ... will move with an uniform ... according to the laws of astronomy the body revolving in the orbit BE will move slower in its aphelion B and swifter in its perihelion E ... accord... ter of ... in the ... een A and C than in the wide space between D and F that is more swiftly in the aphelion than in the perihelion Now these two conclusions contradict each other So at the beginning of the sign of Virgo where the aphelion of Mars is at present the distance between the orbits of Mars and Venus is to the distance between the same orbits at the beginning of the sign of Pisce... orbits ... beginning of Virgo ... the narrower the space is through which the same quantity of matter passes in the same time of one revolution the greater will be the velocity with which it passes through it Therefore if the earth



being relatively at rest in this celestial matter should be carried round by it and revolve together with it about the sun the velocity of the earth at the beginning of Pisces would be to its velocity at the beginning of Virgo in the ratio of 3 to 2 Therefore the sun s apparent diurnal motion at the beginning of Virgo ought to be above 70 minutes and at the beginning of Pisces less than 48 minutes whereas, on the contrary that apparent motion of the sun is really greater at the beginning of Pisces than at the beginning of Virgo as experience testifies and therefore the earth is swifter at the beginning of Virgo than at the beginning of Pisces so that the hypothesis of vortices is utterly irreconcilable with astronomical phenomena and rather serves to perplex than explain the heavenly motions. How these motions are performed in free spaces without vortices may be understood by the first book and I shall now more fully treat of it in the following book.

BOOK THREE

SYSTEM OF THE WORLD

(IN MATHEMATICAL TREATMENT)

—ciples of philosophy principles
ly as we may build our reason

f

that from the same principles I now demonstra ... of
h World Upon this subject I had indeed composed the third book in a
... ring
... ily
... hich
the had been many years accustomed thereto e to pre ... putes
which might be raised upon such account I chose to reduce the substance of
... way) high
the
cone
ound
with such as might cost too much time even to readers of good mathematical learning. It is enough if one carefully reads the Definitions the Laws of Motion and the first three sections of the first book. He may then pass on to this book and consult such of the remaining Propositions of the first two books as the references in this and his occasion shall require

RULES OF REASONING IN PHILOSOPHY

RULE I

We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances

To this purpose the philosophers say that Nature does nothing in vain and more is in vain when less will serve for Nature is pleased with simplicity and affects not the pomp of superfluous causes

RULE II

Therefore to the same natural effects we must as far as possible assign the same causes

As to respiration in a man and in a beast the descent of stones in Europe and in America the light of our culinary fire and of the sun the reflection of light in the earth and in the planets

RULE III

The qualities of bodies which admit neither intensification nor remission of degrees and which are found to belong to all bodies within the reach of our experiments are to be esteemed the universal qualities of all bodies whatsoever

For since the qualities of bodies are only kno

to hold fo

as

not ... the sake of dreams and vain fict ... our own devising nor are we to recede from the analogy of Nature which is wont to be simple and always consonant to itself We no ot ... know the extension of bodie ...

... abundance of bodies are hard we ... by experience and because the hardness of the whole arises from the hardness of the parts we therefore justly infer the hardness of the undivided particles not only of the bodies we feel but of all others That all bodies are impenetrable we gather not from re ...

... bodies which we have seen The extension hardn... and inertia of the whole result from the exten... mobility and inertia of the parts and hence we co... the least particles of all bodies to be also all extended and hard and impenetrable and mov... and endowed with their proper inert... ophy Moreover that the divided ...

separated from one another is matter of observation and in the particles that
are able to distinguish yet lesser parts as is

e conclude that the
and actually sepa

nd astronomical observa
rds the earth and that in
uantity of matter which they severally contain that the

un we must in consequence of this rule universally
whatsoever are endowed with a principle of mutual gravitation For the argument from the appearances concludes with more force for the universal gravitation of all bodies than for their impenetrability of which among those in the celestial regions we have no experiments nor any manner of observation Not that I affirm gravity to be essential to bodies by their *vis insita* I mean nothing but their inertia This is immutable Their gravity is diminished as they recede from the earth

RULE IV

In experimental philosophy we are to look upon propositions inferred by general

This rule we must follow that the argument of induction may not be evaded by hypotheses

PHENOMENA[1]

PHENOMENON I

That the circumjovial planets by radii drawn to Jupiter s centre describe areas proportional to the times of description and that their periodic times the fixed stars being at rest are as the $\frac{3}{2}$th power of their distances from its centre

This we know from astronomical observations For the orbits of these planets differ but insensibly from circles concentric to Jupiter and their motions in those circles are found to be uniform And all astronomers agree that their periodic times are as the $\frac{3}{2}$th power of the semidiameters of their orbits and so it manifestly appears from the following table

The periodic times of the satellites of Jupiter

$1^d\ 18^h\ 27^m\ 34$ $3^d\ 13^h\ 13^m\ 42$ $7^d\ 3^h\ 42^m\ 36$ $16^d\ 16^h\ 32^m\ 9$

The distances of the satellites from Jupiter s centre

	1	2	3	4	
From the observations of					
Borelli	$5\frac{2}{3}$	$8\frac{2}{3}$	14	$24\frac{2}{3}$	
Townly *by the micrometer*	5 52	8 78	13 47	24 72	*Semi*
Cassini *by the telescope*	5	8	13	23	*diameter of*
Cassini *by the eclipse of the satellites*	$5\frac{2}{3}$	9	$14\frac{23}{60}$	$25\frac{3}{10}$	*Jupiter*
From the periodic times	5 667	9 017	14 384	25 299	

Mr Pound hath determined by the help of excellent micrometers the diameters of Jupiter and the elongation of its satellites after the following manner The greatest heliocentric elongation of the fourth satellite from Jupiter s centre was taken with a micrometer in a 15-foot telescope and at the mean distance of Jupiter from the earth was found about 8 16 The elongation of the third satellite was taken with a micrometer in a telescope of 123 feet and at the same distance of Jupiter from the earth was found 4 42 The greatest elongations of the other satellites at the same distance of Jupiter from the earth are found from the periodic times to be 2 56 47 and 1 51 6

The diameter of Jupiter taken with the micrometer in a 123 foot telescope several times and reduced to Jupiter s mean distance from the earth proved always less than 40 never less than 38 generally 39 This diameter in shorter telescopes is 40 or 41 for Jupiter s light is a little dilated by the unequal refrangibility of the rays and this dilatation bears a less ratio to the diameter of Jupiter in the longer and more perfect telescopes than in those which are shorter and less perfect The times in which two satellites the first

[1] ... of Book III ... ital ... (except in Latin ... an ... ns-

4th third passed over Jupiter s body were observed from the beginning of
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(the earth came
as observed al.o
r Jupiter s body
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nearly and then
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PHENOMENON II

The periodic times of the satellites of Saturn

$1^d\ 21^h\ 18^m\ 27^s$ $2^d\ 17^h\ 41^m\ 22^s$ $4^d\ 12^h$ .5m 12s $15^d\ 22^h\ 41^m\ 14$
79d 7h 48m 00

The distances of the satellites from Saturn s centre in semidiameters of its ring

From observations	$1^{19}/_{20}$	2½	3½	8	24
From the periodic times	1 93	2 47	3 45	8	23.35

The greatest elongation of the fourth satellite from Saturn s centre is commonly determined from the observations to be eight of those semidiameters

Saturn s centre in semidiameters of the ring are 2 1 2 69 3 75 8 7 and 25.35
— h h

scopes the apparent magnitudes of the heavenly bodies bear a greater proportion to the dilatation of light in the extremities of those bodies than in horter telescopes If then we reject all the spurious light the diameter of Saturn will not amount to more than 16

PHENOMENON III

That the five primary planets Mercury Venus Mars Jupiter and Saturn with their several orbits encompass the sun

Th

PHENOMENA[1]

PHENOMENON I

That the circumjovial planets, by radii drawn to Jupiter's centre, describe areas proportional to the times of description; and that their periodic times, the fixed stars being at rest, are as the $\frac{3}{2}$th power of their distances from its centre.

This we know from astronomical observations. For the orbits of these planets differ but insensibly from circles concentric to Jupiter, and their motions in those circles are found to be uniform. And all astronomers agree that their periodic times are as the $\frac{3}{2}$th power of the semidiameters of their orbits; and so it manifestly appears from the following table.

The periodic times of the satellites of Jupiter.

$1^d\ 18^h\ 27^m\ 34^s$. $3^d\ 13^h\ 13^m\ 42^s$. $7^d\ 3^h\ 42^m\ 36^s$. $16^d\ 16^h\ 32^m\ 9^s$.

The distances of the satellites from Jupiter's centre.

	1	2	3	4	
From the observations of					
Borelli	$5\frac{2}{3}$	$8\frac{2}{3}$	14	$24\frac{2}{3}$	
Townly *by the micrometer*	5.52	8.78	13.47	24.72	*Semi-*
Cassini *by the telescope*	5	8	13	23	*diameter of*
Cassini *by the eclipse of the satellites*	$5\frac{2}{3}$	9	$14\frac{23}{60}$	$25\frac{3}{10}$	*Jupiter*
From the periodic times	5.667	9.017	14.384	25.299	

Mr. Pound hath determined, by the help of excellent micrometers, the diameters of Jupiter and the elongation of its satellites after the following manner. The greatest heliocentric elongation of the fourth satellite from Jupiter's centre was taken with a micrometer in a 15-foot telescope, and at the mean distance of Jupiter from the earth was found about 8′ 16″. The elongation of the third satellite was taken with a micrometer in a telescope of 123 feet, and at the same distance of Jupiter from the earth was found 4′ 42″. The greatest elongations of the other satellites, at the same distance of Jupiter from the earth, are found from the periodic times to be 2′ 56″ 47‴, and 1′ 51″ 6‴.

The diameter of Jupiter taken with the micrometer in a 123-foot telescope several times, and reduced to Jupiter's mean distance from the earth, proved always less than 40″, never less than 38″, generally 39″. This diameter in shorter telescopes is 40″, or 41″; for Jupiter's light is a little dilated by the unequal refrangibility of the rays, and this dilatation bears a less ratio to the diameter of Jupiter in the longer and more perfect telescopes than in those which are shorter and less perfect. The times in which two satellites, the first

[1] … of Book III … printed in italics (except in Latin …

For to the earth they appear sometimes direct sometimes stationary nay
they are always seen direct and
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equality ng
astronomers and particularly demonstrable in Jupiter us
satellites by the help of these eclipses as we have said the heliocentric longitudes of that planet and its distances from the sun are determined

PHENOMENON VI

That the moon by a radius drawn to the earth's centre describes an area proportional to the time of description

tion of the moon, compared with its
on is a little disturbed
mens, I neglect those

same height on one side or other of the sun when horned they are below or between us and the sun and they are sometime *when directly under* seen like spots traversing the sun s disk That Mars surrounds the sun is as plain from its full face when near its conjunction with the sun and from the gibbous [illegible]

PHENOMENON IV

That the fixed stars being at rest the periodic times of the five primary planets and (whether of the sun about the earth or) of the earth about the sun are as the $\frac{3}{2}$th power of their mean distances from the sun

This proportion first observed by Kepler is now received by all astronomers for the periodic times are the same and the dimensions of the orbits are the same whether the sun revolves about the earth or the earth about the sun And as to the measures of the periodic times all astronomers are agreed about them But for the dimensions of the orbits Kepler and Boulliau above all others have determined them from observations with the greatest accuracy and the mean distances corresponding to the periodic times differ but insensibly from those which they have assigned and for the most part fall in between them as we may see from the following table

The periodic times with respect to the fixed stars of the planets and earth revolving about the sun in days and decimal parts of a day

♄	♃	♂	♁	♀	☿
10759 275	4332 514	686 9785	365 2565	224 6176	87 9692

The mean distances of the planets and of the earth from the sun

	♄	♃	♂
According to Kepler	951 000	519 650	152 350
Boulliau	954 198	522 520	152 350
the periodic times	954 006	520 096	152 369

	♁	♀	☿
According to Kepler	100 000	72 400	38 806
Boulliau	100 000	72 398	38 585
the periodic times	100 000	72 333	38 710

As to Mercury and Venus there can be no doubt about their distances from the sun for they are determined by the elongations of those planets from the sun and for the distances of the superior planets all dispute is cut off by the eclipses of the satellites of Jupiter For by those eclipses the position of the shadow which Jupiter projects is determined from this we have the heliocentric longitude of Jupiter And from its heliocentric and geocentric longitudes compared together we determine its distance

PHENOMENON V

Then the primary planets by radii drawn to the earth describe areas in no wise proportional to the times but the areas which they describe by radii drawn to the sun are proportional to the times of description

will be inversely as D- Th_ will yet more fully appear from comparing this force with the force of gravity as i_ done in the next Proposition.

COR. If we augment the mean centripetal force by which the moon is retained in it_ orb first in the proportion of 177 29/40 to 178 29/40 and then in the proportion of the square of the semidiameter of the earth to the mean distance of the centres of the moon and earth we shall have the centripetal force of the moon at the surface of the earth supposing this force in descending to the earth surface continually to increase inversely as the square of the height

PROPOSITION 4. THEOREM 4

That the moon gravitates towards the earth and by the force of gravity is continually drawn off from a rectil near motion and retained in its orbit.

The mean distance of the moon from the earth in the syzygies in semidiameters of the earth, is according to Ptolemy and most astronomers 59 according to Vendelin and Huygens 60 to Copernicus 60⅓ to Street, 60⅖ and to Tycho 56½ But Tycho and all that follow his tables of refraction, making the refractions of the sun and moon (altogether against the nature of light) to exceed the refraction of the fixed stars and that by four or five

diameters of the earth, near to what others have assigned. Let us assume the mean distance of 60 diameters in the syzygies and suppose one revolution of the moon, in respect of the fixed stars, to be completed in 27d 7h 43m as astronomers have determined and the circumference of the earth to amount to 123,249,600 Paris feet as the French have found by mensuration. And now if we imagine the moon, deprived of all motion to be let go so as to descend towards the earth with the impulse of all that force by which (by Cor Prop 3) it is retained in its orbit it will in the space of one minute of time describe in its fall 15 1/12 Paris feet This we gather by a calculus founded either upon Prop 36 Book 1 or (which comes to the same thing) upon Cor IX, Prop 4 of the same book. For the versed sine of that arc which the moon, in the space of one minute of time would by its mean motion describe at the distance of 60 semidiameters of the earth, is nearly 15 1/12 Paris feet or more accurately 15 feet, 1 inch, and 1 line 4/9. Wherefore since that force in approaching to the earth, increases in the proportion of the inverse square of the distance and, upon that account at the surface of the earth, is 60 60 times greater than at the moon, a body in our regions falling with that force ought in the space of one minute of time to describe 60 60 15 1/12 Paris feet and in the space of one second of time to describe 15 1/12 of those feet or more accurately 15 feet, 1 inch and 1 line 4/9 And with this very force we actually find that bodies here upon earth do really descend for a pendulum oscillating seconds in the latitude of Paris will be 3 Paris feet and 8 lines 1/2 in length, as Mr Huygens has observed. And the space which a heavy body describes by falling in one second of time is to half the length of this pendulum as the square of the ratio of the circumference of a circle to its diameter (as Mr Huygens has also shown) and is therefore 15 Paris feet 1 inch, 1 line 7/9. And therefore the force by which the moon is retained in its orbit becomes, at the very surface of the earth, equal to the force of gravity which we observe in heavy bodies there And therefore (by Rules 1

PROPOSITIONS

Proposition 1. Theorem 1

That the forces by which the circumjovial planets are continually drawn off from rectilinear motions, and retained in their proper orbits, tend to Jupiter's centre; and are inversely as the squares of the distances of the places of those planets from that centre.

The former part of this Proposition appears from Phen. I, and Prop. 2 or 3, Book I; the latter from Phen. I, and Cor. VI, Prop. 4, of the same book.

The same thing we are to understand of the planets which encompass Saturn, by Phen. II.

Proposition 2. Theorem 2

That the forces by which the primary planets are continually drawn off from rectilinear motions, and retained in their proper orbits, tend to the sun; and are inversely as the squares of the distances of the places of those planets from the sun's centre.

The former part of the Proposition is manifest from Phen. V, and Prop. 2, Book I; the latter from Phen. IV, and Cor. VI, Prop. 4, of the same book. But this part of the Proposition is, with great accuracy, demonstrable from the quiescence of the aphelion points; for a very small aberration from the proportion according to the inverse square of the distances would (by Cor. I, Prop. 45, Book I) produce a motion of the apsides sensible enough in every single revolution, and in many of them enormously great.

Proposition 3. Theorem 3

That the force by which the moon is retained in its orbit tends to the earth; and is inversely as the square of the distance of its place from the earth's centre.

The former part of the Proposition is evident from Phen. VI, and Prop. 2 or 3, Book I; the latter from the very slow motion of the moon's apogee; which in every single revolution amounting but to 3° 3' forwards, may be neglected. For (by Cor. I, Prop. 45, Book I) it appears that, if the distance of the moon from the earth's centre is to the semidiameter of the earth as D to 1, the force, from which such a motion will result, is inversely as $D^{2\frac{4}{243}}$, i.e., inversely as the power of D, whose exponent is $2\frac{4}{243}$; that is to say, in the proportion of the distance something greater than the inverse square, but which comes $59\frac{3}{4}$ times nearer to the proportion according to the square than to the cube. But since this increase is due to the action of the sun (as we shall afterwards show), it is here to be neglected. The action of the sun, attracting the moon from the earth, is nearly as the moon's distance from the earth; and therefore (by what we have ... he centripetal force of the moon as 2 to ... 40. And if we neglect so inconsiderable ... which the moon is retained in its orb

Jupiter and Saturn. And since all attraction (by Law III) is mutual, Jupiter will therefore gravitate towards all his own satellites, Saturn towards his, the earth towards the moon, and the sun towards all the primary planets.

COR. II. The force of gravity which tends to any one planet is inversely as the square of the distance of places from that planet's centre.

COR. III. All the planets do gravitate towards one another, by Cor. I and II. And hence it is that *Jupiter* and *Saturn*, when near their conjunction, by their mutual attractions sensibly disturb each other's motions. So the sun disturbs the motions of the moon; and both sun and moon disturb our sea, as we shall hereafter explain.

SCHOLIUM

The force which retains the celestial bodies in their orbits has been hitherto

planets, by Rules 1, 2, and 4.

PROPOSITION 6. THEOREM 6

That all bodies gravitate towards every planet; and that the weights of bodies towards any one planet, at equal distances from the centre of the planet, are proportional to the quantities of matter which they severally contain.

It has been, now for a long time, observed by others, that all sorts of heavy bodies (allowance being made for the inequality of retardation which they suffer from a small power of resistance in the air) descend to the earth *from equal heights* in equal times; and that equality of times we may distinguish to a great accuracy, by the help of pendulums. I tried experiments with gold, silver, lead, glass, sand, common salt, wood, water, and wheat. I provided two wooden boxes, round and equal: I filled the one with wood, and suspended an equal

air. And, placing the one by the other, I observed them to play together forwards and backwards, for a long time, with equal vibrations. And therefore the quantity of matter in the gold (by Cor. I and VI, Prop. 24, Book II) was to the quantity of matter in the wood as the action of the motive force (or *vis motrix*) upon all the gold to the action of the same upon all the wood; that is, as the weight of the one to the weight of the other: and the like happened in the other bodies. By these experiments, in bodies of the same weight, I could manifestly have discovered a difference of matter less than the thousandth part of the whole, had any such been. But, without all doubt, the nature of gravity towards the planets is the same as towards the earth. For, should we imagine our terrestrial bodies taken to the orbit of the moon, and there, together with the moon, deprived of all motion, to be let go, so as to fall together towards the earth, it is certain, from what we have demonstrated before, that, in equal

and 2) the force by which the moon is retained in its orbit is that very same force which we commonly call gravity for were gravity another force different from that then bodies descending to the earth with the joint impulse of both forces would fall with a double velocity and in the space of one second of time would describe $30\frac{1}{6}$ Paris feet altogether against experience

This calculus is founded on the hypothesis of the earth s standing still for if both earth and moon move about the sun and at the same time about their common centre of gravity the distance of the centres of the moon and earth from one another will be $60\frac{1}{2}$ semidiameters of the earth as may be found by a computation from Prop 60 Book I

SCHOLIUM

The demonstration of this Proposition may be more diffusely explained after the following manner Suppose several moons to revolve about the earth as in the system of Jupiter or Saturn the periodic times of these moons (by the argument of induction) would observe the same law which Kepler found to obtain among the planets and therefore their centripetal forces would be inversely as the squares of the distances from the centre of the earth by Prop 1 of this book Now if the lowest of these were very small and were so near the earth as almost to touch the tops of the highest mountains the centripetal force thereof retaining it in its orbit would be nearly equal to the weights of any terrestrial bodies that should be found upon the tops of those mountains as may be known by the foregoing computation Therefore if the same little moon should be deserted by its centrifugal force that carries it through its orbit and be disabled from going onward therein it would descend to the earth and that with the same velocity with which heavy bodies actually fall upon the tops of those very mountains because of the equality of the forces that oblige them both to descend And if the force by which that lowest moon would descend were different from gravity and if that moon were to gravitate towards the earth as we find terrestrial bodies do upon the tops of mountains it would then descend with twice the velocity as being impelled by both these forces conspiring together Therefore since both these forces that is the gravity *of heavy bodies and the centripetal forces of the moons* are directed to the centre of the earth and are similar and equal between them elves they will (by Rules 1 and 2) have one and the same cause And therefore the force which retains the moon in its orbit is that very force which we commonly call gravity becau e otherwise this little moon at the top of a mountain must either be without gravity or fall twice as swiftly as heavy bodies are wont to do

PROPOSITION 5 THEOREM 5

That the circumjovial planets gravitate towards Jupiter the circumsaturnal towards Saturn the circumsolar towards the sun and by the forces of their gravity are drawn off from rectilinear motions and retained in curvilinear orbits

For the revolutions of the circumjovial planets about Jupiter of the circum saturnal about Saturn and of Mercury and Venu and the other circum olar

ort of causes especially since it has been demonstrated that the force upon which those revolutions depend tend to the centres of Jupiter of Saturn and

example we should imagine the terrestrial bodies with us to be raised to the orbit of the moon to be there compared with its body if the weights of such ... the weights of the external parts of the moon as the quantities

against what we have shown above

COR I Hence the weights of bodies do not depend upon their forms and textures for if the weights could be altered with the form they would be greater or less according to the variety of forms in equal matter altogether against experience

COR. II Universally all bodies about the earth gravitate towards the earth ... weights of all at equal distances from the earth s centre are as the

tity of matter then because (according to Aristotle ...) there is no difference between that and other bodies but in *mere* form of matter by a successive change from form to form it might be changed at last into a

might be changed contrary to what was proved in the preceding Corollary

COR. III All spaces are not equally full for if all spaces were equally full

quantity of matter in a given space can by any rarefaction be diminished what should hinder a diminution to infinity?

COR. IV If all the solid particles of all bodies are of the same density and cannot be rarefied without pores then a void space or vacuum must be granted By bodies of the same density I mean those whose inertias are in the proportion of their bulks

COR V The power of gravity is of a different nature from the power of magnetism for the magnetic attraction is not as the matter attracted. Some bodies are attracted more by the magnet others less most bodies not at all The power of magnetism in one and the same body may be increased and

rude observations.

PROPOSITION 7 THEOREM 7

That there is a power of gravity pertaining to all bodies proportional to the several quantities of matter which they contain

That all the planets gravitate one towards another we have proved before as well as that the force of gravity towards every one of them considered

times they would describe equal spaces with the moon and of consequence are to the moon in quantity of matter as their weights to its weight Moreover since the satellites of Jupiter perform their revolutions in times which observe the $^{3}/_{2}$th power of the proportion of their distances from Jupiter's centre their accelerative gravities towards Jupiter will be inversely as the squares of their distances from Jupiter's centre that is equal at equal distances And therefore these satellites if supposed to fall *towards Jupiter* from equal heights would describe equal spaces in equal times in like manner as heavy bodies do on our earth And by the same argument if the circumsolar planets were supposed to be let fall at equal distances from the sun they would in their descent towards the sun describe equal spaces in equal times But forces which equally accelerate unequal bodies must be as those bodies that is to say the weights of the planets *towards the sun* must be as their quantities of matter Further that the weights of Jupiter and of his satellites towards the sun are proportional to the several quantities of their matter appears from the exceedingly regular motions of the satellites (by Cor III Prop 65 Book I) For if some of those bodies were more strongly attracted to the sun in proportion to their quantity of matter than others the motions of the satellites would be disturbed by that inequality of attraction (by Cor II Prop 65 Book I) If at equal distances from the sun any satellite in proportion to the quantity of its matter did gravitate towards the sun with a force greater than Jupiter in proportion to his according to any given proportion suppose of d to e then the distance between the centres of the sun and of the satellite's orbit would be always greater than the distance between the centres of the sun and of Jupiter nearly as the square root of that proportion as by some computations I have found And if the satellite did gravitate towards the sun with a force less in the proportion of e to d the distance of the centre of the satellite's orbit from the sun would be less than the distance of the centre of Jupiter from the sun as the square root of the same proportion Therefore if at equal distances from the sun the accelerative gravity of any satellite towards the sun were greater or less than the accelerative gravity of Jupiter towards the sun but by one $\frac{1}{100}$ part of the whole gravity the distance of the centre of the satellite's orbit from the sun would be greater of less than the distance of Jupiter from the sun by one $\frac{1}{2000}$ part of the whole distance that is by a fifth part of the distance of the utmost satellite from the centre of Jupiter an eccentricity of the orbit which would be very sensible But the orbits of the satellites are concentric to Jupiter and therefore the accelerative gravities of Jupiter and of all its satellites towards the sun are equal among themselves And by the same argument the weights of Saturn and of his satellites towards the sun at equal distances from the sun are as their several quantities of matter and the weights of the moon and of the earth towards the sun are either none or accurately proportional to the masses of matter which they contain But some weight they have by Cor I and III Prop 5

But further the weights of all the parts of every planet towards any other planet are one to another as the matter in the several parts for if some parts did gravitate more others less than for the quantity of their matter then the whole planet according to the sort of parts with which it most abounds would gravitate more or less than in proportion to the quantity of matter in the whole Nor is it of any moment whether these parts are external or internal for if for

the terrestrial bodies with us to be raised to the

ntate towards the earth
arth s centre an as the

Descartes and others)
ut in *mere* form of matter
by a successive chan e from form to form it might be changed at last into a body of the same condition with those which gravitate most in proportion to their quantity of matter and on the other hand the heaviest bodies acquiring the first form of that body mi ht by degrees quite lose their gravity And therefore the weights would depend upon the forms of bodies and with those forms might be changed contrary to what was proved in the preceding Corollary
ually full

quantity of matter in a given space can by any rarefaction be diminished

granted By bodies of the same density I mean those whose inertias are in the proportion of their bulks

COR V The power of gravity is of a different nature from the power of magnetism for the magnetic attraction is not as the matter attracted Some bodies are attracted more by the magnet others less most bodies not at all The power of magnetism in one and the same body may be increased and dimini hed and is sometimes far stron er for the quantity of matter than the power of gravity and in receding from the magnet decreases not as the square but almost as the cube of the distance as nearly as I could judge from some rude observations

PROPOSITION 7 THEOREM 7

That there is a power of gra ity pertain ng to all bod es proportional to the several quant ties of matter which they cont n

That all the plan ts gravitate one towards another we have proved before as well as that the force of gravity towards every one of them considered

apart is inversely as the square of the distance of places from the centre of the planet And thence (by Prop 69 Book I and its Corollaries) it follows that the gravity tending towards all the planets is proportional to the matter which they contain

Moreover since all the parts of any planet A gravitate towards any other planet B and the gravity of every part is to the gravity of the whole as the matter of the part to the matter of the whole and (by Law III) to every action corresponds an equal reaction therefore the planet B will on the other hand gravitate towards all the parts of the planet A and its gravity towards any one part will be to the gravity towards the whole as the matter of the part to the matter of the whole Q E D

COR I Therefore the force of gravity towards any whole planet arises from and is compounded of the forces of gravity towards all its parts Magnetic and electric attractions afford us examples of this for all attraction towards the whole arises from the attractions towards the several parts The thing may be easily understood in gravity if we consider a greater planet as formed of a number of lesser planets meeting together in one globe for *hence it would appear* that the force of the whole must arise from the forces of the component parts If it is objected that according to this law all bodies with us must gravitate one towards another whereas no such gravitation anywhere appears I answer that since the gravitation towards these bodies is to the gravitation towards the whole earth as these bodies are to the whole earth the gravitation towards them must be far less than to fall under the observation of our senses

COR II The force of gravity towards the several equal particles of any body is inversely as the square of the distance of places from the particles as appears from Cor III Prop 74 Book I

PROPOSITION 8 THEOREM 8

In two spheres gravitating each towards the other if the matter in places on all sides round about and equidistant from the centres is similar the weight of either sphere towards the other will be inversely as the square of the distance between their centres

After I had found that the force of gravity towards a whole planet did arise from and was compounded of the forces of gravity towards all its parts and towards every one part was in the inverse proportion of the squares of the distances from the part I was yet in doubt whether that proportion inversely as the square of the distance did accurately hold [illegible] where the distances of the particles are unequal and their situation dissimilar But by the help of Props 75 and 76 Book I and their Corollaries I was at last satisfied of the truth of the Proposition as it now lies before us

COR I Hence we may find and compare together the weights of bodies towards different planets for the weights of bodies revolving in circles about planets are (by Cor II Prop 4 Book I) directly as the diameters of the circles and inversely as the squares of their periodic times and their weights at the surfaces of the planets or at any other distances from their centres are (by this Proposition) greater or less inversely as the square of the distances Thus

from the periodic times of Venus revolving about the sun in 224ᵈ 16¾ʰ of the utmost circumjovial satellite revolving about Jupiter in 16ᵈ 16⁸⁄₁₅ʰ of the Huygenian satellite about Saturn in 15ᵈ 22⅔ʰ and of the moon about the ... 27ᵈ 7ʰ 43ᵐ compared with the mean distance of Venus from the sun ... of the outmost circumjovial ... Huygenian satellite from the ... earth 10 33 by computa... ...om the centres ... Jupiter ... Saturn and the ... respec... ...reased or dimin... ...ghts are ... weights of equal bodies towards ... distances 10 000 997 791 and 100 from their centres that is at ... surfaces will be as 10 000 943 529 and 435 respectively How much the weights of bodies are at the surface of the moon will be shown hereafter

COR. II Hence likewise we discover the quantity of matter in the several planets for their quantities of matter are as the forces of gravity at equal distances from their centres that is in the sun Jupiter Saturn and the earth as 1 1/1067 1/3021 and 1/169282 respectively If the parallax of the sun be taken greater or less than 10 30 the *quantity of matter* in the earth must be augmented or diminished as the cube of that proportion

COR. III Hence also we find the densities of the planets for (by Prop 72 Book I) the weights of equal and similar bodies towards similar spheres are at the surfaces of those spheres as the diameters of the spheres and therefore the densities of dissimilar spheres are as those weights applied to the diameters of the spheres But the true diameters of the sun Jupiter Saturn and the earth were one to another as 10 000 997 791 and 109 and the weights towards the same as 10 000 943 529 and 435 respectively and therefore their densities are as 100 94½ 67 and 400 The density of the earth which comes out by this computation does not depend upon the parallax of the sun but is determined by the parallax of the moon and therefore is here truly defined. The sun therefore is a little denser than Jupiter and Jupiter than Saturn and the earth four times denser than the sun for the sun by its great heat is kept in a sort of rarefied state The moon is denser than the earth as shall appear afterwards.

...

greater density as they are nearer to the sun. So Jupiter is more dense than ...

...

will make water boil. Nor are we to doubt that the matter of Mercury is adapted to its heat and is therefore more dense than the matter of our earth since in a denser matter the operations of Nature require a stronger heat.

PROPOSITION 9 THEOREM 9

That the force of gravity considered downwards from the surface of the planets decreases nearly in the proportion of the distances from the centre of the planets

If the matter of the planet were of an uniform density this Proposition would be accurately true (by Prop 73 Book I) The error therefore can be no greater than what may arise from the inequality of the den ity

PROPOSITION 10 THEOREM 10

That the motions of the planets in the heavens may subsist an exceedingly long time

In the Scholium of Prop 40 Book II I have shown that a globe of water frozen into ice and moving freely in our air in the time that it would describe the length of its semidiameter would lo e by the resistance of the air $\frac{1}{4586}$ part of its motion and the same proportion holds nearly in all globes however great and moved with whatever velocity But that our globe of earth is of greater density than it would be if the whole consisted of water only I thus make out If the whole consisted of water only whatever was of le s density than water because of

And upon this account

water was less dense t

ing water falling back

condition of our earth

if it was not for its greater density would emerge from the seas and according to its degree of levity would be raised more or less above their surface the water of the seas flowing backwards to the opposite side By the same argument the spots of the sun which float upon the lucid matter thereof are lighter than that matter and however the planets have been formed while they were yet in fluid masses all the heavier matter subsided to the centre Since therefore the common matter of our earth on the surface thereof is about twice as heavy as water and a little lower in mines is found about three or four or even five times heavier it is probable that the quantity of the whole matter of the earth may be five or six times greater than if it consisted all of water especially since I have before shown that the earth is about four times more dense than Jupiter If therefore Jupiter is a little more dense than water in the space of thirty days in which that planet describes the length of 459 of its semidiameters it would in a medium of the same density with our air lo e almost a tenth part of its motion But since the resistance of mediums decreases in proportion to their weight or density o that water which is $13\frac{3}{5}$ times lighter than quicksilver resists less in that proportion and air which is 860 times lighter than water resists less in the same proportion therefore in the heavens where the weight of the medium in which the planets move is im men ely diminished the resistance will almost vanish

It is shown in the Scholium of Prop 22 Book II that at the height of 200

exhausted by the air pump from under the receiver heavy bodies fall within the receiver with perfect freedom and without the least sensible resistance
... let fall together will descend with equal
... they
And
the
inue

HYPOTHESIS I

THAT THE CENTRE OF THE SYSTEM OF THE WORLD IS IMMOVABLE

This is acknowledged by all while some contend that the earth others that the sun is fixed in that centre Let us see what may from hence follow

PROPOSITION 11 THEOREM 11

That the common centre of gravity of the earth the sun and all the planets is immovable

For (by Cor iv of the Laws) that centre either is at rest or moves uniformly forwards in a right line but if that centre moved the centre of the world would move also against the Hypothesis

PROPOSITION 12 THEOREM 12

That the sun is agitated by a continual motion but never recedes far from the common centre of gravity of all the planets

... the sun is to the
from the
r greater
the common centre of gravity of Jupiter and the ... a point a
... By the same argument since the quantity
...
... ally at rest the sun
ly be moved
...
and all the
...

it would recede yet less if the body of the sun were denser and greater and therefore less apt to be moved

PROPOSITION 13 THEOREM 13

The planets move in ellipses which have their common focus in the centre of the sun and by radii drawn to that centre they describe areas proportional to the times of description

We have discoursed above on these motions from the Phenomena Now that we know the principles on which they depen l f — l

th e —

t

t

s of description by Props 1 and 11 and Cor I Prop 13 Book I But the action of l —

est

It is true that the action of Jupiter upon Saturn is not to be neglected for the force of gravity towards Jupiter is to the force of gravity towards the sun (at equal distances Cor II Prop 8) as 1 to 1067 and therefore in the conjunction of Jupiter and Saturn because the distance of Saturn from Jupiter is to the distance of Saturn from the sun almost as 4 to 9 the gravity of Saturn towards Jupiter will be to the gravity of Saturn towards the sun as 81 to 16 1067 or as 1 to about 211 And hence arises a perturbation of the orbit of Saturn in every conjunction of this planet with Jupiter so sensible that astronomers are puzzled with it As the planet is differently situated in the c conjunctions its eccentricity is somet m

om so great a force may be almost avoided (except in the mean motion) by placing the lower focus of its orbit in the common centre of gravity of Jupiter and the sun (according to Prop 67 Book I) and therefore that error when it is greatest scarcely exceed two minutes and the greatest error in the mean motion scarcely exceeds two minutes yearly But in the conjunction of Jupiter and Sat n h of gravity of the sun towards Sat towards the sun are almost as 1 re the difference of the forces of gravity of the

Saturn is to the force of gravity

as 1 to 2409 But the greatest pov

is proportional to this difference and therefore the perturbation of the orbit of Jupiter is much less than that of Saturn s The perturbations of the other orbits are yet far less except that the orbit of the earth is sensibly disturbed by the moon The common centre of gravity of the earth and moon moves in an ellipse about the sun in the focus thereof and by a radius drawn to the sun describes areas proportional to the times of description But the earth in the meantime by a menstrual motion is revolved about this common centre

PROPOSITION 14 THEOREM 14

The aphelions and nodes of the orbits of the planets are fixed

The aphelions are immovable by Prop 11 Book I and so are the planes of the orbit by Prop 1 of the same book. And if the planes are fixed the nodes must be so too It is true that some inequalities may arise from the mutual actions of the planets and comets in their revolutions but these will be so small that they may be here passed by

COR. I The fixed stars are immovable seeing they keep the same position to the aphelions and nodes of the planets.

COR. II And since these stars are liable to no sensible parallax from the ... of their immense ... ion that the ... y their con trary attractions destroy their mutual action ...

SCHOLIUM

Since the planets near the sun (viz. Mercury Venus the earth and Mars) are so small that they can act with but little force upon one another therefore their aphelions and nodes must be fixed except so far as they are disturbed by the actions of Jupiter and Saturn and other higher bodies And hence we may find by the theory of gravity that their aphelions move forwards a little in ... of their several distances ... pace of a hundred years ... tars, the aphelions of the ... years be carried forwards ... motions are so inconsider able that we have neglected them in this Proposition

PROPOSITION 15 PROBLEM 1

To find the principal diameters of the orbits of the planets.

They are to be taken as the ⅔th power of the periodic times by Prop 15 Book I and then to be severally augmented in the proportion of the sum of the masses of matter in the sun and each planet to the first of two mean proportionals between that sum and the quantity of matter in the sun by Prop 60 Book I.

PROPOSITION 16 PROBLEM 2

To find the eccentricities and aphelions of the planets

This Problem is resolved by Prop 18 Book I

PROPOSITION 17 THEOREM 15

That the diurnal motions of the planets are uniform and that the libration of the moon arises from its diurnal motion

The Proposition is proved from the first Law of Motion and Cor XXII Prop 66 Book I Jupiter with respect to the fixed stars revolves in 9^h 56^m Mars in 24^h 39^m Venus in about 23 the earth in 23^h 56^m the sun in $25\frac{1}{2}^d$ and the moon in 27^d ... 43^m These things appear by the Phenomena The spots in the sun s body return to the same situation on the sun s disk, with

respect to the earth in 27½ days and therefore with respect to the fixed stars the sun revolves in about 25½ days But because the lunar day arising from its uniform revolution about its axis is menstrual *that is equal to the time of its periodic revolution in its orbit* hence the same face of the moon will be always nearly turned to the upper focus of its orbit but as the situation of that focus requires will deviate a little to one side and to the other from the earth in the lower focus and this is the libration in longitude for the libration in latitude arises from the moon s latitude and the inclination of its axis to the plane of the ecliptic This theory of the libration of the moon Mr N Mercator in his *Astronomy* published at the beginning of the year 1676 explained more fully out of the letters I sent him The utmost satellite of Saturn seems to revolve about its axis with a motion like this of the moon respecting Saturn continually with the same face for in its revolution round Saturn as often as it comes to

most satellite of Jupiter seems to revolve about its axis with a like motion because in that part of its body which is turned from Jupiter it has a spot which always appears as if it were in Jupiter s own body whenever the satellite passes between Jupiter and our eye

PROPOSITION 18 THEOREM 16

That the axes of the planets are less than the diameters drawn perpendicular to the axes

The equal gravitation of the parts on all sides would give a spherical figure to the planets if it was not for their diurnal revolution in a circle By that circular motion it comes to pass that the parts receding from the axis endeavor to ascend about the equator and therefore if the matter is in a fluid state by its ascent towards the equator it will enlarge the diameters there and by its descent towards the poles it will shorten the axis So the diameter of Jupiter (by the concurring observations of astronomers) is found shorter between pole and pole than from east to west And by the same argument if our earth was not higher about the equator than at the poles the seas would subside about the poles and rising towards the equator would lay all things there under water

PROPOSITION 19 PROBLEM 3

To find the proportion of the axis of a planet to the diameters perpendicular thereto

Our countryman Mr Norwood measuring a distance of 905 751 feet of London measure between London and York in 1635 and observing the difference of latitudes to be 2 28 determined the measure of one degree to be 367 196 feet of London measure that is 57 300 Paris toises M Picard measuring an arc of one degree and 22 55 of the meridian between Amiens and

toises and the difference of the latitudes of Collioure and Dunkirk was 8 degrees and 31 11⅚ Hence an arc of one degree appears to be 57 061 Paris toises And from these measures we conclude that the circumference of the

earth i 123 249 600 and its semidiameter 19 615 800 Paris feet upon the supposition that the earth is of a spherical figure

body falling in a second of time describes 15
1 $3_{\overline{9}}$ lines. The weight of the
air Let us suppose the weight
ht then that heavy body fall
lines in one second of time
A body in every sidereal day of uniformly revolving in a circle
800 feet from the centre in one second of time describes
0 0.736561 feet or 54064
cend in the latitude of Paris
or arising from the diurnal

force with
50 10
hat is as
by their
falling by
ll describe
al force of

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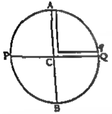

would be to the force of gravity in the same place Q to ards a phere described about the centre C with the radius PC or QC as 126 to 125 And by the same argument the force of gravity in the place A to ard the pheroid generated by the rotation of the ellipse APBQ about the axis AB is to the force of gravity in the same place A towards the sphere described about the centre C with the radius AC as 125 to 126 But the force of gra ty in the pl ce A towards the earth is a mean proportional bet een the forces of gravity towards the pheroid and this phere because the sphere by

porti n is con erted into the said pheroid and the force of gravity in A in either case is diminished nearly in the same proportion Therefore the force of gravity in A to vards the sphere described about the centre C with the radius

respect to the earth in 27½ days, and therefore with respect to the fixed stars the sun revolves in about 25 1/ days But because the lunar day arising from its uniform revolution about its axis is menstrual *that is equal to the time of its periodic revolution in its orbit* hence the same face of the moon will be always nearly turned to the upper focus of its orbit but as the situation of that focus requires will deviate a little to one side and to the other from the earth in the lower focus and this is the libration in longitude for the libration in latitude arises from the moon s latitude and the inclination of its axis to the plane of the ecliptic This theory of the libration of the moon Mr N Mercator in his *Astronomy* published at the beginning of the year 1676 explained more fully out of the letters I sent him The utmost satellite of Saturn seems to revolve about its axis with a motion like this of the moon respecting Saturn continually with the same face for in its revolution round Saturn as often as it comes to the eastern part of its orbit it is scarcely visible and generally quite disappears this is probably occasioned by some spots in that part of its body which is then turned towards the earth as M Cassini has observed So also the utmost satellite of Jupiter seems to revolve about its axis with a like motion because in that part of its body which is turned from Jupiter it has a spot which always appears as if it were in Jupiter s own body whenever the satellite passes between Jupiter and our eye

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grees and

toises And from these measures we conclude that the circumference of the

$\frac{4}{..}$ ... to 1 or as 1 to 9½ nearly Therefore the diameter of Jupiter from east to west is to it diameter from pole to pole nearly as 10⅓ to 9⅓ Therefore ... lesser diameter lying between the poles

to each other as ... formly dense But ...

The time			Great diameter	Lesser diameter	The diameters to each other
	days	hours	Part	Parts	
January	28	[illegible]	13 40	1 [illegible]	As 1 to 11
March	[illegible]		13 1	1 [illegible]	13[illegible] to 1[illegible]
March	9	7	13 1	1 08	1[illegible] to 11[illegible]
April	9	9	1 3[illegible]	11 48	14[illegible] to 13½

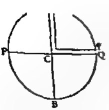
So that the theory agrees with the phenomena for the planets are more heated by the sun s rays towards their equators and therefore are a little more ... poles

...sioned by the diurnal ...r there than it does at ...e) will appear by the experiments of pendulums related under the following Proposition

Proposition 20 Problem 4

To find and compare together the weights of bodies in the different regions of our earth

Because the weights of the unequal legs of the canal of water ACQca are equal and the weights of the parts proportional to the whole legs and alike



P C Q B

equal bodies alike situated in the legs of the canal Their weights are inversely as the legs, that is inversely as the distances of the bodies from the centre of the ...

weights in all other places round the whole surface of the earth are inversely as the distances of the places from the centre and therefore on the hypothesis of the earth s being a spheroid, are given in proportion

From this arises the theorem that the increase of weight in passing from the equator to the poles is nearly as the versed sine of double the latitude or

AC to the force of gravity in A towards the earth as 126 is to $125\frac{1}{2}$ And the force of gravity in the place Q towards the sphere described about the centre C with the radius QC to the force of gravity in the place A towards the sphere described about the centre C with the radius AC in the proportion of the diameters (by Prop 72 Book I) that is as 100 to 101 If therefore we compound those three proportions 126 to 125 126 to $125\frac{1}{2}$ and 100 to 101 into one the force of gravity in the place Q towards the earth will be to the force of gravity in the place A towards the earth as 126 126 100 to 125 $125\frac{1}{2}$ 101 or as 501 to 500

Now since (by Cor III Prop 91 Book I) the force of gravity in either leg of the canal ACca or QCcq is as the distance of the places from the centre of the earth if those legs are conceived to be divided by transverse parallel and equidistant surfaces into parts proportional to the wholes the weights of any number of parts in the one leg ACca will be to the weights of the same number of parts in the other leg as their magnitudes and the accelerative forces of their gravity conjointly that is as 101 to 100 and 500 to 501 or as 505 to 501 And therefore if the centrifugal force of every part in the leg ACca arising from the diurnal motion was to the weight of the same part as 4 to 505 so that from the weight of every part conceived to be divided into 505 parts the centrifugal force might take off four of those parts the weights would remain equal in each leg and therefore the fluid would rest in an equilibrium But the centrifugal force of every part is to the weight of the same part as 1 to 289 that is the centrifugal force which should be $\frac{4}{505}$ parts of the weight is only $\frac{1}{289}$ part thereof And therefore I say by the rule of proportion that if the centrifugal force $\frac{4}{505}$ make the height of the water in the leg ACca to exceed the height of the water in the leg QCcq by $\frac{1}{100}$ part of its whole height the centrifugal force $\frac{1}{289}$ will make the excess of the height in the leg ACca only $\frac{1}{229}$ part of the height of the water in the other leg QCcq and therefore the diameter of the earth at the equator is to its diameter from pole to pole as 230 to 229 And since the mean semidiameter of the earth according to Picard s mensuration is 19 615 800 Paris feet or 3923 16 miles (reckoning 5000 feet to a mile) the earth will be higher at the equator than at the poles by 85 472 feet or $17\frac{1}{10}$ miles And its height at the equator will be about 19 658 600 feet and at the poles 19 573 000 feet

If the density and periodic time of the diurnal revolution remaining the same the planet was greater or less than the earth the proportion of the centrifugal force to that of gravity and therefore also of the diameter between the poles to the diameter at the equator would likewise remain the same But if the diurnal motion was accelerated or retarded in any proportion the centrifugal force would be augmented or diminished nearly in the same proportion squared and therefore the difference of the diameters will be increased or diminished in the same squared ratio very nearly And if the density of the planet was augmented or diminished in any proportion the force of gravity tending towards it would also be augmented or diminished in the same proportion and the difference of the diameters on the contrary would be diminished in proportion as the force of gravity is augmented and augmented in proportion as the force of gravity is diminished Therefore since the earth in respect of the fixed stars revolves in $23^h\ 56^m$ but Jupiter in $9^h\ 56^m$ and the squares of their periodic times are as 29 to 5 and their densities as 100 to $94\frac{1}{2}$ the difference of the diameters of Jupiter will be to its lesser diameter as

that pendulum with the length of the pendulum at Paris (which was 3 Paris feet and 8⅗ lines) he found it horter by 1¼ lines

Afterwards our friend Dr Halley about the year 1677 arriving at the island of St Helena found his pendulum clock to go slower there than at London without marking the difference But he shortened the rod of his clock by more than ⅛ of an inch or 1½ lines and to effect this because the length of the screw at the lower end of the rod was not sufficient he interposed a wooden ... t and the ball

Hayes found the length of a ... al Observatory of Paris to be ... he island of Goree they found ... t and 6⅝ lines differing from ... ine to the islands of Guad

... month of July 1697 at the Royal ...

seconds was shorter at Lisbon by 2½ lines ... reckoned those differences 1⅔ and ... s of the times 2ᵐ 13 ... gross that we cannot confide in them

centre than in mines near the surface unless perhaps the heats of the torrid zone have a little extended the length of the pendulums

For M P ... rd has observed that a rod of iron which in frosty weather in the winter season was one foot long when heated by fire was lengthened into one

which comes to the same thing as the square of the sine of the latitude and the arcs of the degrees of latitude in the meridian increase nearly in the same proportion And therefore since the latitude of Paris is 48 50 that of places under the equator 00 00, and that of places under the poles 90°, and the versed sines of double those arcs are 1 133 400 000 and 20 000 the radius being 10 000 and the force of gravity at the pole is to the force of gravity at the equator as 230 to 229 and the excess of the force of gravity at the pole to the force of gravity at the equator is as 1 to 229 the excess of the force of gravity in the latitude of Paris will be to the force of gravity at the equator as $\frac{11334}{20000}$ to 229 or as 5667 to 2 290 000 And therefore the whole forces of gravity in those places will be one to the other as 2 295 667 to 2 290 000 Therefore since the lengths of pendulums vibrating in equal times are as the forces of gravity and in the latitude of Paris the length of a pendulum vibrating seconds is 3 Paris feet and 8½ lines or rather because of the weight of the air 8⅚ lines the length of a pendulum vibrating in the same time under the equator will be shorter by 1 087 lines And by a like calculus the following table is made

Latitude of the place	*Length of the pendulum*		*Measure of one degree in the meridian*	*Latitude of the place*	*Length of the pendulum*		*Measure of one degree in the meridian*
degrees	*feet*	*lines*	*toises*	*degrees*	*feet*	*lines*	*toises*
0	3	7 468	56637	6	3	8 461	57022
5	3	7 482	56642	7	3	8 494	57035
10	3	7 526	56659	8	3	8 528	57048
15	3	7 596	56687	9	3	8 561	57061
20	3	7 692	56724	50	3	8 594	57074
25	3	7 812	56769	55	3	8 750	57137
30	3	7 948	56823	60	3	8 007	57196
35	3	8 099	56882	65	3	9 044	57250
40	3	8 261	56945	70	3	9 162	57295
1	3	8 294	56958	75	3	9 258	5733
2	3	8 327	56971	80	3	9 329	57360
3	3	8 361	56984	85	3	9 372	57377
4	3	8 394	56997	90	3	9 387	57382
45	3	8 428	57010				

By this table therefore it appears that the inequality of degrees is so small that the figure of the earth in geographical matters may be considered as spherical especially *if the earth be a little denser towards the plane of the equator than towards the poles*

Now several astronomers sent into remote countries to make astronomical observations have found that pendulum clocks do accordingly move slower near the equator than in our climates And first of all in the year 1672 M Richer took notice of it in the island of Cayenne for when in the month of August he was observing the transits of the fixed stars over the meridian he found his clock to go slower than it ought in respect of the mean motion of the sun at the rate of 2ᵐ 28 a day Therefore fitting up a simple pendulum to vibrate in seconds which were measured by an excellent clock he observed the length of that simple pendulum and this he did over and over every week for ten months together And upon his return to France comparing the length of

appears from Prop 65 Book 1 But then their motions will be in
the sun and they will suffer such in
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by a radius drawn to the earth de-
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PROPOSITION 23 PROBLEM 5

To derive the unequal motions of the satellites of Jupiter and Saturn from the motions of our moon

compounded of the squared

than in the latter but in the latter it was greater than the heat of the external parts of a human body for metals exposed to the summer sun acquire a very considerable degree of heat But the rod of a pendulum clock is never exposed to the heat of the summer sun nor ever acquires a heat equal to that of the external parts of a human body and therefore though the 3 foot rod of a pendulum clock will indeed be a little longer in the summer than in the winter season yet the difference will scarcely amount to $\frac{1}{4}$ line Therefore the total difference of the lengths of isochronal pendulums in different climates cannot be ascribed to the difference of heat nor indeed to the mistakes of the French astronomers For although there is not a perfect agreement between their observations yet the errors are so small that they may be neglected and in this they all agree that isochronal pendulums are shorter under the equator than at the Royal Observatory of Paris by a difference not less than $1\frac{1}{4}$ lines nor greater than $2\frac{2}{3}$ lines By the observations of M Richer in the island of Cayenne the difference was $1\frac{1}{4}$ lines That difference being corrected by those of M des Hayes becomes 1 1/ lines or $1\frac{3}{4}$ lines By the less accurate observations of others the same was made about 2 lines And this disagreement might arise partly from the errors of the observations partly from the dissimilitude of the internal parts of the earth and the height of mountains partly from the different temperatures of the air

I take an iron rod 3 feet long to be shorter by a sixth part of one line in winter time with us here in England than in the summer Because of the great heats under the equator subtract this quantity from the difference of $1\frac{1}{4}$ lines observed by M Richer and there will remain 1 1/1 lines which agrees very well with 1 /1000 lines obtained earlier by the theory M Richer repeated his observations made in the island of Cayenne every week for ten months together and compared the lengths of the pendulum which he had there noted in the iron rods with the lengths thereof which he observed in France This diligence and care seems to have been wanting to the other observers If this gentleman's observations are to be depended on the earth is higher under the equator than at the poles and that by an excess of about 17 miles as appeared above by the theory

PROPOSITION 21 THEOREM 17

That the equinoctial points go backwards and that the axis of the earth by a nutation in every annual revolution twice vibrates towards the ecliptic and as often returns to its former position

The Proposition appears from Cor xx Prop 66 Book I but that motion of nutation must be very small and indeed scarcely perceptible

PROPOSITION 22 THEOREM 18

That all the motions of the moon and all the inequalities of those motions follow from the principles which we have laid down

That the greater planets while they are carried about the sun may in the meantime carry other lesser planets revolving about them and that those lesser planets must move in ellipses which have their foci in the centres of the

led forces of both
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But the effects of the luminaries depend upon their distances from the earth for when they are less distant their effects are greater and when more distant their effects are less and that as the cube of their apparent diameter Therefore ... th winter time being then in its perigee has a greater ... the
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syzygies

The effect of either luminary doth likewise depend upon its declination or distance from the equator for if the luminary was placed at the pole it would ... attract all the parts of the waters without any intensification or ...

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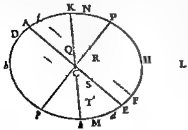
th sun Therefore the greatest tides occur in those syzygies and ... l those quadratures which happen about the time of both equinoxes and the greatest tide in the syzygies is always succeeded by the least tide in the quadratures as we find by experience But because the sun is less distant from the earth in winter than in summer it comes to pass that the greatest and least tides more frequently appear before than after the vernal equinox and more frequently after than before the autumnal

Moreover the effects of the luminaries depend upon the latitudes of places Let ApEP represent the earth covered with deep waters C its centre P p its poles AE the equator F any place without the equator Ff the parallel of the place Dd the correspondent parallel on the other side of the equator L the place of the moon three hours before H the place of the earth directly under it h the opposite place K k the places at 90 degrees distance CH Ch the greatest heights of the sea from the centre of the earth and CK Ck its least heights and if with the axes Hh Kk an ellipse is described and by the revolution of that ellipse about its



the forward motion of the
its nodes as the motion
the same Corollary) ar
found must be diminish to or of about 1 to 2 on account of a cause which I cannot here stop to explain The
of the nodes and of the
the nodes and apog
and apsides of the sa one revolution of the former equations to the motions of the nodes and apogee of our moon in the tim of revolution of the latter equations
is to the variation of our moon in t
their nodes respectively during the
(after parting from) are revolved (the sun by the same Corollary and therefore in the outmost satellite the variation does not exceed 5 12

PROPOSITION 24 THEOREM 19

That the flux and reflux of the sea arise from the actions of the sun and moon

By Cor XIX and XX Prop 66 Book I it appears that the waters of the sea ought twice to rise and twice to fall every day as well lunar as solar and that the greatest height of the waters in the open and deep seas ought to follow the approach of the luminaries to the merid

third
is by t
ular pl or seventh hour and even later The hours I reckon from the approach of each luminary to the merid
place as well under
underst and the 24th
motion employs to c
the day b fore The f oon in raising the sea is greatest in the approach of the luminary to the meridian of the place but the fo ce impressed upon the sea at that t ssion and is afterwards This makes the ea rise is new force becoming too weak to rai e it any more the ea ri es to its greatest height And this will come to pass per hap in one or two hours but more frequently near the shores in about three hours or even more where the sea is shallow

The two luminaries excite two motions which will not appear distinctly but between them will ari e one mixed motion compounded out of both In the conjunction or opposition of the luminaries their forces will be conjoined and bring on the greatest flood and ebb In the quadratures the sun will raise the waters which the moon depres es and depress the waters which the moon rai es and from the difference of their forces the smallest of all tides will follow And because (as experience tells us) the force of the moon is greater than that of the sun the greatest height of the waters will happen about the third lunar hour Out of the syzygies and quadratures the greatest tide which by the single force of the moon ought to fall out at the third lunar hour and by the

h th rd solar hour by the compounded forces of both

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But the effects of the luminaries depend upon their distances from the earth for when they are less di tant their effects are greater and when more di tant their effects are less and that as the cube of their apparent diameter Therefore

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tides more frequently appear before than after the vernal equinox and more frequently afte than before the autumnal

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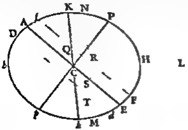

CI Ck its least heights and if with the axes Hh Kk an ellipse is described and by the revolution of that ellipse about its

longer axis Hh a spheroid $HPKhpk$ is formed this spheroid will nearly represent the figure of the sea and CF Cf CD Cd will represent the heights of the sea in the places Ff Dd But further n the
any point N
RT and the
those places therefore in the diurnal revolution of any place the greatest flood will be in F at the third hour after the appulse of the moon to the meridian abo
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the the moon
flood in F For the whe out in the hemisphere KHk o the other in the opposite hemisphere Khk which we may therefore call the
flood
ian
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of the sou in all places with in which the luminaries rise and set But the greatest tide will happen when the moon declines towards the vertex of the place about the third hour after the appulse of the moon to the meridian above the horizon and when the moon changes its declination *to the other side of the equator* that which was the greater tide will be changed into a lesser And the greatest difference of the floo

of the eve in summer exceed those of the morning at Plymouth by the height of one foot but at Bristol by the height of fifteen inches according to the observations of Colepress and Sturmy

But the motions which we have been describing suffer some alteration from that force of reciprocation which the waters being once moved retain a little while *by their inertia* Whence it come some time though the actions of retaining the impressed motion le ence of the alternate tides and makes those tides which immediately succeed after the syzygies greater and those which follow next after th

motions are retarded in their ough shallow channels so that the greatest tides of all in some straits and mouths of rivers are the fourth or even the fifth after the syzygies

Further it may happen that the tide may be propagated from the ocean through different channels towards the same port and may pass quicker through some channels than through others in which case the same tide divided into two or more succeeding one another may compound new motions of different kind Let us suppose two equal tides flowing towards the same port from different places one preceding the other by six hours and suppose

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floods the waters would rise to their least height Thus in the space of t

four hours the waters would come not twice as commonly but once only to their greatest and once only to their least height and their greatest height if

h l vated pole would happen at the sixth or

to the meridian and when the

uld be changed into an ebb An

n the observations of seamen in

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the passage of the moon

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they begin to flo and ebb not twice as in other port but once only every

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ately changed into an ebb and thenceforth the ebb happens a

the flood at the ri in of the moon till the moon again pa ing the equator changes its declination There are two inlets to this port and the neighboring channels on from the seas of China between the continent and the i land of Leu onia the other f om the Indian Sea bet een the continent and the island of Borneo But whether there be really two tides propagated throu h the said channels, one from the Indian Sea in the pace of twelve hours and one from the sea of China in the pace of six hours which therefore happening at the third and ninth lunar hours by being compounded together produce those mot ons or whether there be any other circumstances in the state of those seas I leave to be determined by observations on the neighboring hores

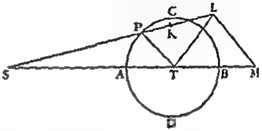
Thus I have explained the causes of the motions of the moon and of the sea. Now it is fit to subjoin something concerning the amount of those motions

PROPOSITION 25 PROBLEM 6

To find the forces with which the sun d sturbs the motions of the moon

Let S represent the sun T the earth P the moon CADB the moon s orbit In SP take SK equal to ST and let SL be to SK as the square of SK to SP draw LM parallel to PT and if ST o SK is upposed to represent the accelerated force of gravity of the earth towards the sun SL will represent the accelerative force of gravity of the moon towards the sun But that force is

compounded of the parts SM and LM of which the f... of SM whic... shown in P... moon are re... ... of gravity, the motion of the earth about that centre will be also disturbed by the like forces but we may consider the sums both of the forces and of the motions as in the moon and represent the sum of the forces by the lines TM and MI which are analogous to them both The force ML (in its mean amount) is to the centripetal force by which the moon may be retained in its orbit revolving about the earth at rest at the



distance PT as the square of the ratio of the periodic time of the moon about the earth to the periodic time of the earth about the sun (by Cor xvii Prop 66 Book i) that is as the square of $27^d\ 7^h\ 43^m$ to $365^d\ 6^h\ 9^m$ or as 1000 to 178725 or as 1 to $178\frac{9}{40}$ But in Prop 4 of this book we found that if both earth and moon were revolved about their common centre of gravity the mean distance of the one from the other would be nearly $60\frac{1}{2}$ ... diameters of the earth ... in its orbit about the ... the earth is to the f... distance of 60 semid... ... and this force is to the force of gravity with us very nearly as 1 to 60 60 Therefore the mean force ML is to the force of gravity on the surface of our earth as 1 $60\frac{1}{2}$ to 60 60 60 $178\frac{9}{40}$ or as 1 to 638092 6 hence by the proportion of the lines TM ML the force TM is also given and these are the forces with which the sun disturbs the motions of the moon Q E I

Proposition 26 Problem 7

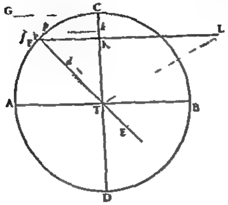
To find the hourly increment of the area which the moon by a radius drawn to the earth describes in a circular orbit

W... e moon describes by a radius dr... de cription excepting so far as ... is disturbed by the action of the sun and here we propose to investigate the inequality of the moment or hourly increment *of that area or motion so disturbed* To render the calculus more easy we shall suppose the orbit of the moon to be circular and neglect all inequalities but that only which is now under consideration and because of the immense distance of the sun we shall further suppo e that the lines SP and ST ... means the force LM ... the f... T...

... at direction accelerates or retards *the description of the area* in proportion as it accelerates or retards the moon That

that is as $\frac{3I\,h\ Ih}{TP}$ Let the time be represented by

CTP or even by the
and supposing the



AC to be divided into an infinite number of equal parts Pp

will be as $\frac{3I\,I\quad h}{TP}$ that is as EL and compounding the two

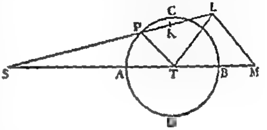
will vary as the sum of all the forces EL impressed upon the moon in the whole time CP and therefore also as the velocity generated by that sum that is as the acceleration of the description of the area CTP or as the increment of the moment *thereof* The force by which the moon may in its periodic time CADB of 27 7h 43m be retained revolving about the earth at rest at the distance TP would cause a body falling in the time CT to describe the length ½CT

the earth at rest as 100 is to 72 ... 72 of

gen rate a velocity equal to $\frac{11049}{1}$ parts of the velocity of the moon but in the time CPA will generate a greater velocity in the proportion of CA to CT or TP Let the greatest EL force in the octants be represented by the area FI hk or by the rectangle ½TP Pp which is equal thereto and the velocity

CP will be ...

compounded of the parts SM and LM of which the force LM, and that part of SM which is represented by TM disturb the motion of the moon as we have shown in Prop 66 Book I and its Corollaries Forasmuch as the earth and moon are revolved about their common centre of gravity the motion of the earth about that centre will be also disturbed by the like forces, but we may consider the sums both of the forces and of the motions as in the moon and represent the sum of the forces by the lines TM and ML which are analogous to them both The force ML (in its mean amount) is to the centripetal force by which the moon may be retained in its orbit revolving about the earth at rest at the



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and this force is to the force of us very nearly as 1 to 60 60 Therefore the mean force ML is to the force of gravity on the surface of our earth as 1 $60\frac{1}{2}$ to 60 60 60 $178\,{}^{29}/_{40}$ or as 1 to 638092 6 hence by the proportion of the lines TM ML the force TM is also given and these are the forces with which the sun disturb the motions of the moon Q E I

PROPOSITION 26 PROBLEM 7

To find the hourly increment of the area which the moon by a radius drawn to the earth describes in a circular orbit

We h e moon describes by a radius dra de cription excepting so far as t disturbed b the sun and here we propose to increment *of that area or* mo we shall suppose the orb to be circul

compos
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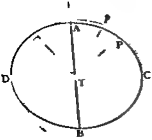
the square of the velocity I estimate the curvatures of lines compared one with another according to the evanescent ratio of the sines or tangents of their angles of contact to equal radii supposing those radii to be infinitely diminished But the attraction of the moon towards the earth in the syzygies is the ... earth ... bove the force of the sun 2PK (see

attracted towards the earth And these attraction ...

are nearly as $\frac{178725}{AT^2} - \frac{2000}{CT \times AT}$ and $\frac{178725}{CT^2} + \frac{1000}{AT \times CT}$ or as $178725 AT^2 \times CT^2 - 2000AT^2 \times CT$ and $178725 AT^2 \times CT^2 + 1000CT^2 \times AT$ For if the accelerative gravity of the moon towards the earth be represented by the number 178725 the mean force ML, which in the quadratures is PT or TK and draws the moon towards the earth, will be 1000 and the mean force TM in the syzygies will be 3000 from which if we subtract the mean force ML, there will remain 2000 the force by which the moon in the syzygies is drawn from the earth and which we above called 2PK. But the velocity of the moon in the ... veloc t in the quadratures C and D as CT is to ... uch the moon by a radius drawn to ... the moment of that area described in the quadratures conjointly that is as 11073CT is to 10973AT Take the ... d th former ratio directly and the curvature ... to the curvature thereof in the ... $\times -120406729 \times 2000AT^4 \times CT$ is ... $611,329 \times 1000CT^4 \times AT$ that is as ... $2191371AT \times CT + 12261CT^3$

Because the figure of the moon orbit is unknown let us in its stead assume the ellipse DBCA in the centre of which we suppose the earth to be situated and the greater axis DC to lie between the quadratures as the lesser AB between the syzygies But since the plane of this ellipse is revolved about the earth by an angular motion and the orbit ... whose curvature we now examine should be described in a plane void of such motion we are to consider the figure which the moon while it is revolved in that ellipse describes in this ...



in such manner that the angle PTp may be equal to the apparent motion of the sun from the time of the last quadrature in C or (which comes to the same thing) that the angle CTp may be to the angle CTP as the time of the synodic revolution of the moon to the time of the periodic revolution thereof or as $29^d\ 12^h\ 44^m$ to 27

or as the quadrantal arc CA is to the radius TP and therefore the latter velocity generated in the whole time will be $\frac{100}{11}$ parts of the velocity of the moon To this velocity of the moon which is proportional to the mean moment of the area (supposing this mean moment to be repre ented by the number 11 915) we add and subtract the half of the other velocity the sum 11 915+50 or 11 96ʋ will represent the greatest moment of the area in the syzygy \ and the difference 11 915−50 or 11 865 the least moment thereof in the quadratures Therefore the areas which n or ᶦ in the syzygies and ~ to the least moment

ᵕ ᴄ uapezium *k*KCG is to the tri

the same thing as the square of the sine PK is to the square of the radius TP (that is as P*d* to TP) the sum will represent the moment of the area when the moon is in any intermediate place P

But these things take place only in the hypothesis that the sun and the earth are at rest and that the synodical revolution of the moon is finished in 27^d 7^h 43^m But since the moon s synodical period is really 29^d 12^h 44^m the increments of the moments must be enlarged in the same proportion as the time is that is in the proportion of 1 080 853 to 1 000 000

the whole ~

become

rature of icon in the syzygy as 11 023−50 to 11 023+ʋʋ or as 10 973 to 11 073 and to the moment thereof when the moon is in any intermediate place P as 10 973 to 10 973+1 *d* that is supposing TP = 100

The area therefore which the moon by a radius drawn to the earth describes in the several little equal parts of time is nearly as the sum of the number 219 46 and the versed sine of the double distance of the moon from the nearest quadrature considered in a circle which hath unity for its radius Thus it is when the variation in the octants is in its mean quantity But if the variation there is greater or less that versed sine must be augmented or diminished in the same proportion

PROPOSITION 27 PROBLEM 8

From the hourly motion of the moon to find its distance from the earth

The area which the moon by a radi ᶦ ~

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COR I Hence the ~

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COR II Hence also the orbit of the moon may be more exactly defined from the phenomena than hitherto could be done

PROPOSITION 28 PROBLEM 9

To find the diameters of the ᶦ ᶦ ~ ᶦ

The curvature of

pendicular to the or ᵕ of attraction and inversely as

TA of the ellipse to its semidiameter TC or as 69 to 70 But the description of the area CTP as the moon advances from the quadrature to the syzygy ought to be in such manner accelerated that the moment of the area in the moon s syzygy may be to the moment thereof in its quadrature as 11 073 to 10 973 and
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And this is its magnitude in the mean distance of the sun from the earth neglecting the differences which may arise from the curvature of the great orbit and the stronger action of the sun upon the moon when horned and new
h n f m the earth the

en) and inversely as the cube of the ratio of the di tance of th u ut the earth And therefore in the apogee of the sun the greatest variation is 33 14 and in its perigee 3 11 if the eccentricity of the sun is to the transverse semidiameter of the great orbit as $16\frac{5}{16}$ to 1000

to the determination of astronomers from the phenomena.

PROPOSITION 30 PROBLEM 11

7^h 43^m *If therefore in this proportion we take the angle CTa to the* right angle CTA and make T*a* of equal length with TA we shall have *a* the lower and C the upper apse of this orbit C*pa* But by computation I find that the difference between the curvature of this orbit C*pa* at the vertex *a* and the curvature of a circle described about the centre T with the interval TA is to the difference between the curvature of the ellipse at the vertex A and the curvature of the same circle as the square of the ratio of the angle CTP to the angle CT*p* and that the curvature of the ellipse in A is to the curvature of that circle as the square of the ratio of TA is to TC and the curvature of that circle is to the curvature of a circle described about the centre T with the radius TC as TC is to TA but that the curvature of this *last arch* is to the curvature of the ellipse in C as the square of the ratio of TA is to TC and that the difference between the curvature of the ellipse in the vertex C and the curvature of this last circle is to the difference between the curvature of the figure T*pa* at the vertex C and the curvature of this same *last* circle as the square of the ratio of the angle CT*p* to the angle CTP All these relations *are easily derived from the sines of the angles of contact and of the differences* of those angles But by comparing those ratios we find the curvature of the figure C*pa* at *a* to be to its curvature at C as $AT^3 - \frac{16824}{100000}CT^2$ AT is to $CT^3 + \frac{16824}{100000}AT^2$ CT where the number $\frac{16824}{100000}$ represents the difference of the squares of the angles CTP and CT*p* divided by the square of the le er angle CTP or (which is all one) the difference of the squares of the times 27^d 7^h 43^m and 29^d 12^h 44^m divided by the square of the time 27^d 7^h 43^m

Since therefore *a* represents the syzygy of the moon and C its quadrature the ratio now found must be the same as the ratio of the curvature of the moon s orb in the syzygies to the curvature thereof in the quadratures which we found above Therefore in order to find the ratio of CT to AT let us multiply the extremes and the means of the resulting proportion and the terms which come out divided by AT CT yield the following equation $2062\ 79CT^4 - 2\ 151\ 969N\ \ CT^3 + 368\ 676N\ \ AT\ \ CT^2 + 36\ 342AT^2\ \ CT^2 - 362\ 047N\ \ AT^2\ \ CT + 2\ 191\ 371N\ \ AT^3 + 4051\ 4AT^4 = 0$ Now if for the half sum N of the terms AT and CT we put 1 and x for their half difference then CT will be $= 1 + x$ and AT $= 1 - x$ And substituting those values in the equation after resolving thereof we shall find $x = 0\ 00719$ and from thence the semidiameter CT $= 1\ 00719$ and the semidiameter AT $= 0\ 99281$ which numbers are nearly as $70\frac{1}{24}$ and $69\frac{1}{24}$ Therefore the moon s distance from the earth in the syzygies is to its distance in the quadratures (etting aside the consideration of eccentricity) as $69\frac{1}{24}$ to $70\frac{1}{24}$ or in round numbers as 69 to 70

PROPOSITION 29 PROBLEM 10

To find the variation of the moon

This inequality is due partly to the elliptic figure of the moon s orbit partly to the inequality of the moments of the area which the moon by a radius drawn to the earth describes If the moon P revolved in the ellipse DBCA about the earth quiescent in the centre of the ellipse and by the radius TP drawn to the earth described the area CTP proportional to the time *of de*

CT *of the ellipse* was *to the least* T

P would be to the tangent of the angle

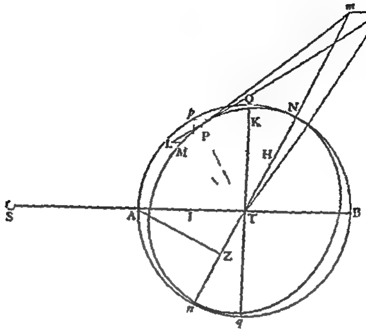
e quadrature C as the semidiameter

and at once impressed in the point P would have generated that whole line and caused the moon to move in the arc whose chord is LP that is to say ... from the plane MPmT into the plane LPlT ... will be equal ... use of the ... time given is also given ... is as the rectangle IT mP And if Tml is a right angle the angle mTl will be as $\frac{ml}{Tm}$ and therefore as $\frac{IT\ Pm}{Tm}$ that is (because Tm and mP TP and PH are proportional) ... $\frac{IT\ PH}{}$ and therefore because TP is given as IT PH But if the ... of the ... ocity ... of the nodes is as IT PH ... angles TPI PTN and STN

If these are right angles as happens when the nodes are in the quadratures and the moon in the syzygy the little line ml will be removed to an infinite distance and the angle mTl will become equal to the angle mPl But in this case the angle mPl is to the angle PTM which the moon in the same time by ... th as 1 to 59 5 5 For the angle ... angle of the moon s deflection ... ravity of the moon should have ... ould by itself have generated in ... to the angle of the moon s deflec ... the force of the sun 3IT should ... the moon is retained in its orbit would have generated in the same time ... and these forces (as we have above shown) are the one to the other as 1 to 59.575 Since therefore the mean hourly motion of the moon (in respect of the fixed stars) is 32m 56 27 12½iv the hourly motion of the node in this case will be 33 10 33iv 12v But in other cases the hourly motion will be to 33 10 33iv 12v as the product of the sines of the three angles TPI PTN and STN (or of the distances of the moon from the quadrature of the moon from the node and of the node from the sun) to the cube of the radius And as often as the sine of any angle is changed from positive to negative and from negative to positive so often must the regressive be changed into a progressive and the progressive into a regressive motion ... that the nodes are progressive as often as the moon

... the quadratures we let fall the perpendiculars ... and ...

of the distance of the moon from the quadrature PH the sine of the distance of the moon from the node and AZ the sine of the distance of the node from the sun and the velocity of the node will be as the product PK PH AZ But



(by Prop 25) is twofold one proportional to the line LM the oth
MT in the scheme of that P
drawn towa d
to the the former force LM
the dir of the moon
non
f out is disturbed is the same with
t or 3IT And this force (by Prop 25) is to the force by which the moon may in its periodic times be uniformly revolved in a circle about the earth at rest as 3IT to the radius of the circle multiplied by the number 178 725 or as IT to the radius thereof multiplied by 59 575 But in this calculus and all that follows I consider all the lines drawn from the moon to the sun as parallel to the line which joins the earth and the sun because what inclination there is almost as much diminishes all effects in some cases as it augments them in others and we are now inquiring after the mean motions of the nodes neglecting such niceties as are of no moment and would only serve to render the calculus more complicated

Now suppo e PM to represent an arc which the moon describes in the least moment of time and ML a little line the half of which the moon by the impulse of the said force 3IT would describe in the same time and joining PL MP let them be produced to m and l where they cut the plane of the ecliptic and upon Tm let fall the perpendicular PH Now since the right line ML is parallel to the plane of the ecliptic and therefore can never meet with the right line ml which lies in that plane and yet both

m will fall upon the line Nn
ich passes through the nodes N n of that orbit And because the force by which the half of the little line LM is generated if the whole had been together

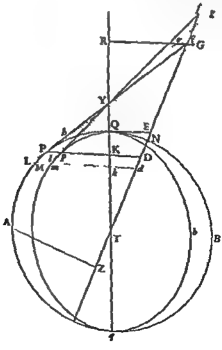
nodes is as AZ^2 and the area PDdM conjointly that is (because the area PDdM described in the syzygies is given) as AZ therefore the mean motion also will be as AZ and therefore when the nodes are without the quadratures this motion will be to 16 3o ʰ 16‴ 36ᵛ as AZ to AT^2 Q E D

PROPOSITION 31 PROBLEM 12

To find the hourly motion of the nodes of the moon in an elliptic orbit

Let *Qpmaq* represent an ellipse described with the greater axis *Qq* and the less axis *ab* QAqB a circle circumscribed T the earth in the common centre of both S the sun *p* the moon moving in this ellipse and *pm* an arc which it describes in the least moment of time N and *n* the nodes joined by the line ... perpendiculars upon the axis *Qq* produced both ways till they ... D and *d* And if the ... al to the ... as the area *pDdm* and AZ conjointly

For let PF touch the circle in P and produced meet TN in F and *pf* touch ... TN ... and both tangents concur ... by the ... with a transverse motion in the meantime while revolving ... describes ... which the moon revolving in the ellipse ... of the same force 3IT or 3PK ... plane of the ecliptic in G and *g* and FG and *fg* be joined of which FG produced may cut *pf* *pg* and TQ in

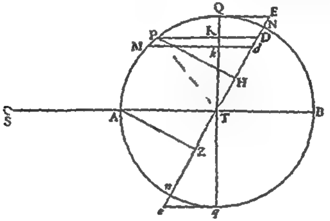


PT is to PK as PM to Kk and therefore because PT and PM are given Kk will be as PK Likewise AT is to PD as AZ is to PH and therefore PH is as the rectangle PD AZ and by compounding those proportions PK PH is as the solid content Kk PD AZ and PK PH AZ as Kk PD AZ^2 that is as the area PD*d*M and AZ^2 conjointly

COR II In any given po

...from the syzygies to the ... or as AZ^2 to AT^2 For if the moon by an uniform motion describes the semicircle QA*q* the sum of all the areas PD*d*M ... of the moon s passage from Q to M ...

... will fall without the ... area *nqe* terminating at the tangent *qe* of the circle which area because the nodes were before regressive but are now progressive,



must be ... area and being itself equal to the area QEN ... While therefore the moon describes a semicir... all the areas PD*d*M will be the area of that semicircle and while the moon describes a complete circle the sum of those areas will be the area of the whole circle But the area PD*d*M when the moon is in the syzygies is the rectangle of the arc PM into the ...

are
ple
circ

... therefore the mean motion by ... uniformly continued they would describe the same space with that which they do in fact describe by an unequal motion is *but* one-half of that motion which they are possessed of in the moon s syzygies Wherefore since their greatest hourly motion if the nodes are in the quadratures is 33 10 33 12 their mean hourly motion in this case will be 16 35 16 36 And seeing the hourly motion of the nodes is everywhere as AZ^2 and the area PD*d*M conjointly and therefore in the moon s syzygies the hourly motion of the

?? nd the area PDdM conjointly that is (because the area ... an motion ... he quadra

Q.E.D.

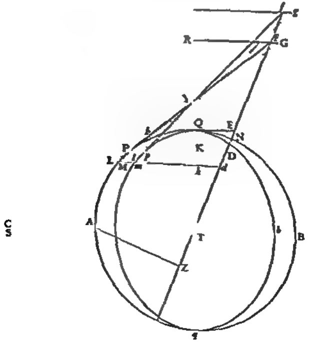
PROPOSITION 31 PROBLEM 1?

To find the hourly motion of the nodes of the moon in an elliptic orbit

Let Qpmaq represent an ellipse described with the greater axis Qq and the lesser ab QAqB a circle circumscribed T the earth in the common centre of both S the sun p the moon moving in this ellipse and pm an arc which it describes in the least moment of time N and n the nodes joined by the line Nn pK and mk perpendiculars upon the axis Qq produced both ways till they meet the circle in P and M and the line of the nodes in D and d And if the moon by a radius drawn to the earth describes an area proportional to the time of description the hourly motion of the node in the ellipse will be as the area pDdm and AZ² conjointly

For let PF touch the circle in P and produced meet TN in F and pf touch the ellipse in p and produced meet the same TN in f and both tangents concur in the axis TQ at Y and let ML represent the space which the moon by the impulse of the above-mentioned force 3IT or 3PK, would describe with a transverse motion in the meantime while revolving in the circle it describes ... space wh h th moon revolving in the ellipse

and FG and fg be joined of which FG produced may cut pl pg and TQ in



c e and R respectively and fg produced may cut TQ in Because the force 3IT or 3PK in the circle is to the force 3IT or $3p$K in the ellipse as PK to pK or as AT to aT the space ML generated by the former force will be to the space ml generated by the latter as PK to pK that is because of the similar figures PYKp and FYRc as FR to cR But (because of the similar triangles PLM PGF) ML is to FG as PI is to PC ...
LK PK GR) as pl is to pc
as lm is to ce and inversely ...
And therefore if fg was to ce as fy to cY that is as fr to cR (that is as fr to FR and FR to cR conjointly that is as fT to FT and FG to ce conjointly) because the ratio of FG to ... on both sides leaves the ratios fg to FG and fT t... to FT and therefore the angles which FG and ... T would be equal to each other But these angles (by ... he motions of the node ... the ellipse the arc pm ... therefore the motions of the nodes in the circle and in the ellipse would be equal to each other Thus I say it would be if fg was to ce as fY to cY that is if fg was equal to $\frac{ce\ fY}{cY}$ But because of the similar triangles fgp cep fg is to ce as fp to cp and therefore fg is equal to $\frac{ce\ fp}{cp}$ and therefore the angle which fg subtends in fact is to the former angle which FG subtends that is to say the motion of the nodes in the ellipse is to the motion of the same in the circle as this fg or $\frac{ce\ fp}{cp}$ to the former fg or $\frac{ce\ fY}{cY}$ that is as fp cY to fY cp or as fp to fY and cY to cp that is if ph parallel to TN meet FP in h as Fh to FY and FY to FP that is as Fh to FP or Dp to DP and therefore as the area Dpmd to the area DPMd And therefore seeing (by Cor I Prop 30) the latter area and AZ^2 conjointly are proportional to the hourly motion of the nodes in the circle the former area and AZ^2 conjointly will be proportional to the hourly motion of the nodes in the ellipse Q E D

COR Since therefore in any given position of the nodes the sum of all the areas pDdm in the time while the moon is carried from the quadrature to any place m is the area mpQEd terminated at the tangent of the ellipse QE and the sum of all those areas in one entire revolution is the area of the whole ellipse the mean motion of the nodes in the ellipse will be to the mean motion of the nodes in the circle as the ellipse to the circle that is as Ta to TA or 69 to 70 And therefore since (by Cor II Prop 30) the mean hourly motion of the nodes in the circle is to 16 35ʰ 16ⁱ 36 as AZ to AT^2 if we take the angle 16 21ᵗʰ 3 30 to the angle 16 35ᵗʰ 16 36 as 69 to 70 the mean hourly motion of the nodes in the ellipses will be to 16 21ᵗʰ 3ⁱ 30ᵛ as AZ^2 to AT^2 that is as the square of the sine of the distance of the node from the sun to the square of the radius

But the moon by a radius drawn to the earth describes the area in the syzygies with a greater velocity than it does that in the quadratures and upon ...

... But the moment of the area in the quadratures of the moon was to the moment thereof in the syzygies as 10 973 to 11 073 and therefore the mean moment in the octants is to the excess in the syzygies and to the

— — of those numbers is to their
moon in the several little
mean time in the octants
and to the defect of the
time in the syzygies arising from this cause ... 11 023 to 50 But, reckon
in from the quadratures to the syzygies I find that the excess of the moments
of the area in the several places above the least moment in the quadratures,
is nearly as the square of the sine of the moon s distance from the quadratures
... difference between the moment in any place and the mean

... ue of the
... ure of the
the nodes
are in the quadratures and we take two places one on one side one on the other equally distant from the octant and other two distant by the same interval, one from the syzygy the other from the quadrature and from the decrements of the motions in the two places between the syzygy and octant we subtract the increments of the motions in the two other places between the ...

nodes is the fourth part of the decrement in the syzygy The whole hourly

space was to this motion as 100 to 11 0 3 and therefore this decrement is 1 ... 43 11 The fourth part of which 4 ... 25‴ 48ᵛ subtracted from the mean hourly motion above found 16 21 3‴ 30ᵛ leaves 16 16 ... 3 ... 42ᵛ their correct mean hourly motion.

If the nodes are without the quadratures and two places are considered one on one side one on the other equally distant from the syzygies the sum of the motions of the nodes, when the moon is in those places will be to the sum of their motions when the moon is in the same places and the nodes in the quadratures as AZ to AT². And the decrements of the motions arising from

the causes but now - -
therefore
to AT^2
in any g
16^{th} 37 ... square of the sine of the distance of the nodes from the syzygies to the square of the radius

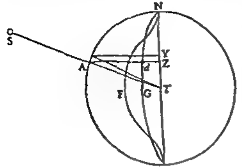
PROPOSITION 32 PROBLEM 13

To find the mean motion of the nodes of the moon

The yearly mean motion is the sum of all the mean hourly motions throughout the course of the year Suppose that the node is in N and that after every hour is elapsed it is drawn back again to its former place so that notwithstanding its proper motion it may constantly remain in the same situation with respect to the fixed stars while in the meantime the sun S by the motion of the earth is seen to leave the node and to proceed till it completes its apparent annual course by an uniform motion Let Aa represent a given least arc which the right line TS always drawn to the sun by its intersection with the circle NAn describes in the least given moment of time ...
motion (from what we have above shown) wi
and ZY are proportional) as the rectangle of
$AZYa$ and the sum of all the mean hourly motions from the beginning will be as the sum of all the areas $aYZA$ that is as the area NAZ But the greatest $AZYa$ is equal to the rectangle of the arc Aa into the radius of the circle and therefore the sum of all these rectangles in the whole circle will be to the like sum of all the greatest rectangles as the area of the whole circle to the rectangle of the whole circumference into the radius that is as 1 to 2 But the hourly motion corresponding to that greatest rectangle was 16 16^{th} 37^{iv} 12^{v} and this motion in the complete course of the sidereal year 365^d 6^h 9 amounts to 39 38 7 50 and therefore the half thereof 19 19 3 55 is the mean motion of the nodes corresponding to the whole circle And the motion of the nodes in the time while the sun is carried from N to A is to 19 49 3 55 as the area NAZ to the whole circle

Thus it would be if the node was after every hour drawn back again to its former place that so after a complete revolution the sun at the year's end would be found again in the same node which it had left when the year began But because of the motion of the node in the meantime the sun must needs meet the node sooner and now it remains that we compute the abbreviation of the time Since then the sun in the course of the year travels 360 degrees and the node in the same time by its greatest motion would be carried 39 38 7 50 or 39 6355 degrees and the mean motion of the node in any place N
to ... to AT^2 the motion of the sun
AT^2 to $39\ 6355AZ^2$ that is as
... suppose the circumference NAn of the whole circle to be divided into little equal parts such as Aa ...
the
the
with
to 9 ... is inversely as the velocity with which the little arc is described and this velocity is the sum of the velocities of both sun

and node If therefore the sector NTA represent the time in which the sun by itself without the motion of the node would describe the arc NA and the indefinitely small part ATa of the sector represent the little moment of the time in which it would describe



the least arc Aa and (letting fall aY perpendicular upon Nn) if in AZ we take dZ of such length that the rectangle of dZ into ZY may be to the least part ATa of the sector as AZ to 9 082 7646AT²+AZ that is to say that dZ may be to ½AZ as AT² to 9 082 7646 AT²+AZ the rectangle of dZ into ZY will represent the decrement of the time arising from the motion of the node while the arc Aa is described and ... always found the curve ... hile the whole arc ... AT above the area ... the node in a less ...

... of ... the node And if the curve

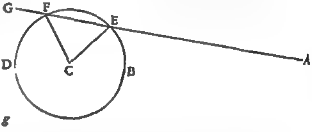
node Now the area of the semicircle is to the area of the figure NeFn found by the method of infinite series nearly as 793 to 60 But the motion corresponding or proportional to the whole circle was 19° 49 3 55 and therefore the motion corresponding to double the figure NeFn is 1 29 58 2 which taken from the former motion leaves 18° 19 5 53 the whole motion of the node with respect to the fixed stars in the interval between two of its conjunctions with the sun and this motion subtracted from the annual motion of the sun 360° leaves 341 40 54 7 the motion of the sun in the interval between the same conjunctions But as this motion is to the annual motion 360° so is the motion of the node but just now found 18 19 5 53 to its annual motion, which will therefore be 19° 18 1 23 and this is the mean motion of the nodes in the sidereal year By astronomical tables it is 19° 21 21 50

f the nodes is somewhat retarded and reduced to its just velocity

PROPOSITION 33 PROBLEM 14

To find the true motion of the nodes of the moon

In the time which is as the area NTA−NdZ (in the preceding Fig) that motion is as the area NAe and hence is given but because the calculus is too difficult it will be better to use the following construction of the Problem About the centre C with any radius CD describe the circle BEFD produce DC to A so as AB may be to AC as the mean motion to half the mean true motion when the nodes are in their quadratures (that is as 19 18 1′ 23 to 19 49 3 ′ 55 ″ and therefore BC is to AC as the difference of those motions 0 31′ 2″ 32 ′ to the latter motion 19 49 3 55 that is as 1 to 38³⁄₁₀) Then through the point D draw the indefinite line Gg touching the circle in D and if we take the angle BCE or BCF equal to the double distance of the sun from the place of the node as found by the mean motion and drawing AE or AF cutting the perpendicular DG in G we



take another angle which shall be to the whole motion of the node in the interval between its syzygies (that is to 9 11 3) as the tangent DG to the whole circumference of the circle BED and add this *last* angle (for which the angle DAG may be used) to the mean motion of the nodes while they are passing from the quadratures to the syzygies and subtract it from their mean motion while they are passing from the syzygies to the quadratures we shall have their true motion for the true motion so found will nearly agree with the true motion which comes out from assuming the times as the area NTA−NdZ and the motion of the node as the area NAe as anyone who chooses to examine and make the computations will find and this is the semimenstrual equation of the motion of the nodes But there is also a menstrual equation but which is by no means necessary for finding of the moon s latitude for since the variation of the inclination of the moon s orbit to the plane of the ecliptic is liable to a twofold inequality the one semimenstrual the other menstrual the menstrual inequality *of this variation* and the menstrual equation of the nodes so moderate and correct each other that in computing the latitude of the moon both may be neglected

COR From this and the preceding Proposition it appears that the nodes are quiescent in their syzygies but regressive in their quadratures by an hourly motion of 16 19ᵗʰ 26ⁱᵛ and that the equation of the motion of the nodes in the octants is 1 30 all of which exactly agree with the phenomena of the heavens

SCHOLIUM

Mr Machin Professor Gresham and Dr Henry Pemberton separately found out the motion of the nodes by a different method Mention has been

I have seen con
in both of them
insert

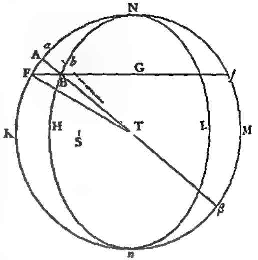
THE MOTION OF THE MOON S NODES

PROPOSITION 1

— k ode is defined by a geometric mean pro-
n and that mean motion with which the
i the node in the quadratures
ne of the moon s nodes at any given
time KTM a perpendicular thereto a right line revolving about the centre
— m l locity with which the sun and the node recede from

sector ATa the exponent of the um of these two velocities into two parts

of the rectangle KHM to HT² But the greatest mean velocity of the node was shown above to be in that very ratio to the velocity of the sun and therefore in



the quadratures the sector ATa is divided into parts proportional to the velocities And because the rectangle KHM is to HT^2 as FBf to BG^2 and the rectangle $AB\beta$ is equal to the rectangle FBf therefore the little area $ABba$ where it is greatest is to the remaining sector TBb as the rectangle $AB\beta$ is to BG^2 But the ratio of these little areas always was as the rectangle $AB\beta$ to BT and therefore the little area $ABba$ in the place A is less than its correspondent little area in the quadratures in the squared ratio of BG to BT that is in the squared ratio of the sine of the sun s distance from the node And therefore the sum of all the little areas $ABba$ namely the space ABN will be as the motion of the node in the time in which the sun hath been going over the arc NA since he left the node and the remaining space namely the elliptic sector NTB will be as the sun s mean motion in the same time And because the mean annual motion of the node is that motion which it performs in the time that the sun completes one period of its course the mean motion of the node from the sun will be to the mean motion of the sun itself as the area of the circle is to the

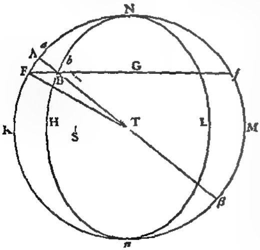
TK to the right line TH which is which comes to the same as the

I ... proportional TH to the right line TS

PROPOSITION 2

The mean motion of the moon s nodes being given to find their true motion

Let the angle A be the distance of the sun from the mean place of the node or the sun s mean motion from the node Then if we take the angle B whose tangent is to the tangent of the angle A as TH to TK that is as the square root of the ratio of the mean hourly motion of the sun to the mean hourly motion of the sun from the node when the node is in the quadrature that angle B will be the distance of the sun from the node s true place For join FT and by the demonstration of the last Proposition the angle FTN will be the distance of the sun from the mean place of the node and the angle ATN the distance from the true place and the tangents of these TH



quation of the moon s nodes and the the octants is to the radius as KH is

to TK+TH But the sine of this equation in any other place A is to the greatest sine as the sine of the sums of the angles FT\ +AT\ is to the radius that i nearly as the sine of double the distance of the sun from the mean place of the node (namely 2FT\) to the radius.

SCHOLIUM

me be 16 16
as
64P

1

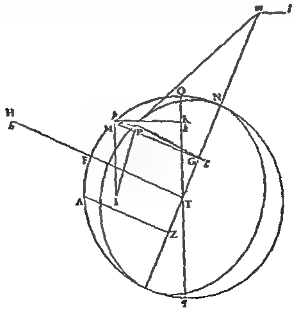
But if the mean motion of the moon ... MO
... observations made use of in the theory of the moon
real year will be 19° 20 31 58 and

1 20 54

PROPOSITION 34 PROBLEM 15

To find the hourly variation of the inclination of the moon s orbit to the plane of the ecliptic

Let A and a represent the syzygies Q and q the quadratures N and n the nodes P the place of the moon in its orbit p the orthographic projection of that place upon the plane of the ecliptic and mTl the momentary motion of the nodes as above If upon Tm we let fall the perpendicular PG and joining



pG
inch
P a
of th
of the inclinatio ... angle GTg as IG to PG and Pp to PG conjointly And therefore if for the moment of time we assume an hour since the angle GTg (by Prop 30) is to the angle 33 10 33 as IT PG AZ to AT³ the angle GPg (or the hourly variation of the inclination) will be to the angle 33 10 33 as IT AZ TG $\frac{Pp}{PG}$ to AT³

And thu
But if the
proportion ... as we have shown above and the variation of the inclination will be also diminished in the same proportion

COR I Upon Nn erect the perpendicular TI and let pM be the hourly motion of the moon in the plane of the ecliptic upon QT let fall the perpendiculars pK Mk and produce them till they meet TI in H and h then IT will be to AT as Kk to Mp and TG to Hp as TZ to AT and therefore IT TG will be equal to $\frac{Kk\ Hp\ TZ}{Mp}$ that is equal to the area HpMh multiplied into the ratio $\frac{TZ}{Mp}$ and therefore the hourly variation of the inclination will be to 33′ 10 33 as the area HpMh multiplied into AZ $\frac{TZ}{Mp}\frac{Pp}{PG}$ is to AT³

COR II And therefore if the earth and nodes were after every hour drawn back from their new and instantly restored to th
situat
varia
aggre ... in the time of one revolution of the point p (with due regard in summing to their proper signs + −) multiplied into AZ TZ $\frac{Pp}{PG}$ to Mp AT³ that is as the whole circle QAqa multiplied into AZ TZ $\frac{Pp}{PG}$ to Mp AT³ that is as the circumference QAqa multiplied into AZ TZ $\frac{Pp}{PG}$ to 2Mp AT²

COR III And therefore in a given position of the nodes the mean hourly variation from which if uniformly continued through the whole month that menstrual variation might be generated is to 33 10 33 as AZ TZ $\frac{Pp}{PG}$ is to 2AT³ or as Pp $\frac{AZ\ TZ}{\frac{1}{2}AT}$ is to PG 4AT that is (because Pp is to PG as the sine of the aforesaid inclination to the radius and $\frac{AZ\ TZ}{\frac{1}{2}AT}$ to 4AT as the sine of double the angle ATn to four times the radius) as the sine of the same inclination multiplied into the sine of double the distance of the nodes from the sun to four times the square of the radius

COR IV Seeing the hourly variation of the inclination when the nodes are in the quadratures is (by this Prop) to the angle 33 10 33‴ as IT AZ TG

$\frac{Pp}{PG}$ is to AT^2 that is as $\frac{IT \cdot TG}{\frac{1}{2}AI} \cdot \frac{Pp}{PG}$ to $2AT$ that is as the sine of double the distance of the moon from the quadratures multiplied into $\frac{Pp}{PG}$ is to twice the

ons during the time that the moon in
the quadrature to the syzygy (that i
he sum of as many angles 33 10 33′
or 5878 as the sum of all the sines of double the distance of the moon from the quadratures multiplied into $\frac{Pp}{PG}$ is to the sum of as many diameters that is as the diameter multiplied into $\frac{Pp}{PG}$ is to the circumference that is if the inclin ation be 5 1 as 7 $\frac{4}{100}$ is to 22 or as 278 to 10 000 And therefore the whole variation composed out of the um of all the hourly variations in the aforesaid time is 163 or 2 43

PROPOSITION 35 PROBLEM 16

liptic
least
circle

For GE^2 is equal to

$GH^2 + HE^2 = BHD + HE^2 = HBD + HE^2 - BH^2 = HBD + BE^2 - 2BH \cdot BE = BE^2 + 2EC \cdot BH = 2EC \cdot AB + 2EC \cdot BH = 2EC \cdot AH$

wherefore ince 2EC is given GE will be as AH Now let AEg represent

tangle GH Gg or GH GE that is as $\frac{}{GE}$ GE or $\frac{}{GE}$ AH that is as AH and

AH is equal to th sine and therefore remains always equal thereto Q E D

double the distance of the nodes from the sun then (by Cor III of the last Prop) the hourly variation of the inclination in the
10 33 as the rectangle of AH
...
it sine 8
whole v
hourly v
...
to the syzygies is to one
h 2079/10 to 1 Therefore compounding all these proportions we shall have the whole variation BD to 33 10 ′ 33‴ as 224 7 2079⁷⁄₁₀ is to 110 000 that is as 29 645 to 1000 and from thence that variation BD will come out 16 23½

And this is the greatest variation of the inclination abstracting from the situation of the moon in its orbit for if the nodes are in the sy clination suffers no change from the
nodes are in the quadratures the
syzygies than when it is in the qu by a difference of 2 43 as we showed (Cor IV of the preceding Prop) and the whole mean variation BD diminished by 1 21½ the half of this excess becomes 15 when the moon is in the quadratures and increased by the same become
moon is in the syzygies If the
...
20 when the nodes
be 4 59 35 when the nodes are in the quadratures and the moon in the syzygies The truth of all this is confirmed by observations

Now if the inclination of the orbit should be required when the moon is in the syzygies and the nodes anywhere between them and the quadratures let AB be to AD as the sine of 4 59 35 is to the sine of 5 17 20 and take the angle AEG equal to double the distance of the nodes from the quadratures and AH will be the sine of the inclination desired To this inclination of the orbit the inclination of the same is equal when the moon
the nodes In other situation of th
the
...

SCHOLIUM

By these comp
by the theor
...

contracted. The moon moves slower in the dilated and faster in the contracted orbit; and the annual equation, by which this inequality is regulated, vanishes in the apogee and perigee of the sun. In the mean distance of the sun from the earth it rises to about 11′ 50″; in other distances of the sun it is proportional to the equation of the sun's centre, and is added to the mean motion of the moon [illegible] from its aphelion to its perihelion, and subtracted [illegible]

[illegible] tricity of the earth's orbit to be [illegible] this equation will be augmented in the same proportion. Suppose the eccentricity 16⅞, and the greatest equation will be 11′ [illegible] 1″.

Farther I found that the apogee and nodes of the moon move faster in the perihelion of the earth, where the force of the sun's action is greater, than in the aphelion thereof, and that inversely as the cube of the ratio of the earth's distance from the sun; and hence arise annual equations of those motions proportional to the equation of the sun's centre. Now the motion of the sun varies inversely as the square of the earth's distance from the sun; and the greatest equation of the centre which this inequality generates is 1° [illegible] 20″, corresponding to the above-mentioned eccentricity of the sun, 16⅞. But if the motion of the sun had been inversely as the cube of the distance, this inequality would have generated the greatest equation 2° 54′ 30″; and therefore the greatest equations which the inequalities of the motions of the moon's apogee and nodes do generate are to 2° 54′ 30″ as the mean diurnal motion of the moon's apogee and the mean diurnal motion of its nodes are to the mean [illegible] of the [illegible] motion of

[illegible]

[illegible] hen the earth is in the opposite semicircle.

By the theory of gravity I likewise found that the action of the sun upon the moon is somewhat greater when the transverse diameter of the moon's orbit passes through the sun than when the same is perpendicular upon the line which joins the earth and the sun; and therefore the moon's orbit is somewhat

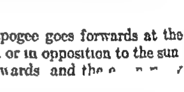
[illegible]

the semiannual, when greatest in the octants of the apogees, arises to about [illegible] 45″, so far as I could determine from the phenomena: and this is its quantity in the mean distance of the sun from the earth. But it is increased and diminished inversely as the cube of the sun's distance, and therefore is nearly [illegible] when that distance is greatest, and [illegible] 56″ when least. But when the moon's apogee is without the octants, it becomes less, and is to its greatest amount as the sine of double the distance of the moon's apogee from the nearest syzygy or quadrature is to the radius.

By the same theory of gravity the ... of the sun upon the moon is somewh... odes passes through the sun than ... joins the sun and the earth and ... equation of the moon s mean motion which I shall call the second semiannual and this is greatest when the nodes are in the octants of the sun and vanishes when they are in the syzygies or quadra... other positions of the nodes is ... of either node from the nearest ... mean motion of the moon if the ... e node which is nearest to him and is subtracted if forward and in the octants where it is of the greatest magnitude it arises to 47 in the mean distance of the sun from the earth as I find from the theory of gravity In other dist... greatest in the ...

... e same theory of gravity the moon s apogee goes forwards at the greatest rate when it is either in conjunction with or in opposition to the sun but in its quadratures with the sun it goes backwards and the ... comes in the former case ... Cor VII VIII and IX Prop ... laries we have named are ... the semiannual equation o... and this semiannual equation in its greatest quantity comes to about 12 18 as nearly as I could determine from the phenomena Our countryman Horrox was the first who advanced the theory of the moon s moving in an ellipse about the earth pla... focus Dr Halley impro... an epicycle wh... motion in this ... of th...

... may 12 18 to the radius TC and t... with the radius CB will be the e... in which the centre of the moon s orbit is placed and revolved according to the order of the letters BDA Set off the angle BCD equal to twice the annual argument or twice the distance of the sun s true place from the place of the moon s apogee once corrected and CTD will be the semiannual equation of the moon s apogee and TD the eccentri... to the pla... of the ...



... place of the moon in its orbit together with its distance from the earth may be determined by the methods commonly known

In the perihelion of the earth where the force of the sun is greatest the centre of the moon s orbit moves faster about the centre C than in the aphelion and that inversely as the cube of the sun s distance from the earth But be-

cause the equation of the un s centre i included in the annual argument the centre of the moon s orbit moves fa ter in its epicycle BDA inversely as the m f h sun s di tance from the earth Therefore that it may move yet m of h orbit a ted that

perigee forwards or which comes to the sun u u to the complement of the sun s true anomaly to 360° and let DF be to DC as twice the eccentricity of the great orbit to the un s mean distance from the th d h su n mean diurnal motion from the moon s apo ee to the sun s 200

as it ought to be

The calculus of this motion is difficult but may be rendered ea ier by the

po nt F to the moon and when greatest amounts to 2 25 But the angle which the line DF contains with the line drawn from the point F to the moon is found ther by subtracting the angle EDF from the mean anomaly of the moon or by adding the di tance of the moon from the sun to the distance of the moon s apogee from the apogee of the sun and as the radius is to the ine of the angle thus found so is 2 25 to the second equation of the centre to be added if the fore-mentioned sum be less than a semi-circle to be subtracted if greater And from the moon s place in its orbit thus corrected its longitude may be found in the syzygies of the luminaries

But the theory of the moon ought to be examined and proved from the phenomena first in the syzygies then in the quadratures and last of all in the

atory of Greenwich to the last day of December ...
the mean motion of the sun ♑ 20 43 40 and of its apogee ♋ 7° 44′ 30 the mean motion of the moon ♒ 15 21 00 of its apogee ♓ 8 20 00 and of its ascending node ♌ 27 24 20 and the difference of meridians between the Observatory at Greenwich and the Royal Observatory at Paris $0^h\ 9^m\ 20^s$ but the mean motion of the moon and of its apogee are not yet obtained with sufficient accuracy

PROPOSITION 36 PROBLEM 17

To find the force of the sun to move the sea

The ... n s force ML or PT to disturb the motions of the moon was (by Prop ... as 1 to 638092 6 ... ble that quantity ... liminished in pro- ... in the proportion of 60½ to 1 and therefore the former for ... face is to the force of gravity as 1 to 38 604 600 and by this force the sea is depressed in such places as are 90 degrees distant from the sun But by the other force which is twice as great the sea is raised not only in the places directly under the sun but in those also which are directly opposed to it and the sum of these forces is to the force of gravity as 1 to 12 868 200 And because the same force excites the same motion whether it depresses the waters in those places which are 90 degrees distant from the sun or ... them in the places which are directly under and directly opposed to the ... the sun to disturb the sea and ... mployed ... the ... n in the places directly under and directly oppo ... the sun ... d from the sun ... lace where the sun is at the same time both ... from the earth In other positions of the sun its force to raise the sea is directly as the versed sine of double its altitude above the horizon of the place and inversely as the cube of the distance from the earth

COR Since the centrifugal force of the parts of the earth arising from the ... 1 is to 289 raises the ... the poles by 85 472 ... we have now shown ...

1 to 44 ...

PROPOSITION 37 PROBLEM 18

To find the force of the moon to move the sea

The force of the moon to move the sea is to be deduced from its ratio to the force of the sun and this ratio is to be determined from the ratio of the motions

9 to 5 ... 's mean thereof 8 feet ... will be

Suppose the greatest difference of those heights to be ... to L−S as 20½ to 11½ or as 41 to 23 a proportion that agrees well enough ... we are rather to de- ... ve procure some- ... to 5 ... atest tides do not

after the syzygies or rather (as Sturmy observes) are the third after the day of

proceeds from the motion of the moon than in the syzygies and quadratures themselves in the proportion of the radius to the cosine of double this distance or of an angle of 37 degrees that is in the ratio of 10 000 000 to 7 986,355 and therefore in the preceding analogy in place of S we must put 0 7986355S

But further the force of the moon in the quadratures must be diminished on account of its declination from the equator for the moon in those quadratures or rather in 18½ degrees past the quadratures declines from the equator by about 23° 13 and the force of either luminary to move the sea is diminished as it declines from the equator nearly as the square of the cosine of the declination and therefore the force of the moon in those quadratures is only 0 8570327L hence we have L+0 7986355S to 0 8570327L−0 7986355S as 9 to

Further yet the diameters of the orbit in which the moon should move setting aside the consideration of eccentricity are one to the other as 69 to 0 and therefore the moon s distance from the earth in the syzygies is to its distance in the quadratures other things being equal as 69 to 70 and its distance when $18\frac{1}{2}$ degrees advanced beyond the syzygies where the greatest tide was excited and when $18\frac{1}{2}$ degrees passed by the quadratures where the least tide was produced are to its mean distance as 69 098747 and 69 897345 to 69[illegible] But the force of the moon to move the sea varies inversel[illegible]
distance and the [illegible]
are to its force
we have 1 0175[illegible]
5 and S to L a [illegible] ...ce the force of the sun is to the force of gravity as 1 to 12 868 200 the moon s force will be to the force of gravity as 1 to 2 871 400

COR I Since the waters attracted by the sun s force rise to the height of 1 foot and $11\frac{1}{30}$ inches the moon s force will raise the same to the height of 8 feet and $7\frac{5}{}$ inches and the joint forces of both will raise the same to the height of $10\frac{1}{2}$ feet and when the moon is in its perigee to the height of $12\frac{1}{}$ feet and more especially when the wind sets the same way as the tide And a force of that amount is abundantly sufficient to produce all the motions of the sea and agrees well with the ratio of those motions for in such seas as lie free and open from east to west as in the P[illegible]

[illegible] ...said to be greater than in the Atlantic and Æthiopic seas for to have a full tide raised an extent of sea from east to west is required of no less than 90 degrees In the Ethiopic sea the waters ri e to a less height within the tropics than in the temperate zones becau e of the narrowness of the sea between Africa and the southern parts of America In the middle of the open sea the waters cannot rise without falling together and at the same time upon both the eastern and western shores when notwithstanding in our narrow seas they ought to fall on those shores by alternate turns upon this account there is commonly but a small flood and ebb in such islands as lie far distant from the continent On the contrary in some ports where to fill and empty the bays alternately the waters are with great violence forced in and out through shallow channels the flood [illegible]

[illegible] ...places the sea is hurri[illegible]
with such violence [illegible]
leave them dry for [illegible]
stopped till it has r[illegible] ...sed the waters to 30 40 or 50 feet and above And a like account is to be given of long and shallow channels or straits such as the Magellanic straits and those channel[illegible]

[illegible] ...ly rise and fall without that precipitation of influx and efflux the ratio of the tides agrees with the forces of the sun and moon

COR. II Since the moon s force to move the sea is to the force of gravity as
force is inappreciable in tatical or hydro-

COR. III Because the force of the
of the sun as 4 4815 to 1 and those forces (by Cor XIV Prop 66 Book I) are as the densities of the bodies of the sun and moon and the cubes of their apparent diameters conjointly the density of the moon will be to the density
d rectl as 4 4815 to 1 and inversely as the cube of the moon s
pparent
0
nd
to
4000 or as 11 to 9 Therefore the body of the re
earthly than the earth itself

COR IV And since the true diameter of the moon (from the observations of astronomers) is to the true diameter of the earth as 100 to 365 the mass of matter in the moon will be to the mass of matter in the earth as 1 to 39 88

COR V And the accelerative gravity on the surface of the moon will be about three times less than the accelerative gravity on the surface of the earth

COR. VI And the distance of the moon s centre from the centre of the earth will be to the distance of the moon s centre from the common centre of gravity of the earth and moon as 40 88 to 39 88

COR VII And the mean distance of the centre of the moon from the centre

mon centre of gravity of the earth and moon as 40 788 to 39 88 which latter

and the moon falling by this force in one minute of time would describe 14 8538067 feet And at the 60th part of the distance of the moon from the

compose one mean semidiameter of the earth a heavy body would describe in falling 15 111 5 or 15 feet 1 inch and $4\frac{1}{11}$ lines in the same time This will be the descent of bodies in the latitude of 45 degrees And by the foregoing table to be found under Prop 20 the descent in the latitude of Paris will be a little greater by an excess of about $\frac{2}{3}$ parts of a line Therefore by this computation heavy bodies in the latitude of Paris falling in a vacuum will describe

15 Paris feet 1 inch $4\frac{5}{33}$ lines very nearly in one second of time And if the gravity be diminished by
arising in that latitude fr
there will describe in one
this velocity heavy bodies do really fall in the latitude of Paris as we have shown above in Props 4 and 19

COR VIII The mean distance of the centres of the earth and moon in the syzygies of the moon is equal to 60 of the greatest semidiameters of the earth subtracting only about one 30th part of a semidiameter and in the moon s quadratures the mean distance of the same centres is $60\frac{5}{6}$ such semidiameters of the earth for these two distances are to the mean distance of the moon in the octants as 69 and 70 to $69\frac{1}{2}$ by Prop 28

COR IX The mean distance of the centres of the earth and moon in the syzygies of the moon is 60 mean semidiameters of the earth and a 10th part of one semidiameter and in the moon s quadratures the mean distance of the same centres is 61 mean semidiameters of the earth subtracting one 30th part of one semidiameter

COR X In the moon s syzygies its mean horizontal parallax in the latitudes of 0 30 38 45 52 60 90 degrees is 57 20 57 16′ 57 14″ 57′ 12 57 10 57 8 57 4 respectively

In these computations I do not consider the magnetic attraction of the earth whose quantity is very small and unknown if this quantity should ever be found out and the measures of degrees upon the meridian the lengths of isochronous pendulums in different parallels the laws of the motions of the ea and the moon s parallax with the apparent diameters of the sun and moon should be more exactly determined from phenomena we should then be enabled to bring this calculation to a greater accuracy

PROPOSITION 38 PROBLEM 19

To find the figure of the moon s body

If the moon s body were fluid like our sea the force of the earth to raise that fluid in the nearest and remotest parts would be to the force of the moon by which our sea is raised in the places under and opposite to the moon as the accelerative gravity of the moon towards the earth is to the accelerative gravity of the earth towards the moon and the diameter of the moon is to the diameter of the earth conjointly that is as 39 788 to 1 and 100 to 365 conjointly or as 1081 to 100 Therefore since our sea by the force of the moon is raised to $8\frac{3}{5}$ feet the lunar fluid would be raised by the force of the earth to 93 feet and upon this account the figure of the moon would be a spheroid whose greatest diameter produced would pass through the centre of the earth and exceed the diameters perpendicular thereto by 186 feet Such a figure therefore the moon possesses and must have had from the beginning Q E I

COR Hence it is that the same face of the moon always is turned toward the earth nor can the body of the moon possibly rest in any other position but would return always by a libratory motion to this situation but those librations however must be exceedingly slow because of the weakness of the forces which excite them so that the face of the moon which should be always directed to the earth may for the reason assigned in Prop 17 be turned towards

the other focus of the moon s orbit without being immediately drawn back and turned a ain tov ards the earth

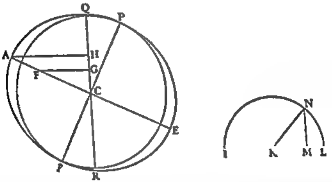
LEMMA 1

— C th pol s

l
i

and the plane QR

For let there be described from the centre K with the diameter IL the



to the sums of the squares of the sine KM and both sums together will be equal to the sums of the quares of as many semidiameters KN and therefore the sum of the squares of all the ines KM will be but half so great as the sum of the squares of as many emidiameters KN

r

A Then the force by which the particle F recede from the plane QR will (by supposition) be as that perpend cular FG and this force multiplied by the distance CG will represent the pov er of the particle F to turn the earth round its centre And therefore the pov er of a particle in the place F will be to the po ver of a particle in the place A as FG GC is to AH HC that is as FC^2 to AC^2 and therefore the whole power of all the particles F in their proper places

15 Paris feet 1 inch $4^{25}/_{33}$ lines very nearly in one second of time And if the gravity be diminished by taking away a quantity equal to the centrifugal force arising in that latitude from
there will describe in one secor
this velocity heavy bodies dc
shown above in Props 4 and 19

COR VIII The mean distance of the centres of the earth and moon in the syzygies of the moon is equal to 60 of the greatest semidiameters of the earth subtracting only about one 30th part of a semidiameter and in the moon s quadratures the mean distance of the same centres is $60^5/_6$ such semidiameters of the earth for these two distances are to the mean distance of the moon in the octants as 69 and 70 to $69\frac{1}{2}$ by Prop 28

COR IX The mean distance of the centres of the earth and moon in the syzygies of the moon is 60 mean semidiameters of the earth and a 10th part of one semidiameter and in the moon s quadratures the mean distance of the same centres is 61 mean semidiameters of the earth subtracting one 30th part of one semidiameter

COR X In the moon s syzygies its mean horizontal parallax in the latitudes of 0 30 38 45 52 60 90 degrees is 57 20 57 16 ', 57' 14 57 12 57 10 57 8 57 4 respectively

In these computations I do not consider the magnetic attraction of the earth whose quantity is very small and unknown if this quantity should ever be found out and the measures of degrees upon the meridian the lengths of isochronous pendulums in different parallels the laws of the motions of the sea and the moon s parallax with the apparent diameters of the sun and moon should be more exactly determined from phenomena we should then be enabled to bring this calculation to a greater accuracy

PROPOSITION 38 PROBLEM 19

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If the moon s body were fluid like our sea the force of the earth to raise that fluid in the nearest and remotest parts would be to the force of the moon by which our sea is raised in the places under and opposite to the moon as the accelerative gravity of the moon towards the earth is to the accelerative gravity of the earth towards the moon and the diameter of the moon is to the diameter of the earth conjointly that is as 39 788 to 1 and 100 to 365 conjointly or as 1081 to 100 Therefore since our sea by the force of the moon is raised to $8^3/_5$ feet the lunar fluid would be raised by the force of the earth to 93 feet and upon this account the figure of the moon would be a spheroid whose greatest diameter produced would pass through the centre of the earth and exceed the diameters perpendicular thereto by 186 feet Such a figure therefore the moon possesses and must have had from the beginning Q E I

COR Hence it is that the same face of the moon always is turned toward the earth nor can the body of the moon possibly rest in any other position but would return always by a libratory motion to this situation but the librations however must be exceedingly slow because of the weakness of the forces which excite them so that the face of the moon which should be always directed to the earth may for the reason assigned in Prop 17 be turned towards

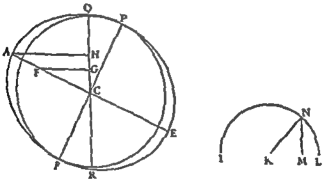
the other focus of the moon s orbit without being immediately drawn back and turned again towards the earth

LEMMA I

ent C the poles

ance from
ll the par
form

For let there be described from the centre K with the diameter IL



F will be to the power of the like number of particles in the place A as the sum of all the FC^2 is to the sum of all the AC^2 that is (by what we have demonstrated before) as 1 to 2 Q E D

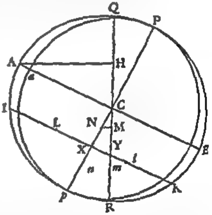
And because the action of those part cle ...

... body of the earth round an axis which lies as well in the plane QR as in that of the equator

LEMMA 2

The ... powe ... the sa ... of the like number of particles uniformly disposed round the whole circumference of the equator AE in the fashion of a ring to turn the whole earth about with the like circular motion as is 2 to 5

For let IK be any lesser circle parallel to the equator AE and let Ll be any two eq ... if upon ... we let ... lm the total forces by which these particles recede from the plane QR will be proportional to the perpendiculars LM lm Let the right line Ll be drawn parallel to the plane Pape and bisect the same in X and through the point X draw Nn parallel to the plane QR and meeting the perpendiculars LM lm in N and n and upon the plane QR let fall the perpendicular XY And the contrary forces of the particles L and l to wheel about the earth contrariwise are as LM MC and lm mC that is as LN MC+NM MC and ln mC−nm mC or LN MC+NM MC and LN mC−NM mC and LN Mm−NM (MC+mC) the difference of the two is the force of both taken together to turn the earth round The positive part of this difference LN Mm or 2LN NX is to 2AH HC the force of two



particles of the same size situated in A as LX^2 to AC^2 and the negative part NM (MC+mC) or 2XY CY is to 2AH HC the force of the same two particles situated in A as CX^2 to AC^2 And therefore the difference of the parts that is the force of the two particles L and l taken together to wheel the earth about is to the force of two particles equal to the former and situated in the pl ... earth round as LX^2-CX^2 is to AC^2 ... IK is suppo ed to be divided into an ... l the LX^2 will be to the like number of ... ne number of AC as IX^2 is to $2AC^2$ and the same number of CX^2 to as many AC^2 as $2CX^2$ is to $2AC^2$ Therefore the united forces of all the particles in the circumference of the circle IK are to the joint forces of as many particles in the place A as IX^2-2CX^2 is to 2AC and therefore (by Lem 1) to the united forces of as many particles in the circumference of the circle AE as IX^2-2CX^2 is to AC^2

Now if Pp the diameter of the sphere is conceived to be divided into an infinite number of equal part upon which a like number of circles IK are supposed to stand the matter in the circumference of every circle IK will be as IX and therefore the force of that matter to turn the earth about will be as IX and therefore the force of that matter to turn the earth about will be as ... force of the same matter if it was situated in the

whose fluxion is AC −4 AC C ...

whose fluxion is $AC^4 - AC^2 CX^2$ and therefore by the method of fluxions as $AC^4 CX - \frac{1}{3}AC^2 CX^3 + \frac{1}{5}CX^5$ is to $AC^4 CX - \frac{1}{3}AC^2 CX^3$ that is if for CX we write the whole Cp or AC as $\frac{1}{15}AC$ is to $\frac{2}{3}AC^5$ that is, as 2 is to 5 Q E D

LEMMA 3

... I say in the third place that the motion of the ... icles ... nded ... three

... bed sphere revolved toget ...

circumference of a circle to double its diameter

HYPOTHESIS II

IF THE OTHER PARTS OF THE EARTH WERE TAKEN AWAY AND THE REMAINING RING WAS CARRIED ALONE ABOUT THE SUN IN THE ORBIT OF THE EARTH BY ...

WOULD BE THE SAME WHETHER THE RING WERE FLUID OR WHETHER IT CONSISTED OF A HARD AND RIGID MATTER

PROPOSITION 39 PROBLEM 20

To find the precession of the equinoxes

in such an orbit which motion in a whole sidereal year becomes 20° 11 46

Since therefore the nodes of the moon in such an orbit would be yearly transferred 20° 11′ 46″ backwards and if there were more moons the motion of the nodes of every one (by Cor xvi Prop 66 Book i) would be as its periodic time if upon the surface of the earth a moon was revolved in the time of a sidereal day the annual motion of the nodes of this moon would be to 20° 11′ 46″ as 23^h 56^m the sidereal day is to 27^d 7^h 43^m the periodic time of our moon that is as 1436 m to 39 343 And the same thing would happen to the nodes of a ring of moons encompassing the earth whether these moons did not mutually touch each the other or whether they were molten and formed into a continued ring or whether that ring should become rigid and inflexible

Let us then suppose that this ring is in quantity of matter equal to the whole exterior earth *PapAPepE* which lies without the sphere *Pape* (see Fig Lem 2) and because this sphere is to that exterior earth as aC^2 is to AC^2-aC^2 that is (seeing PC or aC the least semidiameter of the earth is to AC the greatest semidiameter of the same as 229 is to 230) as 52 441 is to 459 if this ring encompassed the earth round the equator and both together were revolved ... the motion of the ring (by Lem 3) would be to ... 085 275 con... would be to the sum of the motion ... 813 ... the whole and communicates ... on to ... in ... of ... d ... d

of both ring and sphere will be ... h 4590 to 489 813 conjointly that is as 100 to 292 369 But the ... the nodes of a number of moons (as we explained above) and therefore by which the equinoctial points of the ring recede (that is the forces 3IT in Fig Prop 30) are in the several particles as the distances of those particles from the plane QR and by these forces the particles recede from that plane and ... spread all over the surface of ...

exterior part of the earth ... about the earth round any diameter of the equator and there ... equinoctial points would become less than before in the proportion of 2 to 5 Therefore the annual regress of the equinoxes now would be to 20° 11′ 46″ ... 10 is to 73 092 that is would be 9″ 56‴ 50iv

But because the plane of the equator is inclined to that of the ecliptic this ... to be diminished in the ratio of the sine 91 706 (which is the cosine of 23½ ... the remaining motion will now be 9″ 7‴ 20iv ... sun

But the force of the ... early as 4 4815 to 1 and the force of the moon to move the equ... of the sun in the same proportion Whence the annual precession of the equinox ... from the force of the moon comes out 40″ 52‴ 52iv and the total ... of both will be 50″ 00‴ 12iv ... for the precession of the equinoxes by ... yearly

If the height of the earth at the equator exceeds its height at the poles by more than $17\frac{1}{6}$ miles the matter thereof will be more rare near the surface than at the centre and the precession of the equinoxes will be augmented by the excess of height and diminished by the greater rarity

And now we have described the system of the sun the earth moon and planets it remains that we add something about the comets.

LEMMA 4

The comets are more remote than the moon and are in the regions of the planets

As the comets were placed by astronomers beyond the moon because they were found to have no diurnal parallax so their annual parallax is a convincing proof of their descending into the regions of the planets for all the comets

d rect course according to the order of the signs about the

those … again

appearance appear swifter than they ought to be if the earth is between them and the sun and slower and perhaps retrograde if the earth is in the other side of its orbit And these appearances proceed chiefly from the diverse situations which the earth acquires in the course of its motion after the same manner as it happens to the planets which appear sometimes retrograde some-

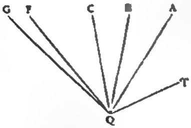
progressive according as the

n the contrary

by an angular

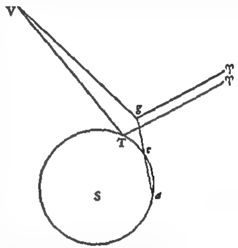
earth to

appear accelerated and from this apparent acceleration or retardation or regressive motion the distance of the comet may be inferred in this manner



Let TQA TQB TQC be three observed longitudes of the comet about the time of its first appearing and TQF its last observed longitude before its disappearing Draw the right line ABC whose parts AB BC intercepted between the right lines QA and QB QB and QC may be one to the other as the two times between the three first observations. Produce AC to G so that AG may be to AB as the time between the first and last observations is to the time between the first and second and join QG Now if the comet did move uniformly in a right line and the earth either stood still or was likewise carried forwards in a right line by an uniform motion the angle TQG would be the longitude of the comet at the time of the last observation. Therefore the angle FQG which is the difference of the longitude proceeds from the inequality

of the motions of the comet and the earth and if the earth and comet move contrary ways this angle is added to the angle TQG and accelerates the apparent motion of the comet but if the comet mo … earth … ren… pro … e earth is justly to be esteemed the parallax of the comet there being neglected thereby some little increment or decrement that may arise from the unequal motion of the comet in its orbit From this parallax we thus deduce the distance of the comet Let S represent the sun acT the great orbit a the earth s place in the first observation c the place of the earth in the third observation T the place of the earth in the last observation and T♈ a right line drawn to the beginning of Aries Set off the angle ♈TV equal to the angle TQ♈ that is equal to the longitude of the comet at the time when the earth is in T join ac and produce it to g so that ag may be to ac as AG is to AC and g will be the place at which the earth would have arrived in the time of the last observation if it had continued to move uniformly in the right line ac Therefore if we draw g♈ parallel



to T♈ and make the angle ♈gV equal to the angle ♈QG this angle ♈gV will be equal to the longitude of the comet seen from the place g and the angle TVg will be the parallax which arises from the earth s being transferred from the place g into the place T and therefore V will be the place of the comet in the plane of the ecliptic And this place V is commonly lower than the orbit of Jupiter

The same thing may be deduced from the incurvation of the way of the comets for these bodies move almost in great circles while their velocity is great but about the end of their course when that part of their apparent motion which arises from the parallax bears a greater proportion to their whole apparent motion they commonly deviate from those circles and when the earth goes to one side they deviate to the other and this deflection because of its corresponding with the motion of the earth must arise chiefly from the parallax and the quantity thereof is so considerable as by my computation to place the disappearing comets a good deal lower than Jupiter Hence it follows that when they approach nearer to us in their perigees and perihelions they often descend below the orbits of Mars and the inf…

Th…

… distance of the comet to the distance of a planet directly as their diameters and inversely as the square root of their lights Thus in the comet of the year 1682 Mr Flamsteed observed with a

to the tenth part of this measure an
1?' but in the light and plendor of its head it surpassed that of the comet in
tars of the first or second
about four times more
ual to the l ght of th
about 21 and there-

Saturn
l than
ar and had
been compared to the tars of the first magnitude of its head
m ans of a tele-
som times
he diameters
ameter of the
nucleus or central star is but about a tenth or perhaps fifteenth part of the
diameter of the head it appears that these stars are generally of about the

fixed tars for if it were the comet could receive no more light from our sun
than our planets do from the fixed stars

obscured by this moke the nearer mu t it be allowed to come to the un that
it may vie with the planets in the quantity of light which it reflects Hence it is

obliged to allow that the moke arising from their heads is prop gated thr u h
such a vast xtent of pace and with such a velocity and expansion as will seem
altogether incred ble in the latter case the hole light of both head and tail is
to be ascribed to the central nucleus But then if we suppose all this light to be
united and condensed within the di k of the nucleus certainly the nucleus will
by far exceed Jupiter itself in splendor especi lly when it emits a very large

that light was supposed to be gathered together into one star it would sometimes exceed not one Venus only but a great many such united into one

Lastly the same thing is inferred from the light of the heads which increases in the recess of the comets from the earth towards the sun and decrease in their return from the sun towards the earth Thus the comet of the year 1665 (by the observations of Hevelcke) from the time that it was first seen was always losing of its apparent motion and therefore had already passed its perigee but yet the splendor of its head was daily increasing till being hid under the sun s rays the comet ceased to appear The comet of the year 1683 (by the observations of the same Hevelcke) about the end of July when it first appeared moved at a very slow rate advancing only about 40 or 45 minutes in its orbit in a day s time but from that time its diurnal motion was continually upon the increase till September 4 when it arose to about 5 degrees and therefore in all this interval of time the com

the earth

a micromet

coma whic ptember 2 he observed to be 9 7 and therefore its head appeared far less about the beginning than towards the end of the motion though about the beginning because nearer to the sun it appeared far more lucid than towards the end as the same He

month of December and that of the year 1680 about the end of the same month did both move with their greatest velocity and were therefore then in their perigees but the greatest splendor of their heads was seen two weeks before when they had just got clear of the sun s rays and the greatest splendor of their tails a little earlier when yet nearer to the sun The head of the former comet (according to the observations of Cysat) on December 1 appeared greater than the stars of the first magnitude and on December 16 (then in the perigee) it was diminished but little in magnitude but much diminished in the splendor and brightness of its light On January 7 Kepler being uncertain about the head left off observing On December [illegible] the head of the latter comet was seen and observed by Mr Flamsteed when but 9 degrees distant from the sun which is scarcely to be done in a star of the third magnitude On December 15 and 17 it appeared as a star of the third magnitude its luster being diminished by the brightness of the clouds near the setting sun On December 26 when it moved with the greatest velocity being almost in its perigee it was less than the mouth of Pegasus a star of the third magnitude On January 3 it appeared as a star of the fourth On January 9 as one of the fifth On January 13 it was hid by the splendor of the moon then in her increase On January 25 it was scarcely equal to the stars of the seventh magnitude If we compare equal intervals of time taken on one side of the perigee and then on the other we shall find that the head of the comet which at both intervals of time was far but yet equally removed from the earth and should therefore have shone with equal splendor appeared brightest on the

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comets tends to be regular and to appear greatest when the heads move fast-

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for they are for the most part nearer to the sun

COR. III Hence also it is evident that the celestial spaces are void of resist ance for though the comets are carried in oblique paths and sometimes contrary to the course of the planets yet they move every way with the greatest

motion for the opinion of some writers that they are no other than meteors an opinion based on the continual changes that happen to their heads seems to have no foundation for the heads of comets are encompassed with huge atmospheres and the lowermost parts of these atmospheres must be the densest and

to each other and the solid body of Jupiter is hardly to be seen through them and much more must the bodies of comets be hid under their atmospheres which are both deeper and thicker

PROPOSITION 40 THEOREM 20

That the comets move in some of the conic sections having their foci in the centre of the sun and by radii drawn to the sun describe areas proportional to the times

This Proposition appears from Cor I Prop 13 Book I compared with Props. 8 12 and 13 Book III

COR. I Hence if comets revolve in orbits returning into themselves the orbits will be ellipses and their periodic times will be to the periodic times of

comet would be to the time of the revolution of Saturn that is to 30 years as $4\sqrt{4}$ (or 8) is to 1 and would therefore be 240 years

COR II But their orbits will be so near to parabolas that parabolas may be used for them without sensible error

COR III And therefore by Cor VII Prop 16 Book I the velocity of comet will always be to the velocity of ... the same d... the distar... comet fror... greatest se... ... of the ellipse which the earth describes ... radius of the great orbit or the 100 000 000 parts and then the earth by it ... 1 720 212 of those parts and 71 675½ by ... comet at the same mean distance of the ... from the sun with a velocity which is to the velocity of the earth as $\sqrt{2}$ to 1 would by its diurnal motion describe 2 432 747 parts and 101 364½ parts by its hourly motion But at greater or less distances both the diurnal and hourly motion will be to this diurnal and hourly motion inversely as the square root of the distances and is therefore given

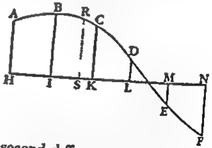
COR IV Therefore if the latus rectum of the parabola is four times the radius of the great orbit and the square of that radius is supposed to ... 100 000 000 parts the area wh... ... d ... I ... I ... greater inversely as the square root of that ratio ... the diurnal and

LEMMA 5

To find a curved line of the parabolic kind which shall pass through any given number of points

Let those points be A B C D E F &c and from the same to any right line HN given in position let fall as many perpendiculars AH BI CK DL EM FN &c

b	$2b$	$3b$	$4b$	$5b$
	$2c$	$3c$	$4c$	
d	$2d$	$3d$		
	$2e$			
	f			



CASE 1 If HI IK KL &c the ... of ... their second differences c $2c$ $3c$ $4c$ ... so say so as AH$-$BI may be$=b$ BI$-$ ... $-$EM$+$FN$=5b$ &c then $b-2b=c$ ... on to the last difference which ... pendicular RS ... in order to find ... LM &c to be ... $+$SK$=r$ ¼r into$+$SL$=s$ ⅕s into$+$SM$=t$ ... p ½p into$-$IS$=q$ ⅓q into ... ME the last perpendicular but one and prefixing negative signs before the terms HS IS &c which lie from S towards A and positive signs before the

terms SK SL &c which lie on the other side of the point S and observing well the signs RS will be $= a + bp + cq + dr + es + ft + \&c$

Case 2 But if HI IK &c the intervals of the points H I K L, &c are unequal take b $2b$ $3b$ $4b$ $5b$ &c the first differences of the perpendiculars AH BI CK &c divided by the intervals between those perpendiculars c, $2c$ $3c$ $4c$ &c their second differences divided by the intervals between every two d $2d$ $3d$ &c their third differences divided by the intervals between every three e $2e$ &c. their fourth differences divided by the intervals between every four and so forth that is in such manner that b may be $= \frac{AH - BI}{HI}$ $2b = \frac{BI - CK}{IK}$ $3b = \frac{CK - DL}{KL}$ &c then $c = \frac{b - 2b}{HK}$ $2c = \frac{2b - 3b}{IL}$ $3c = \frac{3b - 4b}{KM}$ &c. then $d = \frac{c - 2c}{HL}$ $2d = \frac{2c - 3c}{IM}$ &c And those differences being found let AH be $= a$, $-HS = p$ p into $-IS = q$ q into $+SK = r$ r into $+SL = s$ s into $+SM = t$ proceedin in this manner to ME the last perpendicular but one and the ordinate RS will be $= a + bp + cq + dr + es + ft + \&c$

Cor. Hence the areas of all curves may be nearly found for if some number of points of the curve to be squared are found and a parabola be supposed to be drawn through those points, the area of this parabola will be nearly the same with the area of the curvilinear figure proposed to be squared but the parabola can be always squared geometrically by methods generally known.

Lemma 6

Certain observed places of a comet being given to find the place of the same at any intermediate given time

Let HI IK, KL, LM (in the preceding Fig) represent the times between the observations HA, IB KC LD ME, five observed longitudes of the comet and HS the given time between the first observation and the longitude required. Then if a regular curve ABCDE is supposed to be drawn through the points A, B C D E, and the ordinate RS is found out by the preceding Lemma, RS will be the longitude required.

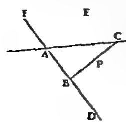
By the same method from five observed latitudes, we may find the latitude at a given time.

[illegible]

observations ought to be used.

Lemma 7

Through a given point P to drawn a right line BC whose parts PB PC cut off by two right lines AB AC given in position may be one to the other in a given ratio



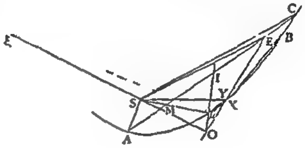
From the given point P suppose any right line PD to be drawn to either of the right lines given as AB and produce the same towards AC the other given right line as far as E, so as PE may be to PD in the given ratio. Let EC be parallel to AD Draw CPB and PC will be to PB as PE to PD

Q.E.F

LEMMA 8

Let ABC *be a parabola having its focus in* S *By the chord* AC *bisected in* I *cut off the segment* ABCI *whose diameter is* Iμ *and vertex* μ *In* Iμ *produced take* μO *equal to one half of* Iμ *Join* OS *and produce it to* ξ *so that* Sξ *may be equal to* 2SO *Now supposing a comet to revolve in the arc* CBA *draw* ξB *cutting* AC *in* E *I say the point* E *will cut off from the chord* AC *the segment* AE *nearly proportional to the time*

For if we join EO cutting the parabolic arc ABC in Y and draw μX touching the same arc in the vertex μ and meeting EO in X the curvilinear area AEXμA will be to the curvilinear area ACYμA as AE to AC and therefore



... joining BX the triangle SEB will be equal to the triangle XEB
Theref...
subtra...
area A...
But th...
ASBYμA is to the area ASCYμA as the time of description of the arc AB is to the time of description of the whole arc AC and therefore AE is to AC nearly in the proportion of the times Q E D

COR When the point B falls upon the vertex μ of the parabola AE is to AC accurately in the proportion of the times

SCHOLIUM

If we join $\mu\xi$ cutting AC in δ and in it take ξn in proportion to μB as 27MI to 16 Mμ and draw Bn this Bn will cut the chord AC in the ratio of the times more accurately than before but the point n is to be taken beyond or on this side the point ξ according as the point B is more or less distant from the principal vertex of the parabola than the point μ

LEMMA 9

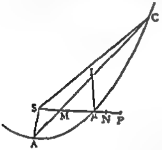
The right lines Iμ *and* μM *and the length* $\frac{AI^2}{4S\mu}$ *are equal among themselves*

For 4Sμ is the latus rectum of the parabola belonging to the vertex μ

LEMMA 10

... may be one-third of μI and SP may be to SN

For if the comet with the velocity ...
supposed to move uniformly forwards in the right line which touches the parabola in μ the area which it would describe by a radius drawn to the point S ...



height Sμ inversely as the square root of ... to Sμ that is in the ratio of Sμ to SN it follows that the length described with this velocity will be to the length in the same time described in the tan

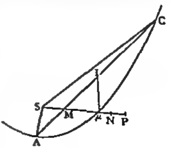
Sμ+⅔Iμ would in the same time nearly describe the chord AC

LEMMA 11

If a comet void of all motion was let fall from the height SN or Sμ+⅓Iμ towards

qual to the length Iμ

For in the same time that the comet would require to describe the parabolic arc AC it would (by the last Lemma) with that velocity which it hath in the height SP describe the chord AC and therefore (by Cor VII Prop 16 Book I) if it was



that circle the length of which would be to the chord of the parabolic arc AC in the ratio of 1 to $\sqrt{2}$ Therefore if with that weight which in the height SP it hath towards the sun it should fall from that height towards the sun it would (by Cor IX Prop 16 Book I) in half the said time describe a space equal to the square of half the said chord divided by four times the height SP that is it would describe the space

$\frac{AI^2}{4SP}$ But since the weight of the comet towards the sun in the height SN is to the weight of the same towards the sun in the height SP as SP to $S\mu$ the comet by the weight which it hath in the height SN in falling from that height towards the sun would in the same time describe the space $\frac{AI^2}{4S\mu}$ that is a space equal to the length $I\mu$ or μM Q E D

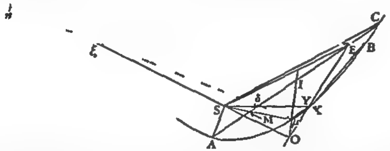
PROPOSITION 41 PROBLEM 21

From three given observations to determine the orbit of a comet moving in a parabola

This being a Problem of very great difficulty I tried many methods of resolving it and several of those Problems the composition whereof I have given in the first book tended to this purpose But afterwards I contrived the following solution which is somewhat more simple

Select three observations distant one from another by intervals of time nearly equal but let that interval of time in which the comet moves more slowly be somewhat greater than the other namely so that the difference of the times may be to the sum of the times as the sum of the times is to about 600 days or that the point E may fall nearly upon M and may err therefrom rather towards I than towards A If such direct observations are not at hand a new place of the comet must be found by Lemma 6

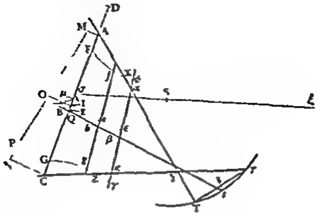
Let S represent the sun T t τ three places of the earth in the earth s orbit TA tB τC three observed longitudes of the comet V the time between the first observation and the second W the time between the second and the third X the length which in the whole time V+W the comet might describe with that velocity which it has in the mean distance of the earth from the sun which length is to be found by Cor III Prop XL Book III and tV a perpendicular upon the chord $T\tau$ In the mean observed longitude tB take at pleasure



the point B for the place of the comet in the plane of the ecliptic and from thence towards the sun S draw the line BE which may be to the perpendicular tV as the product of SB and St^2 is to the cube of the hypothenuse of the right-angled triangle whose sides are SB and the tangent of the latitude of the

and C will be nearly the places of the comet in the plane of the ecliptic in the first and third observations if B was its place rightly assumed in the second

Upon AC bisected in I erect the perpendicular Ii Through B imagine the



line Bι drawn parallel to AC Imagine the line Sι drawn cutting AC in λ, and complete the parallelogram iIλμ Take Iσ equal to 3Iλ and through the sun S im mne th l ne σξ drawn equal to 3Sσ+3iλ. Then canceling the letters A, E drawn

)

AEC by the same rule as before that i. so that its parts AE and EC may be one to the o her as the times V and W between the observa ons. Thus A and C will be the places of the comet more accurately

Upon AC bisected in I erect the perpendiculars AM CN IO of which AM and CN may be the tangent of the latitudes in the first and third observation to the radii TA and τC Join MN cutting IO and O Draw the rectangular parallelogram iIλμ as before In IA produced take ID equal to

(

By the same method as the points E A C G were found from the assumed point B from other points b and β assumed at plessure find out the new points

spectiv ly to CG eg εγ through the points F f and φ draw the circumference f a circle Ffφ cutting the right line AT in X and the point X will be another place of the comet in the plane of the ecliptic. And at the points X and Z erectin the tangents of the latitudes of the comet to the radii TX and Z two places of the comet in its own orbit will be determined Lastly if (by Prop 19 Book 1) to the focus S a parabola is described passing through those two places this parabola will be the orbit of the comet Q.E.I.

The demonstration of this construction follows f om the preceding Lemmas, because the right line AC is cut in E in the proportion of the times by Lemma

7 as it ought to be by Lemma 8 and BE by Lemma 11 is a portion of the right line BS or Bξ in the plane of the ecliptic intercepted between the arc ABC and the chord AEC and MP (by Cor Lem 10) is the length of the chord of that arc which the comet should describe in its proper orbit between the first and third observations and therefore is equal to MN providing B is a true place of the comet in the plane of the ecliptic

But it will be convenient to assume the points B b β not at random but nearly true If the angle AQt at which the projection of the orbit in the plane of the ecliptic cuts the right line tB is roughly known at that angle with Bt draw the line AC which may be to $\frac{4}{3}$Tτ as the square root of the ratio of SQ to St and drawing the right line SEB so as its part EB may be equal to the length Vt the point B will be determined which we are to use for the first time Then canceling the right line AC and drawing anew AC [illegible] ccording to the preceding [illegible] point b by [illegible] in tB take the Yb may be [illegible] distance YB in a ratio compounded of the ratio of MP to MN and the square root of the ratio of SB to Sb And by the same method you may find the third point β if you please to repeat the operation the third time but if this method is followed two operations generally will be sufficient for if the distance Bb happens to be very small after the points F f and G g are found draw the right lines Ff and Gg and they will cut TA and τC in the points required X and Z

EXAMPLE

Let the comet of the year 1680 be proposed The following table shows the motion thereof as observed by Flamsteed and calculated afterwards by him from his observations and corrected by Dr Halley from the same observations

	Time Apparent h m	Time True h m	Sun's longitude	Comet Longitude	Comet Latitude north
1680 *Dec* 12	4 46	4 46 0	♑ 1 51 23	♑ 6 32 30	8 28 [illegible]
21	6 32½	[illegible] 36 59	11 06 44	♒ 5 08 12	21 42 13
24	[illegible] 12	6 17 52	14 09 26	18 49 23	25 23 5
26	5 14	5 20 44	[illegible] 09 22	[illegible] 24 13	27 00 52
29	7 55	8 03 02	19 19 43	♓ 13 10 41	28 09 58
30	8 02	8 10 26	20 21 09	17 38 20	8 11 53
1681 *Jan* 5	[illegible] 51	6 01 38	26 22 18	♈ 8 48 53	26 1[illegible] 7
9	6 49	7 00 53	♒ 0 29 02	18 44 04	24 11 [illegible]6
10	5 54	[illegible] 06 10	1 27 43	20 40 50	23 43 52
13	[illegible] 56	7 08 55	4 33 20	25 59 48	22 17 28
25	7 44	7 58 42	16 45 36	♉ 9 35 0	17 56 30
30	8 07	8 21 53	[illegible] 49 58	13 19 51	16 42 18
Feb [illegible]	6 20	6 34 51	24 46 59	1[illegible] 13 53	16 04 1
5	6 50	7 04 41	27 49 51	16 59 0[illegible]	15 27 3

To these you may add some observations of mine

These observations were made by a telescope of 7 feet with a micrometer and threads placed in the focus of the telescope by these instruments we determined the positions both of the fixed stars among themselves and of the

	Apparent time	Comet Longitude	Comet Latitude north
	m		
1681 *Feb*	8 30	♉ 6 18 35	1 46 46
	8 15	7 04 30	1 36 1
Mar 1	11 0	♈ 5 4	1 23 40
	8 0	25 1 48	1 19 38
	11 [illegible]	29 18 0	1 03 16
7	9 0	♊ 0 4 0	11 [illegible] 0
9	8 30	0 43 4	11 45

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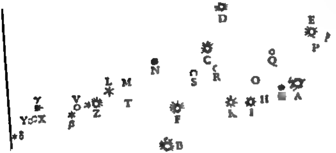

n) in the heel of the same foot and D E F G H I K L M N O Z β γ δ other smaller stars in the same foot and let p P, Q R S T V X represent

to 9 and produced did pass through the star H Thus were the positions of the fixed stars determined in respect to one another

The fixed star	Their longitudes	Latitude north	The fixed star	Their longitude	Latitude north
A	♉ 6 41 50	12 8 36	L	♉ 29 33 34	1 7 48
B	8 40 3	11 17 54	M	29 18 [illegible]	12 7 0
C	27 58 30	12 40 25	N	8 48 29	1 31 9
E	6 27 17	1 5 7	Z	29 44 48	11 57 13
F	8 28 37	11 5 22	α	29 5 3	11 55 48
G	26 56 8	1 4 58	β	♊ 0 8 23	11 48 56
H	7 11 45	1 1	γ	0 40 [illegible]	11 55 18
I	27	11 53 11	δ	1 3 20	11 30 4
K	♈ 4 7	11 [illegible] 6			

Mr Pound has since observed a second time the positions of these fixed stars amongst themselves and obtained their longitudes and latitudes according to the preceding table

The positions of the comet to these fixed stars were observed to be as follows

Friday February 25 O S at $8\frac{1}{2}^h$ P M the distance of the comet in p from the star E was less than $\frac{3}{13}$AE and greater than $\frac{1}{5}$AE and therefore nearly equal to $\frac{3}{14}$AE and the angle ApE was a little obtuse, but almost right For on pE the distance of the comet from that

tance of the comet in P from the star E was greater than $\frac{1}{4\frac{1}{2}}$AE and less than $\frac{1}{5\frac{1}{4}}$AE and therefore nearly equal to $\frac{1}{4\frac{7}{8}}$ of AE or $\frac{8}{39}$ AE But the distance of the comet from the perpendicular let fall from the star A upon the right line PE was $\frac{4}{5}$PE

Sunday February 27 $8\frac{1}{4}^h$ P M the distance of the comet in Q from the star O was equal to the distance of the stars O and H and the right line QO produced passed between the stars K and B I could not by reason of intervening clouds determine the position of the star to greater accuracy

Tuesday March 1 11^h P M the comet in R lay exactly in a line between the stars K and C so as the part CR of the right line CRK was a little greater than $\frac{1}{3}$CK and a little less than $\frac{1}{3}$CK$+\frac{1}{8}$CR and therefore$=\frac{1}{3}$CK$+\frac{1}{16}$CR or $\frac{16}{45}$CK

Wednesday March 2 8^h P M the distance of the comet in S from the star C was nearly $\frac{4}{9}$FC the distance of the star F from the right line CS produced was $\frac{1}{24}$FC and the distance of the star B from the same right line was five times greater than the distance of the star F and the right line NS produced passed between the stars H and I five or six times nearer to the star H than to the star I

Saturday March 5 $11\frac{1}{6}^h$ P M when the comet was in T the right line MT was equal to $\frac{1}{2}$ML and the right line LT produced passed between B and F four or five times nearer to F than to B cutting off from BF a fifth or sixth part thereof towards F and MT produced passed on the outside of the space BF towards the star B four times nearer to the star B than to the star F M was a very small star scarcely to be seen by the telescope but the star L was greater and of about the eighth magnitude

Monday March 7 $9\frac{1}{2}^h$ P M the comet being in V the right line Vα produced did pass between B and F cutting off from BF towards F $\frac{1}{10}$ of BF and was to the right line Vβ as 5 to 4 And the distance of the comet from the right line $\alpha\beta$ was $\frac{1}{2}$Vβ

Wednesday March 9 $8\frac{1}{2}^h$ P M the comet being in X the right line γX was equal to $\frac{1}{4}\gamma\delta$ and the perpendicular let fall from the star δ upon the right line γX was $\frac{2}{5}$ of $\gamma\delta$

The same night at 12^h the comet being in Y the right line γY was equal to $\frac{1}{3}$ of $\gamma\delta$ or a little less as perhaps $\frac{5}{16}$ of $\gamma\delta$ and a perpendicular let fall from the star δ on the right line γY was equal to about $\frac{1}{6}$ or $\frac{1}{7}$ $\gamma\delta$ But the comet being then extremely near the horizon was scarcely discernible and therefore its place could not be determined with the same certainty as in the foregoing observations

From these observations by constructions of figures and calculations I

deduced the longitudes and latitudes of the comet and Mr Pound by cor ... termined more correctly the places

Now in order to determ ... those three which Flamsteed made (Dec. 21 Jan ... supposing ... obser ... first ... 28 4 ... d the

node in ♋ and ascending ... the perihelion of the comet) ... th latitude 34 south its ... d by a radius drawn to the ... of the earth ...

operations and partly by scale and compass to the ... servations as may be seen in the following table

The C ...

	Distance from ☉	Longitude computed	Latitude computed
Dec 1	[illegible]	♑ 6° 3[illegible]	8° 18½
29	8403	♓13 13⅔	8 00
Feb 5	16669	♉17 00	15 [illegible]
Mar 5	1 3	[illegible]9 19¾	12 4

in the table on page 348

Th
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from
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the places of the comet as follows y Mr Pound Dr Halley has determined and from the

Nov 3 17ʰ 2ᵐ apparent time at London the comet was in ♌ 29 51, with 1 17 45 latitude north

Nov 5 15ʰ 58ᵐ the comet was in ♍ 3 23 with 1 6 latitude north

Nov 10 16ʰ 31ᵐ the comet was equally distant from which are designated σ and τ in P line that joins them but was ve

True time			*The c t* *D t nc f n th*	*L g i t c mpt t l*	*Lat t d comp t l*	*E rr* *Lo git d*	*L t t de*
	d	h m					
Dec	12	4 46	28028	♑ 6 29 25	8 26 0 bor	−3 5	−2 0
	21	6 37	61076	♒ 5 6 30	21 43 20	−1 12	+1
	24	6 18	70008	18 48 20	25 22 40	−1 3	−0 25
	26	5 20	75576	28 22 45	27 1 36	−1 28	+0 44
	29	8 3	84021	♓13 12 40	28 10 10	+1 59	+0 12
	30	8 10	86661	17 40 5	28 11 20	+1 45	−0 33
Jan	5	6 1 ½	101440	♈ 8 49 49	26 15 15	+0 56	+0 8
	9	7 0	110959	18 44 36	24 12 54	+0 32	+0 58
	10	6 6	113162	20 41 0	23 44 10	+0 10	+0 18
	13	7 9	120000	26 0 21	22 17 30	+0 33	+0 2
	25	7 59	145370	♉ 9 33 40	17 57 55	−1 0	+1 5
	30	8 22	155303	13 17 41	16 42	−2 10	−0 11
Feb	2	6 35	160951	15 11 11	16 4 15	−2 42	+0 14
	5	7 4 ½	166686	16 58 55	15 29 13	−0 41	+ 0
	25	8 41	202570	26 15 46	12 48 0	−2 49	+1 10
Mar		11 39	216205	9 18 35	12 5 40	+0 35	+ 14

alogue this star σ was then in ♍ 14 15 with 1 11 latitude north nearly and τ in ♍ 17° 3½′ with 0 34 latitude south and the middle point between those stars was ♍ 15 39¼ with 0° 33½′ latitude north Let the distance of the comet from that right line be about 10 or 12 and the difference of the longitude of the comet and that middle point will be 7 and the difference of the latitude nearly 7½′ and thence it follows that the comet was in ♍ 15 32 with about 26 latitude north

The first observation from the posit on of small fixed st
accurate enc
might be an but hardly greater The long
found in the first and mo as the least accurate there
said parabolic orbit come
its distance from the sun and

Moreover Dr Halley observing that a remarkable comet had appeared four times at equal intervals of 575 years (that is in the month of September

and that ... death at which time by reason of the inconvenient situation of the ... bit ... rth ... 57 ... ane ... lion ... lion ... the

Dec [illegible] 23h 9m the distance of the perihelion from the ... the

True time	Longitude observed	Latitude observed	Longitude computed	Latitude computed	Error in longitude	Error in latitude
m						
Nov 3 16 47	♌29 51 0	1 17 45	♌[illegible] 51 22	1 17 3[illegible]	+0 22	−0 13
5 15 37	♍ 3 23 0	1 6 0	♍ 3 4 3[illegible]	1 6 9	+1 32	+0 9
10 [illegible] 18	15 32 0	0 27 0	15 33 2	0 [illegible]5 7	+1	−1 53
16 17 00			♎ 8 16 45	0 53 7 S		
[illegible] 21 34			18 5[illegible] [illegible]	1 [illegible]6 54		
[illegible]0 17 0			28 10 36	1 53 35		
23 17 5			♏13 22 42	2 [illegible] 0		
Dec 1 4 46	♑ 6 3[illegible] 30	8 [illegible] 0	♑ 6 31 20	8 [illegible] 6 N	−1 10	+1 6
1 6 37	♒ 5 8 1	21 42 13	♒ 5 6 14	1 44 42	−1 [illegible]	+2 [illegible]
1 6 18	18 49 23	5 23 5	18 47 30	25 23 35	−1 53	+0 30
26 5 1	28 24 13	[illegible]7 0 5[illegible]	28 1 4	27 1	−2 31	+1 9
[illegible] 8 3	♓13 10 41	28 9 58	♓13 11 14	[illegible] 10 38	+0 33	+0 40
30 8 10	17 38 0	[illegible]8 11 [illegible]	17 38 7	28 11 37	+0 7	−0 1[illegible]
Jan. 5 6 1¼	♈ 8 48 53	[illegible]6 15 7	♈ 8 48 51	[illegible] 14 57	−0	−0 10
9 7 1	18 44 4	4 11 56	18 43 51	4 1 17	−0 13	+0 21
10 6 6	20 40 50	23 43 3[illegible]	[illegible]0 40 [illegible]3	23 43 [illegible]5	−0 [illegible]7	−0 7
13 7 9	25 [illegible] 48	2[illegible] 17 28	[illegible]6 0 8	22 16 32	+0 [illegible]0	−0 [illegible]
25 7 59	♉ 9 35 0	17 56 30	♉ 9 34 11	17 56 6	−0 49	−0 4
30 8 2[illegible]	13 19 51	16 4 18	[illegible] 18 8	16 40 5	−1 23	−[illegible] 13
Feb 2 6 35	15 13 53	16 4 1	15 [illegible] 59	16 2 17	−1 54	−1 [illegible]
5 7 4½	16 59 6	15 27 3	16 59 17	15 [illegible]7 0	+0 11	−0 3
2[illegible] 8 41	[illegible]6 [illegible] 3[illegible]	1 46 46	26 16 59	1 45 [illegible]	−1 3[illegible]	−1 [illegible]
Mar 1 11 10	[illegible]7 5 42	12 23 40	27 51 47	1 [illegible] [illegible]8	−0 55	−1 1
5 11 30	29 18 0	1 3 16	29 [illegible]0 11	1 [illegible]0	+ 11	−0 [illegible]6
9 8 38	♊ 0 43 4	11 45 5	♊0 4 43	11 45 3[illegible]	−0 21	−0 17

plane of the ecliptic 9° 17′ 35″ and its conjugate axis [illegible] 481 [illegible] he computed the motions of the comet in this elliptic orbit The places of the comet as deduced from the observations and as arising from computation made in this orbit may be seen in the preceding table

The observations of this comet from the beginning to the end agree as perfectly with the motion of the comet in the orbit just now described as the motions of the planets do with the theories from whence they are calculated and by this agreement plainly evince that it was one and the same comet that appeared all that time and also that the orbit of that comet is here rightly defined

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the places of the comet as follows by I found Dr Halley has determined

Nov 3 17h 2m apparent time at London the comet was in ♌ 29 51′, with 1 17 45 latitude north

Nov 5 15h 58m the comet was in ♍ 3 23 with 1 6′ latitude north

Nov 10 16h 31m the comet was equally distant from two stars in ♌, which are designated σ and τ in Bayer but it had not quite touched the right line that joins them but was very little distant from it In Flamsteed s catalogue this star σ was then in ♍ 14 15 with 1 11 latitude north nearly and τ in ♍17 3½ with 0 34 latitude south and the middle point between those stars was ♍ 15 39¼ with 0 33½ latitude north Let the distance of the comet from that right line be about 10 or 12 and the difference of the longitude of the comet and that middle point will be 7 and the difference of the latitude nearly 7½ and thence it follows that the comet was in ♍ 15 32 with about 26 latitude north

T ime			D t ce f th s	The (t L gitud computed	Lat t d comp t d	Errors L gitud	Errors Lat t de
	d	h m					
Dec	12	4 46	28028	♑ 6 29 25	8 26 0 bor	−3 5	−2 0
	21	6 37	61076	♒ 5 6 30	21 43 20	−1 12	+1 [illegible]
	24	[illegible] 18	70008	18 48 20	25 22 40	−1 3	−0 25
	26	5 20	75576	28 22 45	27 1 36	−1 28	+0 44
	29	8 3	84021	♓13 12 40	28 10 10	+1 59	+0 12
	30	8 10	86061	17 40 5	28 11 20	+1 45	−0 33
Jan	5	6 1½	101440	♈ 8 49 49	26 15 15	+0 56	+0 8
	9	7 0	110959	18 44 36	24 12 54	+0 32	+0 58
	10	6 [illegible]	113162	20 41 0	23 44 10	+0 10	+0 18
	13	7 9	120000	26 0 21	22 17 30	+0 33	+0 2
	25	7 59	145370	♉ 9 33 40	17 57 55	−1 0	+1 25
	30	8 22	155303	13 17 41	16 42 [illegible]	−2 10	−0 11
Feb	2	6 35	160951	15 11 11	16 4 15	−2 42	+0 14
	5	7 4½	166686	16 58 55	15 29 13	−0 41	+2 [illegible]
	25	[illegible] 41	202570	26 15 46	12 48 0	−2 49	+1 10
Mar	5	11 39	216205	29 18 35	12 5 40	+0 35	+ 14

The 6 t ob a

ob rvation which was the least accurate there might be an error of 6 or 7 but hardly greater The longitude of the comet as found in the first and most accurate observation being computed in the aforesaid parabolic orbit comes out ♌ 29° 30 32 its latitude north 1 25 7 and its distance from the sun 115 546

Moreover Dr Halley observing that a remarkable comet had appeared four times at equal intervals of 575 years (that is in the month of September

Nov 22 the comet was een by Montenari in ♏ 2° 33 but at Boston in New England it as found in about ♏ 3 and ... fore that 1 30 The same day at 5ʰ

south side of Cor Leonis at a very small di tance ...

throu h Cor Leonis and Spica ♍ did cut the ecliptic in ♍ 3 46 at an angle of 2° 51 and if the comet had been in this line and in ♏ 3 its latitude would have been 2° 26 but since Hooke and Montenari agree that the comet was at some small di tance from this line to ards the north its latitude mu t have been some hat less On the 20th by the observation of Montenari its latitude was almost the same with that of Spica ♍ that is about 1 30 But by the

sun to ards the north

Nov 23 o at 5 morning at Nuremberg (that i at 4½ at London) Mr Zimmerman saw the comet in ♏ 8° 8 with 2° 31 south latitude its place bein obtained by taking its di tances from fixed tars

Nov 24 before sunrise the comet was seen by Montenari in ♏ 12° 52 on the north ide of the ri ht line through Cor Leoni and Spica ♍ and therefore its latitude as ome hat less than 2° 38 and ince the latitude as we said by the concurring observations of Montenari Ango and Hooke as con

tudes as are also the observat ons of Gallet Those are better which were made by takin the position of the comet to the fixed stars by Montenari Hooke An o and the observer in New England and ometimes by Ponthio and Cellio The same day at 5 morning at Ballasore the comet was observed in ♏ 11 45 and therefore at 5ʰ morning at London was in ♏ 13° nearly And by the theory the comet was at that time in ♏ 13 22 42

... com t was in ♏ 16½ nearly

morning at Rome (that is $5^h\ 10^m$ at London) by threads directed to the fixed stars observed the comet in ♎ 8 30 with latitude 0 40 south Their observations may be seen in a treatise which Ponthio published concerning this comet Cellio who was present and communicated his observations in a letter to Cassini saw the comet at the same hour in ♎ 8 30, with latitude 0° 30 south It was likewise seen by Gallet at the same hour at Avignon (that is at $5^h\ 42^m$ morning at London) in ♎ 8 without latitude But by the theory the comet was at that time in ♎ 8 16 45 and its latitude was 0 53 7 south

Nov 18 at $6^h\ 30^m$ in the morning at Rome (that is at $5^h\ 40^m$ at London) Ponthio observed the comet in ♎ 13 30 with latitude 1 20 south and Cellio in ♎ 13 30 with latitude 1 00 south But at $5^h\ 30^m$ in the morning at Avignon Gallet saw it in ♎ 13 00 with latitude 1 00 south In the University of La Fleche in France at 5^h in the morning (that is at $5^h\ 9^m$ at London) it was seen by Ango in the middle between two small stars one of which is the middle of the three which lie in a right line in the southern hand of Virgo Bayer's Ψ and the other is the outmost of the wing Bayer's θ Hence the comet was then in ♎ 12 46 with latitude 50 south And I was informed by Dr Halley that on the same day at Boston in New England in the latitude of 42½ at 5^h in the morning (that is at $9^h\ 44^m$ in the morning at London) the comet was seen near ♎ 14 with latitude 1 30 south

Nov 19 at $4\frac{1}{2}^h$ at Cambridge the comet (by the observation of a young man) was distant from Spica ♍ about 2 towards the northwest Now the Spike was at that time in ♎ 19 23 47 with latitude 2 1 59 south The same day at 5^h in the morning at Boston in New England the comet was distant from Spica ♍ 1 with the difference of 40 in latitude The same day in the island of Jamaica it was about 1 distant from Spica ♍ The same day Mr Arthur Storer at the river Patuxent near Hunting Creek in Maryland in the confines of Virginia in latitude 38½ at 5^h in the morning (that is at 10^h at London) saw the comet above Spica ♍ and very nearly joined with it the distance between them being about ¾ of one degree And from these observations compared I conclude that at $9^h\ 44^m$ at London the comet was in ♎ 18 50 with about 1 25 latitude south Now by the theory the comet was at that time in ♎ 18 52 15 with 1 26 54 latitude south

Nov 20 Montenari Professor of Astronomy at Padua at 6^h in the morning at Venice (that is $5^h\ 10^m$ at London) saw the comet in ♎ 23 with latitude 1 30 south The same day at Boston it was distant from Spica ♍ by about 4 of long[illegible] 24 nearly [illegible] in the morning observed the com[illegible] Cellio in ♎ 28 Ango at 5^h in the morning in ♎ 27° 45 Montenari in ♎ 27 51 The same day in the island of Jamaica it was seen near the beginning of ♏ and of about the same latitude with Spica ♍ that is 2 2 The same day at 5^h morning at Ballasore in the East Indies (that is at $11^h\ 20^m$ of the night preceding at London) the distance of the comet from Spica ♍ was taken 7 35 to the east It was in a right line between the Spike and the Balance and therefore was then in ♎ 26 58 with about 1 11 latitude south and after $5^h\ 40^m$ (that is at 5^h in the morning at London) it was in ♎ 28 12 with 1 16 latitude south Now by the theory the comet was then in ♎ 28 10 36 with 1 53 35 latitude south

— 9° 54 Nov 19 in Maryland
the observation of Mr Flam
distance intercepted between
the tail of the serpent of Ophiucnus and the .a δ in the south wing of Aquila
th tars λ, a δ in Bayer s tables Therefore the end of
it 34½° north Dec 11 it ascended
minating in ♑ 26° 43 with latitude
he middle of Sagitta nor did it reach
much farther terminating in ♒ 4 with latitude 42½° north nearly But these
part of the tail for
ner ky at Rome
ose to 10° above the
d towards the north
tail was 3° broad
t ward the upper end and therefore the middle thereof was 2° 15 distant

tance from either of the two was equal to the distance of the one from the other and therefore did terminate in ♓ 24 with latitude 47½° Dec 20 it reached to a contact with Scheat on its left and exactly filled up the space between the two tars in the northern foot of Andromeda, being 54 in length and therefore terminated in ♉ 19 with 35° of latitude Jan 5 it touched the star π in the breast of Andromeda on its right side and the tar μ of the girdle on its left and according to our observations, was 40° long but it was curved

very faint and very hardly to be seen but the axis thereof was exactly directed to the bright star in the eastern houlder of Auriga and therefore deviated from the opposition of the sun towards the north by an angle of 10° Lastly Feb 10 with a telescope I observed the tail 2° long for that fainter light which I poke of did *not appear through the glasses. But Ponthio* writes that on Feb he saw the tail 12° long Feb 25 the comet was without a tail and so continued till it disappeared

Now if one reflects upon the orbit described and duly considers the other

From all this it is plain that these observations agree with the theory so far as they agree with one anoth

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 ginning of Capricorn parts of the heavens but and therefore the way of the comet did very much deviate f including an arc of about 98 great circle for in the month of

t

f other by an apparent angle of above 30 ascended again from the sun declining one the comet This comet traveled over nine signs namely from the l as observed by Montenari beginning of ♊ beside the

l

motion being retarded between Nov 26 and Dec 12 to wit in the space of 15½ days it described only 40 But the motion thereof being afterwards accelerated it described near 5 a day till its motion began to be again retarded And the theory which justly corresponds with a motion so unequal and through so great a part of the heaven the theory of th

ical observati

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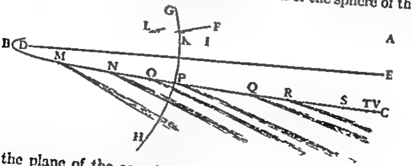
comet d

drawing its the orbit of the comet several places In this orbit DF the line of the nodes GH the intersection of the sphere of the earth s



orbit with the plane of the comet s orbit I the place of the comet Nov 4 1680 K the place of the same Nov 11 L th place Dec 12 N its place Dec 21 O its pl ing Q its place Jan 25 R its place Feb March 5 and V its place March 9 In de its place made mining the length of the tail I

Nov

it elf 11 the tail just began to show app above ½ degree long through a 10-foot telescope Nov 17 the tail was seen by Ponthio more than 15 long Nov 18 in New England the tail appeared 30 long and directly opposite to the sun extending

must be some reflecting matter in those parts where the tails of the comets are seen for otherwise since all the celestial spaces are equally illuminated by the [illegible] light no part of the heavens could appear with more splendor than an[illegible]

planets to us is a demonstration that the ether or celestial medium is [illegible] dowed with any refractive power for as to what is alleged that the fixed stars have been sometimes seen by the Egyptians environed with a coma because that has but rarely happened, it is rather to be ascribed to a casual refraction of cloud. and so the radiation and scintillation of the fixed stars to the refractions both of the eyes and air for upon laying a telescope to the eye those [illegible] immediately disappear By the tremulous agita[illegible]

perceptible refraction. But, to obviate an objection that may be made from the appearing of no tail in such comets as shine but with a faint light as if the secondary rays were then too weak to affect the eyes and for that reason it is that the tails of the fixed stars do not appear we are to consider that by the [illegible]

in light to the stars of the second magnitude and yet emitted a notable tail extending to the length of 40° 50° 60° or 70° and upward. and afterward on the 27th and 28th of January when the head appeared but as a star of the [illegible]th magnitude yet the tail (as we said above) with a light that was clearly perceptible though faint was stretched out to 6° or 7° in length and with a languishing light that was more difficult to see even to 12° and upwards But on the 9th and 10th of February when to the naked eye the head appeared no more through a telescope I viewed the tail of 2° in length. But further if the [illegible]

part. But the comet of the year 1680 December 2[illegible] 8[illegible] P.M. a London, was seen in ♓ 8° 41 with latitude north 28° 6 while the sun was in ♑ 18° 2[illegible] And the comet of the year 15[illegible] December 29[illegible] was in ♓ 8° 41 with latitude north 28° 40′ and the sun as before in about ♑ 18° 26 In both cases the situation of the earth was the same and the comet appeared in the same place of the heavens yet in the former case the tail of the comet (as well by my observations as by the observations of others) deviated from the opposition of the sun towards the north by an angle of 4½ degrees whereas in the latter there was (according to the observations of Tycho) a deviation of 21 degrees towards the south. The crumbling therefore of the heavens being thus dis-

its rays that is inversely as the square of the distance of the places from the sun Therefore since on Dec 8 when the comet was in its perihelion the distance thereof from the centre of the sun was to the distance of the earth from the same as about 6 to 1000 the sun s heat on the comet was at that time to the heat of the summer sun with us as 1 000 000 to 36 or as 28 000 to 1 But the heat of boiling water is about three times greater than the heat which dry earth acquires from the summer sun as I have tried and the heat of red hot iron (if my conjecture is right) is about three or four times greater than the heat of boiling water And therefore the heat which dry earth on the comet while in its perihelion might have received from the rays of the sun was about 2000 times greater than the heat of red hot iron But by so fierce a heat vapors and exhalations and every volatile matter must have been immediately consumed and dissipated

This comet therefore must have receive[illegible]

[illegible] ...ould retain its heat longer in the ratio of its diameter because the surface (in proportion to which it is cooled by the contact of the ambient air) is in that ratio less in respect of the quantity of the included hot matter and therefore a globe of red hot iron equal to our earth that is about 40 000 000 feet in diameter would scarcely cool in an equal number of days or in above 50 000 years But I suspect that the duration of heat may on account of some latent causes increase in a yet less ratio than that of the diameter and I should be glad that the true ratio was investigated by experiments

It is further to be observed that the comet in the month of December just after it had been heated by the sun did emit a much longer tail and much more splendid than in the month of November before when it h[illegible] arrived at its perih[illegible]

always ar[illegible]

of the sun [illegible] ...by the comet conduces to the greatness of the tail from this I think I may infer that the tail is nothing else but a very fine vapor which the head or nucleus of the comet emits by its heat

But we have had three several opinions about the tails of comets for some will have it that they are nothing else but the beams of the sun s light transmitted through the comets heads which they suppose to be transparent others that they proceed from the refraction wh[illegible]

f[illegible]

[illegible] ...ist is the opinion of such as are yet unacquainted with optics for the beams of the sun are seen in a darkened room only in consequence of the l[illegible]

ticles of dust and smoke [illegible]

reason in air impregnat[illegible]

brightness and impress [illegible]

more faint and are less e[illegible] but in the heavens where there is no matter to reflect the light they can never be seen at all Light is not seen as it is in the beam but as it is thence reflected to our eyes for vision can be produced in no other way than by rays falling upon the eyes and therefore there

times greater than water of the same weight and therefore a cylinder of air 850 feet high is of equal weight with a cylinder of water of the same breadth and but one foot high But a cylinder of air reaching to the top of the atmosphere is of equal weight with a cylinder of water about 33 feet high and therefore if from the whole cylinder of air the lower part of 850 feet high is taken ... part will be of equal weight with a cylinder of

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reckoned from the earth s surface the air is more rare than ... us in a far greater ratio than that of the whole space within the orbit of Saturn to a spherical space one inch in diameter and therefore if a sphere of our air of

at greater distances is immensely rarefied and the coma or atmosphere of

the particles of their air and vapors towards each other it may happen that the air in the celestial spaces and in the tails of comets is not so vastly rarefied yet from this computation it is plain that a very small quantity of air and vapor is abundantly sufficient to produce all the appearances of the tails of comets for that they are indeed of a very notable rarity appears from the

of their splendor Nor is the brightness of the tails of most comets ordinarily greater than that of our air an inch or two in thickness reflecting in a darkened room the light of the sunbeams let in by a hole of the window shutter

And we may pretty nearly determine the time spent during the ascent of the vapor from the comet s head to the extremity of the tail by drawing a right line from the extremity of the tail to the sun and marking the place where that right line intersects the comet s orbit for the vapor that is now in the extremity of the tail if it has ascended in a right line from the sun must have begun to rise from the head at the time when the head was in the point of

proved it remains that the phenomena of the tails of comets must be derived from some reflecting matter

And that the tails of comets do arise from their heads and tend towards the parts opposite to the sun is further confirmed from the laws which the tails observe As that lying in the planes of the comets orbits which pass through the sun they constantly deviate from the opposition of the sun towards the parts which the comets heads in their progress along these orbits have left That to a spectator placed in those planes they appear in the parts directly opposite to the sun but as the spectator recedes from those planes their deviation begins to appear and daily becomes greater That the deviation other things being equal appears less when the tail is more ol l … t of the comet as well as when the h …
sun …
Tha
devi … s
grea
things being e … the curvature is hardly to be perceived … angle of deviation is less near the comet s head but greater towards the other end of the tail and that because the convex side of the tail regards the parts from which the deviation is made and which lie in a right line drawn out infinitely from the sun through the com … l that the tails that are long …
m…

… omena of the tails of comets … ons of their heads and by no means upon the places of the heavens in which their heads are … erefore the tails of comets do … a
heads whi
mol

… heavens where all bodies … sun smoke and vapor must (as we have …) ascend from the sun and either … rest …
the … vapor had risen before and tl … least where the vapor ascends with most velo … namely near the smoking body when th …
obliquity vare …

their
arise f … motions of our clouds in part obscur … ose tails or perhaps from parts of the Milky Way which might have been confounded with and mistaken for parts of the tails of the comets as they passed by

But that the atmospheres of comets may furnish a supply of vapor great enough to fill so immense spaces we may easily understand from the rarity of our own air for the air near the surface of our earth possesses a space 850

times greater than water of the same weight and therefore a cylinder of air 850 feet high is of equal weight with a cylinder of water of the same breadth, and but one foot high But a cylinder of air reaching to the top of the atmosphere is of equal weight with a cylinder of water about 33 feet high and therefore if from the whole cylinder of air the lower part of 850 feet high is taken away the remaining upper part will be of equal weight with a cylinder of water 32 feet high and from thence (and by the hypothesis confirmed by — m he p tl far

greater ratio than that of the whole space within the orbit of Saturn to a spherical space one inch in diameter and therefore if a sphere of our air of but one inch in thickness was equally rarefied with the air at the height of one semidiameter of the earth from the earth's surface it would fill all the regions of the planets to the orb of Saturn and far beyond it Therefore since the air at greater distances is immensely rarefied and the coma or atmosphere of comets is ordinarily about ten times higher reckoning from their centres, than the surface of the nucleus and the tails rise yet higher they must therefore be exceedingly rare and though on account of the much thicker atmospheres of comets and the great gravitation of their bodies toward the sun as well as of the particles of their air and vapors towards each other it may happen that the air in the celestial spaces and in the tails of comets is not so vastly rarefied yet from this computation it is plain that a very small quantity of air and vapor is abundantly sufficient to produce all the appearances of the tails of comets for that they are indeed of a very notable rarity appears from the shining of the stars through them The atmosphere of the earth illuminated by

tails of comets likewise illuminated by the sun, without the least diminution of their splendor Nor is the brightness of the tails of most comets ordinarily greater than that of our air an inch or two in thickness reflecting in a darkened room the light of the sunbeam let in by a hole of the window shutter

And we may pretty nearly determine the time spent during the ascent of the vapor from the comet's head to the extremity of the tail by drawing a right line from the extremity of the tail to the sun and marking the place where that right line intersects the comet's orbit for the vapor that is now in the extremity of the tail, if it has ascended in a right line from the sun must have begun to rise from the head at the time when the head was in the point of intersection

the axis the orbit parallel to the length of the tail or rather (because of the curvilinear motion of the comet) diverging a little from the line or length of the tail. And by means of this principle I found that the vapor which, January was in the extremity of the tail had begun to rise from the head before

proved it remains that the phenomena of the tails of comets must be derived from some reflecting matter

And that the tails of comets do arise from their heads and tend towards the parts *opposite to the sun is further confirmed from the laws which the tails* observe As that lying in the planes of the comets orbits which pass through the sun they constantly deviate from the opposition of the sun towards the parts which the comets heads in their progress along these orbits have left That to a spectator placed in those planes they appear in the parts directly opposite to the sun but as the spectator recedes from those planes their deviation begins to appear and daily becomes greater That the deviation other things being equal appears less when the tail is more oblique to the orbit of the comet as well as when the head of the comet approaches nearer to the sun especially if the angle of deviation is estimated near the head of the comet That the tails which have no deviation appear straight but the tails which deviate are likewise bended into a certain curvature That this curvature is greater when the deviation is greater and is more sensible when the tail other things being equal is longer for in the shorter tails the curvature is hardly to be perceived That *the angle of devi* [illegible]
greater towards the other end of the [illegible]
the tail regards the parts from which [illegible]
right line drawn out infinitely from the sun through the comet s *head* And that the tails that are long and broad and shine with a stronger light appear more resplendent and more exactly defined on the convex than on the concave side Upon these accounts it is plain that the phenomena of the tails of comets depend upon the motions of their heads and by no means upon the places of the heavens in which their heads are seen and that therefore the tails of comets do not proceed from the refraction of the heavens but from their own heads which furnish the matter that forms the tail For as in our air the smoke of a heated body ascends either perpendicularly if the body is at rest or obliquely if the body is moved obliquely so in the heavens where all bodies gravitate towards the sun smoke and vapor must (as we have already said) ascend from the sun and either rise perpendicularly if the smoking body is at rest or obliquely if the body in all the progress of its motion is always leaving those places from which the upper or higher parts of the vapor had risen before and that obliquity will be least where the vapor ascends with most velocity namely near the smoking body when that is near the sun But because the obliquity varies the column of vapor will be incurvated and because the vapor in the preceding side is something more recent *that is has ascended something more late from the body* it will therefore be somewhat more dense on that side [illegible] defined I add
[illegible] of comets and
[illegible] ause they may
arise from the mutations of our air and the motions of our clouds in part obscuring those tails or perhaps from parts of the Milky Way which might have been confounded with and mistaken for parts of the tails of the comets as they passed by

But that the atmospheres of comets may furnish a supply of vapor great enough to fill so immense spaces we may easily understand from the rarity of our own air for the air near the surface of our earth possesses a space 850

times greater than water of the same weight and therefore a cylinder of air 850 feet high is of equal weight with a cylinder of water of the same breadth and but one foot high. But a cylinder of air reaching to the top of the atmosphere is of equal weight with a cylinder of water about 33 feet high and therefore if from the whole cylinder of air the lower part of 850 feet high is taken away the remaining upper part will be of equal weight with a cylinder of [illegible] d from thence (and by the hypothesis confirmed by

22 Book II I found that at the height of one [illegible] reckoned from the earth's surface the air is more rare than with us in a far greater ratio than that of the whole space within the orbit of Saturn to a spherical space one inch in diameter and therefore if a sphere of our air of but one inch in thickness was equally rarefied with the air at the height of one semidiameter of the earth from the earth's surface it would fill all the regions of the planets to the orb of Saturn and far beyond it Therefore since the air at greater distances is immensely rarefied and the coma or atmosphere of comets is ordinarily about ten times higher reckoning from their centres than the surface of the nucleus and the tails rise yet higher they must therefore be exceedingly rare and though on account of the much thicker atmospheres of comets and the great gravitation of their bodies toward the sun as well as of the particles of their air and vapors towards each other it may happen that the air in the celestial spaces and in the tails of comets is not so vastly rarefied yet from this computation it is plain that a very small quantity of air and [illegible]

of their splendor Nor is the brightness of the tails of most comets ordinarily greater than that of our air an inch or two in thickness reflecting in a dark

that right line intersects the comet's orbit for the vapor that is now in the extremity of the tail if it has ascended in a right line from the sun must have begun to rise from the head at the time when the head was in the point of intersection It is true the vapor does not rise in a right line from the sun but

December 11 and therefore had spent in its whole ascent 45 days but that the whole tail which appeared on December 10 had finished its ascent in the space of the two days then elapsed from the time of the comet s being in its perihelion The vapor therefore about the beginning and in the neighborhood of the sun rose with the greatest velocity and afterwards continued to ascend with a motion constantly retarded by its own gravity and the higher it ascended the more it added to the length of the tail and while the tail continued to be seen it was made up of almost all that vapor which had risen since the time of the comet s being in its perihelion nor did that part of the vapor which had risen first and which formed the extremity of the tail cease to appear till its too great distance as well from the sun from which it received its light as from our eyes rendered it invisible Whence also it is that the tails of other comets which are short do not rise from their heads with a swift and continued motion and soon after disappear but are permanent and lasting columns of vapors and exhalations which ascending from the heads with a slow motion of many days and partaking of the motion of the heads which they had from the beginning continue to go along together with them throu h the heavens From this again we have another argument proving the cele l spaces to be free and without ... l

Kepler ascribes the ascent of the tails of the comets to the atmospheres of their heads and their direction towards the ... osite to the sun to the action of ... matter of the comets tails and ... ppose that in so free spaces so ... atter as that of the ether may yield to the action of the rays of the sun s light though those rays are not able sensibly to move the gro s substances in our parts which are clogged with so palpable a resistance An other author thinks that there may be a sort of particles of matter endowed with a principle of levity as well as others are ... the matter of ... from the su ... terrestrial b ... bodies and therefore can be neither more nor less in the same quantity of matter I am inclined to believe that this ascent may rather proceed from the rarefaction of the matter of the comets tails The ascent of smoke in a chimney is due to the impulse of the air with which it is entangled The air rarefied by heat ascends because its specific gravity is diminished and in its ascent carries along with it the smoke which floats in it and why may not the tail of a comet rise from the sun after the same manner? For the sun s rays do not act upon the mediums which they pervade otherwise than by reflection and refraction and those reflecting particle heated by this action heat the matter of the ether which is involved with them That matter is rarefied by the heat which it acquires and becau e by this rarefaction the specific gravity with which it tended towards the sun before is diminished it will ascend therefrom and carry along with it the reflecting particles of which the tail of the comet is compo ed But the ascent of the vapors is further promoted by their circumgyration about the sun in con e quence thereof they endeavor to recede from the sun while the sun s atmos-

phere and the other matter of the heavens are either altogether quiescent or are only moved with a slower circumgyration derived from the rotation of the
the tails of the comets in the
bent into a greater curvature

must always accompany the head and freely a
tation of the vapors towards the sun can no more force the tails to abandon the heads and descend to the sun than the gravitation of the heads can oblige them to fall from the tail They must by their common gravity either fall together toward the sun or be retarded together in their common ascent

common gravitation

The tails therefore that rise in the perihelian positions of the comet will go along with their heads into far remote parts and together with the heads will either return again from thence to us after a long course of years or rather will be there rarefied and by degrees quite vanish away for afterward in the

comets are broader at their upper extremity than near their heads And it
be
and
heir
atmosphere for as the seas are absolutely necessary to the constitution of our earth that from them the sun by its heat may exhale a sufficient quantity of vapors which being gathered together into clouds may drop down in rain for watering of the earth and for the production and nourishment of vegetables or being condensed with cold on the tops of mountains (as some philosophers with reason judge) may run down in springs and rivers so for the conservation of the seas and fluids of the planets comets seem to be required that from their exhalations and vapors condensed the wastes of the planetary

The atmospheres of comets in their descent towards the sun by running out into the tails are spent and diminished and become narrower at least on that side which regards the sun and in receding from the sun h - h ; run out into the tails they are ar n enl
their ...
heate ... longest and most re-
splent ... aps at the same time the nuclei are environed with a denser and blacker smoke in the lowermost parts of their atmosphere for smoke that is raised by a great and intense heat is commonly the denser and blacker Thus the head of that comet which we have been describing at equal distances both from the sun and from the earth appeared darker after it had passed by its perihelion than it did before for in the month of December it was commonly compared with the stars of the third magnitude but in November with those of the first or second and such as saw both ... have described the first as of anoth
N...

degrees long ... which have come into my hands) writes that in the month of December when the tail appeared of the greatest bulk and splendor the head was but small and far less than that which was seen in the month of November before sun ...
cause of the appe...
quantity of ... gradually spent

And for th... that the heads of other comets which ... put forth tails of the greatest bulk and ... and small For in Brazil March 5 ... a comet near the horizon and towa... with a he... scarcely to be discerned but ...

... length from the ...
... to the horizon ...
only th...
creasi...
up on ... from ...
with a ... splendor th...
h...

the comet of 1 ...
appearing in th... *and obscure* (as that of 1680) *but the* ... *of its tail was very bright and like a huge fiery beam stretched out in a direction between the east and north* as Hevelcke has it al... from Simeon the monk of Durham This comet appeared in the beginning of February about the evening and towards the southwest part of heaven from this and from the position of the tail we infer that the head was near the sun Matthew Paris says *It was distant from the sun by about a cubit from three o clock* (rather six) *till nine putting forth a long tail* Such al... was that re-

and nt comet described by Aristotle *Meteorology* i 6 *The head whereof*

or at least was hid under the

be for having left the sun but

f the head

erwards

ead of)

th moon.

tne

are

in

in that inquiry

PROPOSITION 42 PROBLEM 22

To correct a comet s orbit found as above

OPERATION 1 Assume that position of the plane of the orbit which was determined according to the preceding Proposition and select three places of the comet deduced from very accurate observations and at great distances one from the other Then suppose A to represent the time between the first

operations the three true places of the comet in that assumed plane of the

first observation and the second and E the area between the second and third and let T represent the whole time in which the whole area D+E should be described with the velocity of the comet found by Prop 16 Book I.

t

The atmospheres of comets in their descent towards the sun by running out into the tails are spent and diminished and become narrower at least on that side which regards the sun and in receding from the sun when they less run out into the tails they are again enlarged if Hewelcke has justly marked their appearances But they are seen least of all just after they have been most heated by the sun and on that account then emit the longest and most resplendent tails and perhaps at the same time the nuclei are environed with a denser and blacker smoke in the lowermost parts of their atmosphere for smoke that is raised by a great and intense heat is commonly the denser and blacker Thus the head of that comet which we have been describing at equal distances both from the sun and from the earth appeared darker after it had passed by its perihelion than it did before for in the month of December it was commonly compared with the stars of the third magnitude but in November with those of the first or second [illegible]
have [illegible]
N [illegible]
p [illegible] and at th[illegible]t time it shone with greater bright[illegible] November 20 [illegible]
observed [illegible] its tail being then 2 degrees lo[illegible] (by letters which have come into my hands) writes that in the month of December when the tail appeared of the greatest bulk and splendor the head was but small and far less than that which was seen in the month of November before sun rising and conjecturing at the cause of the appearance he judged it to proceed from the existence of a greater quantity of matter in the head at first which was afterwards gradually spent

And for the same reason I find that the heads of other comets which did put forth tails of the greatest bulk and splendor have appeared but obscure and small For in Brazil March 5 1668 N S 7[h] P M Valentin Estancel saw a comet near the horizon and towards the southwest with a head so small as scarcely to be discerned but with a tail above measure spl[illegible]
reflection there[illegible]
it looked like [illegible]
south almost [illegible] But this excessive splendor continued only three days decreasing apace afterwards and while the splendor was decreasing the bulk of the tail increased also in Portugal it is said to have taken up one quarter of the heavens that is 45 degrees extending from west to east with a very notable splendor though the whole tail was not seen [illegible]
because the he[illegible]
the [illegible]
ther [illegible] been very near to it in its perihelion as the comet of 1680 was And we read in the *Saxon Chronicle* of a like comet appearing in the year 1106 *the star whereof was small and obscure* (as that of 1680) *but the splendor of its tail was very bright and like a huge fiery beam stretched out in a direction between the east and north* as Hewelcke has it also from Simeon the monk of Durham This comet appeared in the [illegible]
F[illegible]
th[illegible]
N[illegible] *from the sun by about a cubit from three o clock* (rather six) *till nine putting forth a long tail* Such also was that re-

shall hereafter call γ was in ♈ 28° 30 15 with ° 8 58 north latitude the second tar of Aries was in ♈ 29° 1, 18 with 8° 28 16 north latitude another star of the seventh magnitude which I call A was in ♈ 28° 24 45 with 8° 28 33 north latitude The comet Feb 7ᵈ 7ʰ 30ᵐ at Paris (that is Feb ,ᵈ 8 3,ᵐ at Danzig) o s made a triangle with those tars γ and A which was right angled in γ and the di tance of the comet from the tar γ was equal to the di tance of the stars γ and A that is 1 19 46 of a great circle and therefore in the parallel of the latitude of the star γ it was 1 20 26 Therefore if from the longitude of the tar γ there be subtracted the longitude 1 20' 26 there will remain the longitude of the comet ♈ 2° 9 49 M Auzout from this observation of hi placed the comet in ♈ 27° 0 nearly and by the drawing in which Dr Hooke delineated its motion it was then in ♈ 26° 59 24 I place it in ♈ 27° 4 46 taking the middle between the two

(

omet at made it γ being

equal to the difference of the longitude o

February 2ᵈ , 30ᵐ at London that i February 22ᵈ 8ʰ 46ᵐ at Danzig the di tance of the comet from the tar A according to Dr Hooke s observation as was delineated by himself in a scheme and also by the observations of M Auzout delineated in like manner by M Petit was a fifth part of the distance between the tar A and the first star of Aries or 15 57 and the distance of the comet from a right line joining the tar A and the first of Aries was a fourth part of the same fifth part that i 4 and therefore the comet was in ♈ 28° 29 46 with 8° 12 36 north latitude

March 1 ʰ 0ᵐ at London that i March 1 8 16ᵐ at Danzig the comet was observed near the second star in Aries the di tance between them being to the di tance between the first and second stars in Aries that is to 1 33 as 4 to 45 according to M C mies And therefore the di vas 8 16 according to Dr ling a mean between both t had gone beyond the second star of Aries about a fourth or a fifth part of the space th t mmonly went over in a day to wit about 1 3 (in which he agrees

with 8° 36 26 north latitude

March , 30ᵐ at Paris that is March 7ʰ 3 ᵐ at Danzig from the

was 45 or 46 or taking a mean quantity 45 30' and therefore the comet as

vards the end of the motion and Hevelius in the drawing of M Auzout s observations which he constructed himself corrected this irregular curvature

OPER 3 Retaining the long

found in this new

passing through them and the two areas of the same described between the observation which call δ and ϵ and let τ be the whole time in which the whole area $\delta+\epsilon$ should be described

Then taking C to 1 as A to B and G to 1 as D to E and g to 1 as d to e and γ to 1 as δ to ϵ let S be the true time between the first observation and the third and observing well the signs $+$ and $-$ let such numbers m and n be found out as will make $2G-2C=mG-mg+nG-n\gamma$ and $2T-2S=mT-mt+nT-n\tau$ And if in the first operation I represents the inclination of the plane of the orbit to the plane of the ecliptic and K the longitude of either node then $I+nQ$ will be the true inclination of the plane of the orbit to the plane of the ecliptic and $K+mP$ the true longitude of the node And lastly if in the first second and third operations the quantities R r and ρ represent the parameters of the orbit and the quantities $\frac{1}{L}$ $\frac{1}{l}$ $\frac{1}{\gamma}$ the transverse diameters of the same then $R+mr-mR+n\rho-nR$ will be the true parameter and $\frac{1}{L+ml-mL+n\lambda-nL}$ will be the true transverse

times of the revolutions of and the transverse diameters of their orbits cannot be accurately enough determined but by comparing comets together which appear at different times If after equal intervals of time several comets are found to described the same orbit we may then the same comet revolved in the s revolutions the transverse diamet orbits will be given and from those diameters the elliptic orbits themselves will be

To this purpo those orbits to phenomena as parabolic orbit of the comet of the year 1680 which I compared above with the observations but likewise from that of the notable comet which appeared in the year 1664 and 1665 and was observed by Hevelcke who from his own observations calculated the longitudes and latitudes thereof though with little accuracy But from the same observations Dr Halley did again compute its places and from those new places determined its orbit finding its ascending node in ♊ 21 13 55 the inclination of the orbit to the plane of the ecliptic 21 18 40 the distance of its perihelion from the node estimated in the comet s orbit 49 27 30 its perihelion in ♌ 8 40 30 with heliocentric latitude south 16 01 45 the comet to have been in its perihelion November 24d 11h 52m p.m. equal time at London or 13h 8m at Danzig O S and that the latus rectum of the parabola was 410 286 of such parts as the sun

In February the beginning of the year 1665 the first star of Aries which I

and so made the latitude of the comet 8° 55′ 30″. And by further correcting this irregularity the latitude may become 8° 56′ or 8° [illegible]

This comet was also seen March 9 and at that time its place must have been in ♉ 0° 18′ with 9° 3½′ north latitude nearly

This comet appeared for three months in which space of time it traveled over almost six signs and in one of the days described almost [illegible]0 degrees Its course did very much deviate from a great circle bending towards the north and its motion towards the end from retrograde became direct and notwithstanding that its course was so uncommon yet by the table it appears that the theory from beginning to end agrees with the observations no less accurately than the theories of the planets usually do with the observations of them but we are to subtract about 2′ when the comet was swiftest which we may effect by taking off 12″ from the angle between the ascending node and the perihelion or by making that angle 49° 2[illegible] 18″ The annual parallax of both [illegible] was very conspicuous and by its quan[illegible] the earth in the earth's orbit

[illegible]he motion of that comet which in the year 1683 appeared retrograde in an orbit whose plane contained almost a right angle with the plane of the ecliptic and whose ascending node (by the computation of Dr Halley) was in ♍ 23° 23′ the inclination of its orbit to the ecliptic 83° 11′ its perihelion in ♊ 25° [illegible]9′ 30″ its perihelion distance from the sun 56020 of such parts as the radius of the earth's orbit contains 100 000 and the time of its perihelion was July 2d 3h 50m And the places thereof computed by Dr Halley in this orbit are compared with the places observed [illegible] wing table

[illegible] the motion of that retrograde comet [illegible]cending node of this (by Dr Halley's

1683 Equatorial time				Sun place			Comet longitude computed			Latitude north computed			Comet longitude observed			Latitude north observed			Difference longitude		Difference latitude	
	d	h	m																			
July	13	12	55	♌ 1	02	30	♎ 13	05	42	29	28	13	♎ 13	6	4[illegible]	29	[illegible]	[illegible]0	+1	00	+0	0[illegible]
	15	11	15		53	1[illegible]	11	37	48	29	34	0	11	39	43	29	34	50	+1	55	+0	50
	17	10	[illegible]0	4	4	4[illegible]	10	7	6	29	33	30	10	8	40	29	34	0	+1	34	+0	30
	23	13	40	10	38	21	5	10	2[illegible]	28	51	4[illegible]	5	11	30	28	[illegible]0	[illegible]	+1	03	−1	14
	[illegible]5	14	5	1	3[illegible]	2[illegible]	3	27	53	24	4	47	3	[illegible]	0	28	23	40	−0	53	−1	7
	31	9	4[illegible]	18	09	22	♊ 27	55	3	26	22	[illegible]	♊ 2[illegible]	[illegible]4	4	[illegible]	22	[illegible]5	−0	3[illegible]	−0	2[illegible]
	31	14	55	18	1	53	27	41	7	26	16	5[illegible]	[illegible]	41	8	26	14	50	+0	1	−	7
Aug	2	14	56	[illegible]0	1	16	25	[illegible]9	32	[illegible]5	16	1	[illegible]	28	4	[illegible]5	17	28	−0	4[illegible]	+1	9
	4	10	49	22	0[illegible]	50	[illegible]	[illegible]	20	24	10	49	23	16	5[illegible]	24	1	19	−1	[illegible]5	+1	30
	6	10	9	23	56	45	[illegible]0	4[illegible]	23	22	47	5	20	40	32	22	49	5	−1	51	+[illegible]	0
	9	10	26	26	[illegible]0	5[illegible]	16	7	5[illegible]	20	6	37	16		[illegible]5	20	6	10	−		−0	[illegible]
	15	14	1	♍	47	13	3	30	48	11	3[illegible]	33	3	[illegible]	1	11	3[illegible]	1	−4	30	−5	32
	16	15	10	3	48	[illegible]	0	43	[illegible]	9	34	16	0	41	55	9	34	13	−1	1	−0	3
	18	1	44	5	45	33	♉ [illegible]4	5[illegible]	53		11	15	♉ 4	49	5	5	9	11	−3	45	−	4
										South						South						
	22	14	44	9	3[illegible]	4[illegible]	11	7	14	5	16	[illegible]8	11	07	1	5	16	58	−0	[illegible]	−0	3
	23	15	[illegible]	10	36	48	7	2	18	8	17	9	7	1	17	8	16	41	−1	1	−0	28
	26	16	2	13	31	10	♈ 4	45	31	16	38	0	♈ 4	44	00	16	38	[illegible]0	−1	31	+0	20

Apparent time at Dantzig	The observed distances of the comet from		The observed places	The places computed in the orbit
December d h m				
3 18 29½	The Lion's heart	46 24 20	Long ♎ 7 01 00	♎ 7 1 29
	The Virgin's spike	22 52 10	Lat S 21 39 0	21 38 50
4 18 1½	The Lion's heart	46 2 45	Long ♎ 6 15 0	♎ 6 16 5
	The Virgin's spike	23 52 40	Lat S 22 24 0	22 24 0
7 17 48	The Lion's heart	44 48 0	Long ♎ 3 6 0	♎ 3 7 33
	The Virgin's spike	27 56 40	Lat S 25 22 0	25 21 40
17 14 43	The Lion's heart	53 15 15	Long ♌ 2 56 0	♌ 2 56 0
	Orion's right shoulder	45 43 30	Lat S 49 25 0	49 25 0
19 9 25	Procyon	35 13 50	Long ♊ 28 40 30	♊ 28 43 0
	Bright star of Whale's jaw	52 56 0	Lat S 45 48 0	45 46 0
20 9 53½	Procyon	40 49 0	Long ♊ 13 03 0	♊ 13 5 0
	Bright star of Whale's jaw	40 04 0	Lat S 39 54 0	39 53 0
21 9 9½	Orion's right shoulder	26 21 25	Long ♊ 2 16 0	♊ 2 18 30
	Bright star of Whale's jaw	29 28 0	Lat S 33 41 0	33 39 40
22 9 0	Orion's right shoulder	29 47 0	Long ♉ 24 24 0	♉ 24 27 0
	Bright star of Whale's jaw	20 29 30	Lat S 27 45 0	27 46 0
26 7 58	The bright star of Aries	23 20 0	Long ♉ 9 0 0	♉ 9 2 28
	Aldebaran	26 44 0	Lat S 12 36 0	12 34 13
27 6 45	The bright star of Aries	20 45 0	Long ♉ 7 5 40	♉ 7 8 45
	Aldebaran	28 10 0	Lat S 10 23 0	10 23 13
28 7 39	The bright star of Aries	18 29 0	Long ♉ 5 24 45	♉ 5 27 52
	Palilicium	29 37 0	Lat S 8 22 50	8 23 37
31 6 45	Andromeda's girdle	30 48 10	Long ♉ 2 7 40	♉ 2 8 20
	Palilicium	32 53 30	Lat S 4 13 0	4 16 25
Jan. 1665 7 7 37½	Andromeda's girdle	25 11 0	Long ♈ 28 24 47	♈ 28 24 0
	Palilicium	37 12 25	Lat N 0 54 0	0 53 0
13 7 0	Andromeda's head	28 7 10	Long ♈ 27 6 54	♈ 27 6 39
	Palilicium	38 55 20	Lat N 3 6 50	3 7 40
24 7 29	Andromeda's girdle	20 32 15	Long ♈ 26 29 15	♈ 26 28 50
	Palilicium	40 5 0	Lat N 5 25 0	5 26 0
February 7 8 37			Long ♈ 27 4 46	♈ 27 24 55
			Lat N 7 3 29	7 3 15
22 8 46			Long ♈ 28 29 46	♈ 28 29 58
			Lat N 8 12 36	8 10 25
March 1 8 16			Long ♈ 29 18 15	♈ 29 18 20
			Lat N 8 36 0	8 36 12
7 8 37			Long ♉ 0 2 48	♉ 0 2 42
			Lat N 8 56 30	8 56 56

From these examples it is abundantly evident that the motions of comets ... urately represented by our theory than the motions of the plan ... m an of th

in ♉ 20°

... from these ... this comet But these things are to be ... pace of 75 years the same comet shall ... other comets seem to ascend to greater heights and to require a longer time to perform their revolutions

But because of the great number of comets of the great distance of their aphelions from the sun and of the slowness of their motions in the aphelions they will by their mutual gravitations disturb each other so that their eccentricities and the times of their revolutions will be sometimes a little increased and sometimes diminished Therefore we are not to expect that the same comet will return exactly in the same orbit and in the same periodic times it will be sufficient if we find the changes no greater than may arise from the causes just spoken of

And hence a reason may be assigned why comets are not comprehended ...

ance from their mutual gravitations and hence it is that the comets which descend the lowest and therefore move the slowest in their aphelions ought also to ascend the highest

The comet which appeared in the year 1680 was in its perihelion less distant from the sun than by a sixth part of the sun's diameter and because of its extreme velocity in that proximity to the sun and some density of the sun's atmosphere it must have suffered some resistance and retardation and therefore ...

computation) was in ♉ 21 16 30 the inclination of its orbit to the plane of the ecliptic 17 56 00 its perihelion in ≏ 2 52 50 t north from the sun 58 328 part of 1

Apparent time			Sun's place	Comet's longitude computed	Latitude north computed	Comet's longitude observed	Latitude north observed	Difference longitude	Difference latitude
	d h	m							
Aug	19 16	38	♍ 7 0 7	♌ 18 14 28	25 50 7	♌ 18 14 40	25 49 55	−0 12	+0 12
	20 15	38	7 55 [illegible]	24 46 23	26 14 42	24 46 22	26 12 52	+0 1	+1 [illegible]
	21 8	21	8 36 14	29 37 15	26 20 3	29 38 02	26 17 37	−0 47	+2 26
	22 8	8	[illegible] 33 55	♍ 6 29 53	26 8 42	♍ 6 30 3	26 7 12	−0 10	+1 [illegible]
	29 08	20	16 22 40	≏ 12 37 54	18 37 47	≏ 12 37 49	18 34 5	+0 5	+3 42
	30 7	45	17 19 41	15 36 1	17 26 43	15 35 18	17 27 17	+0 43	−0 34
Sept	1 7	33	19 16 9	20 30 53	15 13 0	20 27 4	15 9 49	+3 49	+3 11
	4 7	22	22 11 28	25 42 0	12 23 48	25 40 58	12 22 0	+1	+1 48
	5 7	32	23 10 29	27 0 46	11 33 08	26 59 24	11 33 51	+1 2	−0 43
	8 7	16	26 5 58	29 58 44	[illegible] 26 46	29 58 45	9 26 43	−0 1	+0 3
	9 7	26	27 5 9	♏ 0 44 10	8 49 10	♏ 0 44 4	8 48 25	+0 6	+0 45

This theory is also confirmed by the retrograde motion of the comet that appeared in the year 1723 The ascending node of this comet (according to the computation of Mr Bradley Savilian Professor of Astronomy at Oxford) was in ♈ 14 16 the inclination of the orbit to the plane of the ecliptic 49 59 Its perihelion was in ♉ 12 15 20 its perihelian distan e f om parts of which the radius of the earth s orbit con time of its perihelion September 16ᵈ 16ʰ 10ᵐ puted in this orbit by M R d himself his uncle Mr Pc

1723 Equatorial time				Comet's longitude observed	Latitude north observed	Comet's longitude computed	Latitude north computed	Difference longitude	Difference latitude
	d	h	m						
Oct	9	8	[illegible]	♒ 7 22 15	5 2 [illegible]	♒ 7 21 26	5 2 47	+49	−47
	10	6	21	[illegible] 41 12	7 44 13	6 41 42	7 43 18	−50	+55
	12	7	22	[illegible] 39 58	11 55 0	5 40 19	11 54 55	−21	+ 5
	14	8	57	4 59 49	14 43 50	5 0 37	14 44 1	−48	−11
	15	6	35	4 47 41	15 40 51	4 47 45	15 40 55	− 4	− 4
	21	6	22	4 2 32	19 41 49	4 2 21	19 42 3	+11	−14
	22	6	24	3 59 2	20 8 12	3 59 10	20 8 17	− 8	− 5
	24	8	2	3 55 29	20 55 18	3 55 11	20 55 9	+18	+ 9
	[illegible]	8	56	3 56 17	22 20 27	3 56 42	22 20 10	−25	+17
	30	6	20	3 58 9	22 32 28	3 58 17	22 32 12	− 8	+16
Nov	5	5	53	4 16 30	23 38 33	4 [illegible] 23	23 38 7	+ 7	+26
	8	7	[illegible]	4 29 36	24 4 30	4 29 54	24 4 40	−18	−10
	14	[illegible]	20	5 2 16	24 48 46	5 2 51	24 48 16	−35	+30
	20	7	45	5 42 20	25 24 45	5 43 13	25 25 17	−53	−3
Dec	7	6	45	[illegible] 4 13	26 54 [illegible]	8 3 55	26 53 42	+18	+36

GENERAL SCHOLIUM

The hypothesis of vortices is pressed with many difficulties. That every planet by a radius drawn to the sun may describe areas proportional to the times of description, the periodic times of the several parts of the vortices should observe the square of their distances from the sun; but that the periodic times of the planets may obtain the $\frac{3}{2}$th power of their distances from the sun, the periodic times of the parts of the vortex ought to be as the $\frac{3}{2}$th power of their distances. That the smaller vortices may maintain their lesser revolutions

and undisturbed

parts of the sun's

planets about their

axes, which ought to correspond with the motions of these vortices, recede far from all these proportions. The motions of the comets are exceedingly regular, are governed by the same laws with the motions of the planets, and can by no means be accounted for by the hypothesis of vortices; for comets are carried

atmosphere, in these spaces where there is no air to resist their motions, all bodies will move with the greatest freedom; and the planets and comets will constantly pursue their revolutions in orbits given in kind and position, according to the laws above explained; but though these bodies may, indeed, continue in their orbits by the mere laws of gravity, yet they could by no means have at first derived the regular position of the orbits themselves from those laws.

The six primary planets are revolved about the sun in circles concentric with the sun, and with motions directed towards the same parts, and almost in the same plane. Ten moons are revolved about the earth, Jupiter, and Saturn, in circles concentric with them, with the same direction of motion, and nearly in the planes of the orbits of those planets; but it is not to be conceived that mere mechanical causes could give birth to so many regular motions, since the comets range over all parts of the heavens in very eccentric orbits; for by that kind of motion they pass easily through the orbs of the planets,

may pass for new stars Of this kind are such fixed stars as appear on a sudden and shine with a wonderful brightness at first and afterwards vanish by little and little Such was that star which appeared in Cassiopeia's Chair which Cornelis Gemma did not see upon the 8th of November 1572 though he was observing that part of the heavens upon that very night and the sky was perfectly serene but the next night (November 9) he saw it shining much brighter than any of the fixed stars and scarcely inferior to Venus in splendor Tycho Brahe saw it upon the 11th of the same month when it shone with the greatest lustre and from that time he observed it to decay by little and little and in 16 months time it entirely disappeared In the month of November when it first appeared [illegible] Dec[illegible]hat of Venus In the month of of J[illegible]was now become equal to that [illegible] January 1573 it was less than Jupiter and greater than Sirius and about the end of February and the beginning of March became equal to that star In the months of April and May it was equal to a star of the second magnitude in June July and August to a star of the third magnitude in September October and November to those of the fourth magnitude in December and January 1574 to those of the fifth in February to those of the s[illegible] v[illegible]ginning a[illegible]yellow [illegible]May it t[illegible] kind of dusky whiteness like that we observe in Saturn and that color it retained ever after but growing always more and more obscure Such also was the star in the right foot of Serpentarius which Kepler's scholars first observed September 30 O.S. 1604 with a light exceeding that of Jupiter though the night before it was not to be seen and from that time it decreased by little and little and in 15 or 16 months entirely disappeared Such a new star appearing with an unusual splendor is said to have moved Hipparchus to observe and make a catalogue of the fixed stars As to those fixed stars that appear and disappear by turns and increase slowly and by degrees and scarcely ever exceed the stars of the third magnitude they seem to be of another kind which revolve about their axes and having a light and a dark side show those two different sides by turns The vapors which arise from the sun the fixed stars and the tails of the comets may meet at last with and fall into the atmospheres of the planets by their gravity and there be condensed and turned into water and humid spirits and from thence by a slow heat pass gradually into the form of salts and sulphurs and tinctures and mud and clay and sand and stones and coral and other terrestrial substances

GENERAL SCHOLIUM

And the same argument must apply to the celestial spaces above the earth's atmosphere in these spaces where there is no air to resist their motions all bodies will move with the greatest freedom and the planets and comets will constantly pursue their revolutions in orbits given in kind and position according to the laws above explained but though these bodies may indeed continue in their orbits by the mere laws of gravity yet they could by no means have at first derived the regular position of the orbits themselves from those laws

The six primary planets are revolved about the sun in circles concentric with the sun and with motions directed towards the same parts and almost in the same plane Ten moons are revolved about the earth Jupiter and Saturn in circles concentric with them with the same direction of motion and nearly in the planes of the orbits of those planets but it is not to be conceived that mere mechanical causes could give birth to so many regular motions since the comets range over all parts of the heavens in very eccentric orbit for by that kind of motion they pass easily through the orbs of the planets and with great rapidity and in their aphelions where they move the lowest

fixed stars are the centres of other like systems the e being formed by the like wise counsel must be all subject to the dominion of One especially since the

light of the fixed stars is of
every system light passes in
fixed stars should by their
systems at immense distan

This Being gov
all and on
τωρ or Uni
ants
who
God eternal infinite absolutely perfect but a being however perfect without dominion cannot be said to be Lord God for we say my God your God the God of Israel the God of Gods and Lord of Lords but we do not say my Eternal your Eternal the Eternal of Israel the Eternal of Gods we do not say my Infinite or my Perfect these are titles wh
respect to servants The
is God It is the dominion
supreme or imaginary d is a true supreme or imaginary God And from his true dominion it follows that the true God is a living intelligent and powerful Being and from his other perfections that he is supreme or most perfect He is eternal and infinite omnipotent and omniscient that is his duration reaches from eternity to eternity his presence from infinity to infinity he governs all things and knows all things that
is not eternity
but he endure
and by existi he constitutes duration and space Since every particle of space is *always* and every indivisible moment of duration is *everywhere* certainly the Maker and L
ies and
There
duration coexistent parts in space but neither the one nor the other in the person of a man or his thinking principle and much less can they be found in the thinking substance of God Every man so far as he is a thing that has perception is one and the same man during his whole life in all and each of his organs of sense God is the same God always and everywhere He is omnipresent not *virtually* only but also *substantially* for virtue cannot subsist without substance In
is necessarily and by the same necessity he

[1]Dr Pocock derives the Latin

in Deuteronomy 4 39 and 10 14 David in Psalms 139 7 8 9 Solomon in I Kings 8 27 Job 22 12 13 14 Jeremiah 23.23 24 The idolaters supposed the sun moon, and stars the souls of men and other parts of the world to be parts of the Supreme God and therefore to be worshipped but erroneously

A ... here Whence also he is all similar all eye all ear all
...ad to act but in a man
...al in a manner utterly
...o have we no idea of the
...rstands all things He is
...herefore neither be seen
nor heard nor touched nor ought he to be worshipped under the representation

smell only the smells and taste the savors but their inward substances are

to see to speak to laugh to love to hate to desire to give to receive to rejoice to be angry to fight to frame to work to build for all our notions of

natural philosophy

Hitherto we have explained the phenomena of the heavens and of our sea by the *power of gravity* but have not yet assigned the cause of this power This is certain that it must proceed from a cause that penetrates to the very centres of the sun and planets without suffering the least diminution of its force that operates not according to the quantity of the surfaces of the particles upon which it acts (as mechanical causes used to do) but according to the quantity of the solid matter which they contain and propagates its virtue on all sides to immense distances decreasing always as the inverse square of the distances Gravitation towards the sun is made up out of the gravitations towards the several particles of which the body of the sun is composed and in receding from the sun decreases accurately as the inverse square of the distances as f

of the aph

comets if

to discove the cause of those properties of gravity from phenomena and I

does really exist and act according to the laws which we have explained and abundantly serves to account for all the motions of the celestial bodies and of our sea

And no ... e m ... gh ... dd

1

f ... ted and heats bodies and all sensation is excited and the members of animal bodies move at the command of the will namely by the vibrations of this spirit mutually propagated along the solid filaments of the nerves from the outward organs of sense to the brain and from the brain into the muscles But these are things that cannot be explained in few words nor are we furnished with that sufficiency of experiments which is required to an accurate determination and demonstration of the laws by which this electric and elastic spirit operates

OPTICS

CONTENTS

ADVERTISEMENT TO FIRST EDITION

[illegible] d course about light was written at the desire of some [illegible] s [illegible] of [illegible] h [illegible] d which were since put together out of [illegible] d [illegible]

prevailed upon me If any other papers writ on this [illegible] hands they are imperfect and were perhaps written before I had tried all the experiments here set down and fully sati fied myself about the laws of refractions and composition of colours I have here publi hed what I think proper to come abroad wishing that it may not be translated into another language without my consent

The crowns of colours which sometimes appear about the Sun and Moon I have endeavoured to give an account of but for want of sufficient observations leave that matter to be further examined The subject of the third book I have also left imperfect not having tried all the experiments which I intended when I was about these matters nor repeated ome of those which I did try until I had sati fied myself about all their circum tances To communicate what I have tried and leave the rest to others for further enquiry is all my design in publi hing these papers

In a letter written to Mr Le bnitz in the year 16 9 and published by Dr Wallis I mentioned a method by which I had found some general theorems about quarin curvilinear figures or comparing them with the conic sections or other the simplest figures with which they may be compared And some years ago I lent out a manu cript containing such theorems and having since met with some things copied out of it I have on this occasion made it public prefixin to it an Introduction and subjoining a Scholium concerning that method And I have joined with it another small tract concerning the curvilinea figures of the second kind hich was al o written many years ago and made kno n to some friends who have solicited the making it public

I N

April 1 1 04

ADVERTISEMENT TO SECOND EDITION

In this Second Edition of these *Opt* . . .
p . . .
a . . .
d . . . ty for an essential property of bodies . . . questions And to shew that I am not yet satisfied about it for want of experiments I have added one question concerning its cause choosing to propose it by way of a question because

July 16 1717

I N

BOOK ONE

Part I

My design in this book is not to explain the properties of light by hypotheses but to propose and prove them by reason and experiments in order to which I shall premise the following definitions and axioms

DEFINITIONS

DEFINITION I

By the rays of light I understand its least parts and those as well successive in the same lines as contemporary in several lines

For it is manifest that light consists of part both successive and contemporary because in the same place you may top that which comes one moment … ft and in the same time you may
… ht which
… light or
… light or
propagated alone or do or suffer any thing alone which the rest of the light doth not or suffers not I call a *ray* of light

DEFINITION II

…

the luminous body to the body illuminated and the refraction of those rays to be the bendin or breaking of those lines in their passing out of one medium into another And thus may rays and refractions be considered if light be propagated in an instant But by an argument taken from the equations of the times of the eclipses of Jupiter s satellites it seems that light is propagated in time pendin n its passage from the Sun to us about seven minutes of time and therefore I have chosen to define *rays* and *refractions* in such general terms as may agree to light in both cases

DEFINITION III

…

As if light pass out of a glass into air and by being inclined more and more to the common surface of the glass and air begins at length to be totally reflected

by th
cop
mos

DEFINITION IV

The angle of incidence is that angle which the line described by the incident ray contains with the perpendicular to the reflecting or refracting surface at the point of incidence

DEFINITION V

The angle of reflexion or refraction is the angle which the line described by the reflected or refracted ray containeth with the perpendicular to the reflecting or refracting surface at the point of incidence

DEFINITION VI

The sines of incidence reflexion and refraction are the sines of the angles of incidence reflexion and refraction

DEFINITION VII

The light whose rays are all alike refrangible I call Simple Homogeneal and Similar and that whose rays are some more r f ... han others I call compound

I would affirm it so in all

those their other p

DEFINITION VIII

The colours of homogeneal lights I call primary homogeneal *and* simple *and those of heterogeneal lights* heterogeneal *and* compound

For these are always compounded of the colours of homogeneal lights as will appear in the following discourse

AXIOMS

AXIOM I

The angles of reflexion and refraction lie in one and the same plane with the angle of incidence

AXIOM II

The angle of reflexion is equal to the angle of incidence

AXIOM III

If the refracted ray be returned directly back to the point of incidence it shall be refracted into the line before described by the incident ray

AXIOM IV

Refraction out of the rarer medium into the denser is made towards the perpendicular that is so that the angle of refraction be less than the angle of incidence

AXIOM V

The sine of incidence is either accurately or very nearly in a given ratio to the sine

f t refraction as 4 to 3 If out of air into glass ...

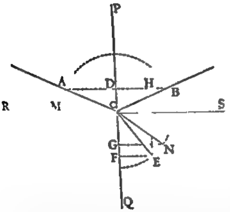
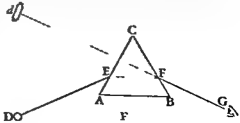


Fig 1

from A in the line ... or refracted and I would know whither this ray shall go after reflexion or refraction I erect upon the surface of the water from the point of incidence the perpendicular CP and produce it downwards to Q and conclude by the first Axiom that the ray after reflexion and refraction shall be found somewhere in the plane of the angle of incidence ACP produced I let fall therefore upon the perpendicular CP the sine of incidence AD and if the reflected ray be desired I produce AD to B so that DB be equal to AD and draw CB For this line CB shall be the reflected ray the angle of reflexion BCP and its sine BD being equal to the angle and sine of incidence as they ought to be by the second Axiom But if the refracted ray be desired I produce AD to H so that DH may be to AD as the sine of refraction to the sine of incidence that is (if

on the line PQ this line EF shall be the sine of refraction of the ray CE the angle of refraction being ECQ and this sine EF is equal to DH and consequently in proportion to the sine of incidence AD as 3 to 4



In like manner if there be a prism of glass (that is a glass bounded with two equal and parallel triangular ends and three plain and well polished sides which meet in three parallel lines running from the three angles of one end to the

three angles of the other end) and ...
cross th...
transv...
throu...

... and EF the line
... ay for the ...
whe...

... or if the sine
... sine of refraction as 17 to 11 the sine
o... the contrary be to the sine of refrac
tio... by the third Axiom

Much after the same manner if ACBD [Fig 3] represent a glass spherically convex on both sides (usually called a *lens* such as is a burning glass or spectacle glass or an object-glass of a telescope) and it be required to know

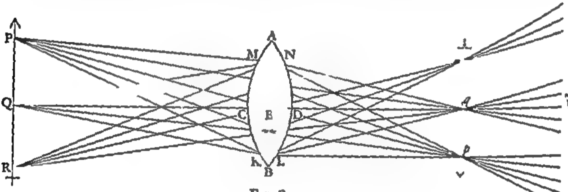


Fig 3

how light falling upon it from any lucid point Q shall be refracted ... present a ray falling upon any point ...
by ...
ray ... Let that ray in going ...
gla... it upon N and then ...

AXIOM VI

... thus either accurately or without ... error And the same thing will happen if the rays be reflected or refracted successively by two or three or more plane or spherical surfaces*

The point from which rays diverge or to ...
their fo...
or re...
abc...
C

of th incident rays and $Q q C$ a perpendicular to that plane And if this perpendicular be produced to q so that qC be equal to QC the point q shall be the focus of the reflected rays or if qC be taken on the same side of the plane with

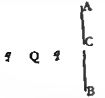


Fig 4

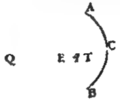


Fig 5

QC and in proportion to QC as the sine of incidence to the sine of refraction the point q shall be the focus of the refracted rays

continual proportionals and the point Q be the focus of the incident rays the point q shall be the focus of the reflected ones.

CASE 3 Let ACB [Fig 6] be the refracting surface of any sphere whose centre is E. In any radius thereof EC produced both ways take ET and Ct equal to one another and severally in such proportion to that radius as the lesser of the

Fig 6

sines of incidence and refraction hath to the difference of those sines And then if in the same line you find any two points Q and q so that TQ be to ET as Et to tq taking tq the contrary way from t which TQ lieth from T and if the point Q be the focus of any incident rays the point q shall be the focus of the refracted ones

And by the same means the focus of the rays after two or more reflexions or refractions may be found.

CASE 4 Let ACBD [Fig 7] be any refracting lens spherically convex or concave or plane on either side and let CD be its axis (that is the line which

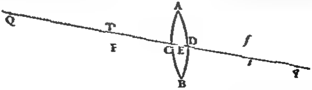


Fig 7

said circle in T and t and therein take tq in such proportion to tE as tT or TF hath to TQ Let tq lie the contrary way from t which TQ doth from T and q shall be the focus of the refracted rays without any sensible error provided the point Q be not so remote from the axis nor the lens so broad as to make any of the rays fall too obliquely on the refracting surfaces

And by the like operations may the reflecting or refracting surfaces be found when the two foci are given and thereby a lens be formed which shall make the rays flow towards or from what place you please

So then the meaning of this Axiom is that if rays fall upon any plane or spherical surface or lens and before their incidence flow from or towards any point Q they shall after reflexion or refraction flow from or towards the point q found by the foregoing rules And if the incident rays flow from or towards several points Q the reflected or refracted rays shall flow from or toward so many other points q found by the same rules Whether the reflected and refracted rays flow from or towards the point q is easily known by the situation of that point For if that point be on the same side of the reflecting or refracting surface or lens with the point Q and the incident rays flow from the point Q the reflected flow towards the point q and the refracted from it and if the incident rays flow towards Q the reflected flow from q and the refracted towards it And the contrary happens when q is on the other side of the surface

AXIOM VII

Wherever the rays which come from all the points of any object meet again in so many points after they have been made to converge by reflection or refraction there they will make a picture of the object upon any white body on which they fall

So if PR [Fig 3] represent any object without doors and AB be a lens placed at a hole in the window shut of a dark chamber whereby the rays that come from any point Q of that object are made to converge and meet again in the point q and if a sheet of white paper be held at q for the light there to fall upon it the picture of that object PR will appear upon the paper in its proper shape and colours For as the light which comes from the point Q goes to the point q so the light which comes from other points P and R of the object will go to so many other correspondent points p and r (as is manifest by the sixth Axiom) so that every point of the object shall illuminate a correspondent point of the picture and thereby make a picture like the object in shape and colour this only excepted that the picture shall be inverted And this is the reason of that vulgar experiment of casting the species of objects from abroad upon a wall or sheet of white paper in a dark room

In like manner when a man views any object PQR [Fig 8] the light which comes from the several points of the object is so refracted by the transparent skins and humours of the eye (that is by the outward coat LFC called the *tunica cornea* and by the crystalline humour AB which is beyond the pupil mk) as to converge and meet again in so many points in the bottom of the eye and there to paint the picture of the object upon that skin (called the *tunica retina*) with which the bottom of the eye is covered For anatomists when they have taken off from the bottom of the eye that outward and most thick coat called the *dura mater* can then see through the thinner coats the pictures of objects lively painted thereon And these pictures propagated by motion along the fibres of the optic nerves into the brain are the cause of vision For accordingly

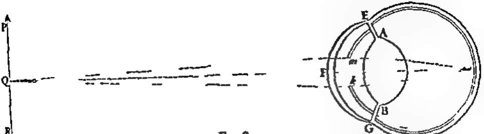


Fig 8

as these pictures are perfect or imperfect the object is seen perfectly or imperfectly If the eye be tin ed with any colour (as in the disease of the jaundice) so as to tinge the pictures in the bottom of the eye with that colour then all r n ed with the same colour If the humours of the eye by old

sight in old men and convex glasses supply ie refraction make the rays converge sooner so as to convene distinctly at the bottom of the eye if the glass have a due degree of convexity And the contrary happens in hort ighted men whose eyes are too plump For the refraction being now too great the rays converge and convene in the eyes before they come at the bottom and therefore the picture made in the bottom and the vision caused thereby will not be distinct unless the object be brought so near the eye as that the place where the converging rays convene may be removed to the bottom or that the plumpness hed l a concave-glass of a

therefore they are accounted to have the most lasting eyes

AXIOM VIII

An object seen by eflexion o refract on ppears in that place from whence the rays after their last reflexion or refraction d verge in falling on the spectator s eye

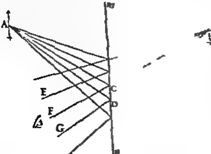


Fig 9

If the object A [Fig 9] be een by reflexion of a looking glass *mn* it shall appear not in its proper place A but behind the glass at *a* from whence any rays AB AC AD which flow from one and the same point of the object do after their reflexion

tator s eyes For the e rays do make the same picture in the bottom of

the eyes as if they had come from the object really placed at *a* without the interposition of the looking glass and all vision is made according to the place and shape of that picture

In like manner the object D [Fig 2] seen through a prism appears not in its proper place D but is thence translated to some other place *d* situated in the last refracted ray FG drawn backward from F to *d*

And so the object Q [Fig 10] seen through the lens AB appears at the place *q* from whence the rays diverge in passing from the lens to the eye Now it is to be noted that the image of the object at *q* is so much bigger or lesser than the object itself at Q as the distance of the image at *q* from the lens AB is bigger or less than the distance of the object at Q from the same lens And if the object be seen through two or more such convex or concave glasses every glass shall make a new image and the object shall appear in the place of the bigness of the last image Which consideration unfolds the theory of microscopes and telescopes For that theory consists in almost nothing else than the describing such glasses as shall make the last image of any object as distinct and large and luminous as it can conveniently be made

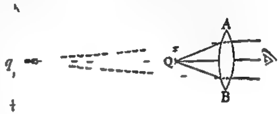


Fig 10

I have now given in Axioms and their explications the sum of what hath hitherto been treated of in Optics For what hath been generally agreed on I content myself to assume under the notion of Principles in order to what I have further to write And this may suffice for an Introduction to readers of quick wit and good understanding not yet versed in Optics although those who are already acquainted with this science and have handled glasses will more readily apprehend what followeth

PROPOSITIONS

PROPOSITION 1 THEOREM 1

Lights which differ in colour differ also in degrees of refrangibility

The Proof by Experiments

Experiment 1 I took a black oblong stiff paper terminated by parallel sides and with a perpendicular right line drawn cross from one side to the other distinguished it into two equal parts One of these parts I painted with a red colour and the other with a blue The paper was very black and the colours intense and thickly laid on that the phenomenon might be more conspicuous This paper I viewed through a prism of solid glass whose two sides through which the light passed to the eye were plane and well polished and contained an angle of about sixty degrees which angle I call the refracting angle of the prism And whilst I viewed it I held it and the prism before a window in such manner that the sides of the paper were parallel to the prism and both those sides and the prism were parallel to the horizon and the cross line was also parallel to it and that the light which fell from the window upon the paper

made an angle with the paper equal to that angle which was made with the same paper by the light reflected from it to the eye Beyond the prism was the wall of the chamber under the window covered over with black cloth and the cloth was involved in darkness that no light might be reflected from thence which in passing by the edges of the paper to the eye might mingle itself with the light of the paper and obscure the phenomenon thereof These things being thus ordered I found that if the refracting angle of the prism be turned upwards so that the paper may seem to be lifted upwards by the refraction its blue half will be lifted higher by the refraction than its red half But if the

from the blue half of the paper through the prism to the eye does in like circumstances suffer a greater refraction than the light which comes from the red half and by consequence is more refrangible

Illustration In the eleventh Figure MN represents the window and DE the paper terminated with parallel sides DJ and HE and by the transverse line FG distinguished into two halves the one DG of an intensely blue colour the other FE of an intensely red And BACcab represents the prism whose refracting planes ABba and ACca meet in the edge of the refracting angle Aa This edge Aa being upward is parallel both to the horizon and to the parallel edges of the paper DJ and HE and the transverse line FG is perpendicular to the plane of the window And de represents the image of the paper seen by refraction upwards in such manner that the blue half DG is carried higher to dg than the red half FE is to fe and therefore suffers a greater refraction If the edge of the refracting angle be turned downward the image of the paper will be refracted downward suppose to δ and the blue half will be refracted lower to δγ th

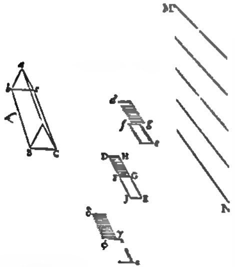


Fig 11

Expe
with red
times a
of the th
ove ther
drawn black lines with a pen bu
This paper thus coloured and h
horizon so that one of the colou and to the right hand and the other to the left. Close before the paper at the confine of the colours below I placed

t

make them converge tow

feet and one or two inche

of the coloured paper upon a white paper placed there e lens and so form the image that a lens at a hole in a window casts the images of objects abroad upon a sheet of white paper in a dark room The aforesaid white paper erected perpendicular to the horizon and to the rays which fell upon it from the lens I moved sometimes towards the lens sometimes from it to find the places where the images of the blue and red parts of the tinct Those places I easily knew by the made by winding the silk about the pape for the images of those fine and slender lines (which by reason of their blackness were like shadows on the colours) were confused and scarce visible unless when the colours on either side of each line were terminated most distinctly Noting therefore as diligently as I could the places where the im the coloured the paper app

I

I

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I appeared distinct there was the distance of an inch and a half the distance of the white paper from the lens when the image of the red half of the coloured paper appeared most distinct being greater by an inch and a half than the distance of the same white paper from the lens when the image of the blue half appeared most distinct In th

therefore more refrangible igure DE signifies the coloured paper DG the lens HJ the white paper in that place

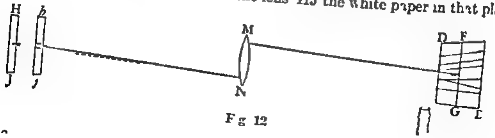

Fig 12

I

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SCHOLIUM The same things succeed notwithstanding that some of the circumstances be varied as in the first experiment when the pri any ways inclined to the upon very black paper down such circumstan

m..re conspicuou.. or a novice mi..ht more easily try them or by which I did try them only The same thin.. I have often done in the follo ing experiment ... one admonition may suffice now from th..e ex ... refrangible than ..ll ... refrangible so ... those of the bl e ... those of the red ... ew and serve to

b t these rays in proportion ... d..min..h the event of the experim nt but are no able to de..troy it For if the red and blue colours were more dilute and weak the di tance o the images would be less than an inch and a half and if they were more intense and full th..t di..tance would be greate as will appear hereafter These experiments may suffice for the colours of natural bodies. For in the colours made by the re..raction of prism.. this Proposition will appear by the experiments which are now to follow in the next Proposition.

PROPO ITION 2 THEOREM 2

The light of the Sun consists of rays differently refrangible.

The Proof by Experiment

EXPER 3 In a very dark chamber at a round hole about one-third part of an ..ch broad made in the ..ut of a window I placed a glass pri..m whereby the beam of the Sun ..ght which came in at that hole might be refracted upward.. toward the opposite wall of the chamber and there form a coloured image of ... hrough the middle of ... edge of the refract ... endicular to the inci-

d nt rays About this axis I turned the pri..m ... ..nd ..aw the refracted light on the wall o coloured image of the Sun first to descend and then to ascend. Between the descent ..nd ascent when the image seemed stationary I stopped the pri..m and fixed it in that posture that it ..hould be moved no more Fo in that posture the refraction.. of the light at the two sides of the refracti.. an..le that is at the entrance of the rays into the pri..m and at their go..g out of it were equal to one another So also in other experiments as often as I would have the refractions on both des the pri..m to be equal to one ... refra..ted ... f its ... the ... that ... ther ... I let ... op-

posite wall of the chamber and observed the figure and dimen..ions of the solar image formed on th paper by that l..ht This image wa.. oblong and no oval but terminated with two rect linear and parallel sides and two semicircular e... f...

in hes and the eighth part of an inch including the penumbra For the image

]

ten inches and a quarter and the length of the rectilinear ... about ... inches and the refracting angle of the prism whereby so great a length was made was 64 degrees With a less angle the length of the image was less the breadth remaining the same If the prism was turned about its axis that way which made the rays emerge more obliquely out of the second refracting surface of the prism the image soon became an inch or two longer or more and if the prism was turned about the contrary way so as to make the rays fall more obliquely on the first refracting surface the image soon became an inch or two shorter And therefore in trying this experiment I was as curious as I could be in placing the prism by the above mentioned rule exactly in such a posture that the refractions of the rays at their emergence out of the prism might be equal to that at their incidence on it This prism had some veins running along within the glass from one end to the other which scattered some of the sun s light irregularly but had no sensible effect in increasing the length of the ... I ... the ... me experiment with other prisms with the ... ich seemed free from such veins ... I found the length of the image ... m the prism the breadth of the hole in the window shut being one quarter of an inch as before And because it is easy to commit a ... I repeated the experiment four ... which is set down al ... which seemed free ... the length of this image at the same distance of 18½ feet was also about 10 inches or 10⅛ Beyond these measures for about ¼ or ⅓ of an inch at either end of the spectrum the light of the clouds seemed to be a little tinged with red and violet but so very faintly that I suspected that tincture might either ... in great measure arise from some rays of the spectrum scattered ...

cemented together in the shape of a prism ... like success of the experiment according to the quantity of the refraction It is further to be observed that the rays went on in right lines from the prism to the image and therefore at their very going out of the prism had all that inclination to one another from which the length of the image proceeded that is the inclination of more than two degrees and a half And yet according to the laws of Optics vulgarly received they could not possibly be so much inclined to one another For let LG [Fig 13] represent the window-shut F the hole made therein through which a beam of the Sun s light was transmitted into the darkened chamber and ABC a triangular imaginary plane whereby the prism is feigned to be cut transversely through the middle of the light Or if

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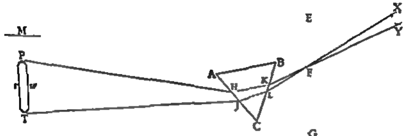


Fig 13

the two rays being equally refracted have the same inclination to one another after refraction which they had before that is the inclination of half a degree

consequence be equal to the breadth rw and therefore the image would be round Thus it would be were the two rays XLJT and YKHP and all the rest which form the image PwTr alike refrangible And therefore seeing by ex

Th ... PT ...
T
u
d
foregoing experiments I measured from the faintest and outmost red at one end to the faintest and outmost blue at the other end excepting only a little penumbra, whose breadth scarce exceeded a quarter of an inch as was said above

EXPER. 4 In the Sun s beam which was propagated into the room through the hole in the window-shut at the distance of some feet from the hole I held the prism in such a posture that its axis might be perpendicular to that beam Then I looked through the prism upon the hole and turning the prism to and

fro about its axis to make the image of the hole ascend and ... between its two cont... that the refractions c... other as in the forr... ...s situation of the prism viewing through it the said hole I observed the length of its refracted image to be many times greater than its breadth and that the most refracted part thereof appeared violet the least refracted red the middle parts bl... green... in order The same th... light and looked thr... beyond it And yet if ... were done regularly according to one certain proportion of the sines of incidence and refraction as is vulgarly supposed the refracted image ought to have appeared round

So then by these t... is a considerable ine... whether it be that s... rays are refracted more and others less constantly or by chance ... disturbed ...

EXPER 5 Considering therefore that ... of ... eve... im... bre... rays or other casual inequality of the refractions sideways I tried what would be the effects of such a second refraction For this end I ordered all things as in the third experiment and then placed a second prism immediately after the first in a cross position to it that it might again refract the beam of the Sun s light which came to it through the first prism In the first prism this beam was refracted upwards and in the second sideways And I found that by the refraction of the second prism the breadth of the image was not increased but its superior part which in the first prism suffered the greater refraction and appeared violet and blue did again in the second prism suffer a greater refraction than its inferior part which appeared red and yellow and this without any dilatation of the image in breadth

AI... by ... the prisms are taken away PT the oblong image of the Sun made by that beam passing through the first prism alone when the second prism is taken away and *pt* the image made by the cross refractions of both prisms together Now if the rays ... several ... of the fi... points ...

... become oblong those rays and their several parts tending towards the several points of the image PT ought to be again

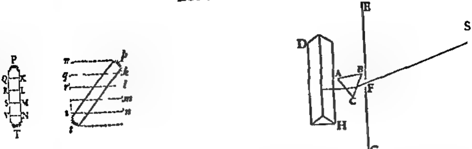


Fig 14

dilated and pread sideways by the transverse refraction of the second pri m
... uch as is represented at πτ For the
be distinguished into five
And by the same irregularity

many other long images lrsm msin nit and all these long images would com
pose the four square images π Thus it ought to be were every ray dilated by
refraction and pread into a triangular superficies of rays diverging from the
point of refraction For the econd refraction would spread the rays one way

in the second prism than the light ...
that is the blue and violet than the red and yellow and therefore was more
refrangible The same light was by the refraction of the first prism tran lated
farther from the place Y to which it tended before refraction and therefore

Sometimes I placed a third prism after the second and sometimes al o a
fourth after the third by all which the ima e might be often refracted ide-
ways but the rays which were more refracted than the rest in the first prism
were also more refracted in all the rest and that without any dilatat on of the
image sideways and therefore those rays for their constancy of a greater
refraction are deservedly reputed more refrangible

But that the meaning of this experiment may more clearly appear it is to be
considered that the rays which are equally refrangible do fall upon a circle
answering to the Sun s disk. For this was proved in the third experiment By a
circle I understand not here a perfect geom trical circle but any orbicular
figure whose length is equal to it bre dth and which as to sense may seem

circular Let therefore AG [Fig 15] represent the circle which all the most refrangible rays propagated from the whole disk of the Sun would illuminate and paint upon the opposite wall if they were alone EL the circle which all the least refrangible rays would in like manner illuminate and paint if they ircles which so many intermediate sorts of rays the wall if they were singly propagated from Sun in successive order the rest being always intercepted and conceive that there are other intermediate circles without number which innumerable other intermediate sorts of rays would successively paint upon the wall if the

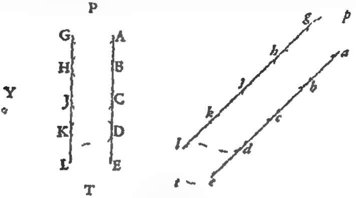


Fig 15

Sun should successively emit every sort apart And seeing the Sun emits all these sorts at once they must all together illuminate and paint innumerable equal circles of all which being according to their degrees of refrangibility placed in order in a continual series that oblong spectrum PT is composed which I described in the third experiment

[Fi

the

cor ought every circle AG BH CJ &c in that spectrum by the cross refraction of the second prism again dilating or otherwise scattering the rays as before er drawn out and of the image PT Y was before by and thus by the refractions of both prisms together would be formed a four square figure *pπtτ* as I described above Wherefore since the breadth of the spectrum PT is not increased by the refraction sideways it is certain that the rays are not split or dilated or otherways irregularly scattered by that refraction but that every circle is by a regular and uniform refraction translated entire into another place as the circle AG by the greatest refraction into the place *ag* the circle BH by a less refraction into the place *bh* the circle CJ b

because the breadths of all the spectrums Y T and *pt* at equal distances from the prisms are equal

I considered further that by the breadth of the hole F through which the light enters into the dark chamber there is a penumbra made in the circuit of the spectrum Y and that penumbra remains in the rectilinear sides of the

spectrums PT and pt I placed therefore at that hole a lens or object-glass of a telescope which might cast the image of the Sun distinctly on Y without any penumbra at all and found that the penumbra of the rectilinear sides of the oblong spectrums PT and pt was also thereby taken away so that those sides

... the ... one of the circles was refracted according to some most regular uniform, and constant law For if there were any irregularity in the refraction the ...

cou ... stra ... som ... con ... by experience Whatsoever penumbra or perturbation should be made in the circles by the cross refraction of the second prism ...

by the refra ... those ... is more or le ... in the same proportion more or less refracted in the second. And seeing all these things continue to succeed after the same manner when the rays are again in a third prism ...

... differ in degree of refrangibility and that in some certain and constant proportion. Which is the thing I was to prove

There is yet another circumstance or two of this experiment by which it becomes still more plain and convincing Let the second prism DH [Fig 16] be placed not immediately after the first but at some distance from it suppose

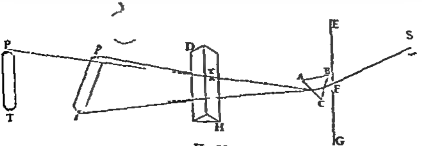


Fig. 16

circular Let therefore AG [Fig 15] represent the circle which all the most refrangible rays propagated from the whole disk of the Sun would illuminate and paint upon the opposite wall if they were alone EL the circle which all the least refrangible rays would in like manner illuminate and paint if they

ircles which so many intermediate sorts of rays

the wall if they were singly propagated from

Sun in successive order the rest being always intercepted and conceive that there are other intermediate circles without number which innumerable other intermediate sorts of rays would successively paint upon the wall if the

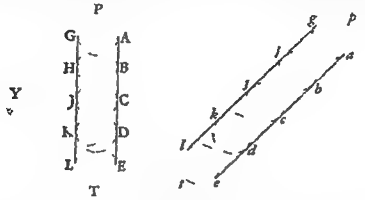


Fig 15

Sun should successively emit every sort apart And seeing the Sun emits all these sorts at once they must all together illuminate and paint innumerable equal circles of all which being according to their degrees of refrangibility placed in order in a continual series that oblong

which I

[Fig 15]

the singl y in the refraction of the first prism

converted into the oblong spectrum PT then ought every circle AG BH CJ

&c in that spectrum by the c

or otherwise scattering the ra

transformed into an oblong fi the image PT

would be now as much augmented was before by

the refraction of the first prism a both prisms

together would be formed a four square figure *pπtτ* as I described above Wherefore since the breadth of the spectrum PT is not increased by the refraction sideways it is certain that the rays are not split or dilated or otherways irregularly scattered by that refraction but that every circle is by a regular and uniform refraction translated entire into another place as the circle AG by the greatest refraction into the place *ag* the circle BH by a less refraction into the place *bh* the circle CJ by a refraction still less into the place *cj* and so of the rest by which means a new spectrum *pt* inclined to the former PT is in like manner composed of circles lying in a right line and these circles must be of the same bigness with the former because the breadths of all the spectrums Y PT and *pt* at equal distances from the prisms are equal

I considered further that by the breadth of the hol

bject-glass of
without any
r sides of the
at those sides
irst image Y
their sides be
accurately plane and well poli hed without ... ... saves or curls
which usually arise from sand holes a little smoothed in polishing with putty
If the glass be only well poli hed and free from veins and the sides not accu
cave as it frequently happens yet may
t penumbras but not in equal distances
it of penumbras I knew more certainly
acted according to some most regular
uniform and constant law For if there were any irregularity in the refraction
the right lines AE and GL which all the circles in the spectrum PT do touch

tion should be made in the circles by the cross refraction of the second pri m
all that penumbra or perturbation would be conspicuous in the right lines *ae*
and *gl* which touch those circles And therefore since there is no such penum
bra or perturbation in those right lines there must be none in the circles Since
the distance between those tangents or breadth of the spectrum is not increased
by the refraction the diameters of the circles are not increased thereby Since
those tangents continue to be right lines every circle which in the first prism
is more or less refracted is exactly in the same proportion more or less refracted
in the second. And seeing all these things continue to succeed after the same

prove

There is yet another circumstance or two of this experiment by which it becomes still more plain and convincing Let the second pri m DH (Fig 16) be placed not immediately after the first but at some distance from it suppose

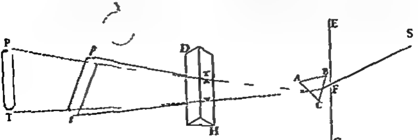


Fig. 16

in the mid way between it and the wall on which the oblong spectrum PT is cast so that the light from the first prism may fall upon it in the form of an oblong spectrum $\pi\tau$ parallel to this second prism and be refracted sideways to form the oblong spectrum *pt* upon the wall And you will find as before that this spectrum *pt* is inclined to that spectrum PT which the first prism forms alone without the second the blue ends P and *p* being farther distant from one another than the red ones T and *t* and by consequence that the rays which go to the blue end π of the image $\pi\tau$ and which therefore suffer the greatest refraction in the first prism are again in the second prism more refracted than the rest

The same thing I tried also by letting the Sun s light into a dark room through two little round holes F and φ [Fig 17] made in the window and with two parallel prisms ABC and $\alpha\beta\gamma$ placed at those holes (one at each) refracting

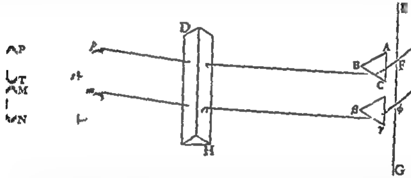


Fig 17

those two beams of light to the opposite wall of the chamber in such manner that the two coloured images PT and MN which they there painted were joined end to end and lay in one straight line the red end T of the one touching the blue end M of the other For if these two refracted beams we e m n l l l prism DH nl a l thereby tra spectrum T n to *mn* the e translated spectrums *pt* and *mn* would not lie in one straight line with their ends contiguous as before but be broken off from one another and become parallel the blue end *m* of the image *mn* being by a greater refraction translated farther f om ts former place MT

MT which n it

third pri

from the a ted in the two first pri ms be either white and circular or coloured and oblong when it fall on the third

EXPER 6 In the middle of two thin boards I made round holes a third part of an inch in diameter and in the window-shut a much broader hole being made to let into my darkened chamber a large beam of the Sun s light I placed a prism behind the shut in that beam to refract it towards the opposite wall and close behind the prism I fixed one

middle of the refracted light n

rest be intercepted by the bo

from the first board I fixed the other board in such manner that the middle of the refracted light which came through the hole in the first board and fell

upon the opposite wall might pass through the hole in this other board and
... by the board might paint upon it the coloured ...

which fell upon the second board to move up and ...
... that board and fall
... places on the op-
...ond prism did pass
... which being most
refracted in the first prism did go the the blue end of the image was again more refracted in the second prism than the light which went to the red end of that image which proves as well the first Proposition as the second And this happened whether the axis of the two prisms were parallel or inclined to one another and to the horizon in any given angles

ILLUSTRATION Let F [Fig 18] be the wide hole in the window-shut through which the Sun shines upon the first prism ABC and let the refracted light fall

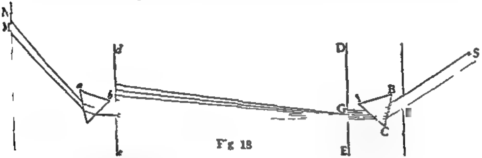


Fig 18

upon the middle of the board DE and the middle part of that light upon the hole G made in the middle part of that board Let this trajected part of that

on end to the other may be made to pass successively through the hole g

two boards and second prism remained unmoved those places by turning the ...

some place on the wall between M and N The unchanged position of the holes in the boards made the incidence of the rays upon the second prism to be the same in all cases. And yet in that common incidence some of the rays were more refracted and others less And those were more refracted in this prism which

by a greater refraction in the first prism were more turned out of the way and therefore for their constancy of being more refracted are deservedly called more refrangible

EXPER 7 At two hole ...
two prisms one at each
...

... and parallel edges and ordered the prisms and paper so that the red colour of one image might fall directly upon one half of the paper and the violet colour of the other image upon the other half of the same paper so that the paper appeared of two colours red and violet much after the manner of the painted paper in the first and second experiments Then with a black cloth I covered the wall behind the paper that no light might be re-

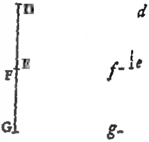


Fig 19

flected from it to disturb the experiment and viewing the paper through a third prism held parallel to it I saw that half of it which was illuminated by the violet light to be divided from the other half by a greater refraction especially when I went a good way off from the paper For when I viewed it too near at hand the two halves of the paper did not appear fully divided from one another but seemed contiguous at one of th ...
paper in the fir ...

Sometimes ...

... illuminated with red and ... be illuminated with all the colours successively (which may be done by causing one of the prisms to be turned about its axis whilst the other remains unmoved) this other half in viewing the thread through the prism will appear in a continual right line with the first half when illuminated with red and begin to be a little divided from it when illuminated with ... remove farther from it when ...

... which plainly shows that the ... are more and more refrangible one than another in this order of their colours red orange yellow green blue indigo deep violet and so proves as well the first Proposition as the second

I caused also the coloured spectrums PT [Fig 17] and MN made in a dark chamber by the refractions of two prisms to lie in a right line end to end as was described above in the fifth experiment and viewing them through a third prism held parallel to their length they appeared no longer in a right line but became broken from one another as they are represent... *mn* the violet end *m* of the ...
lated farther from ...

I further caused ... to become coincident in an inve... order of their colours the red end of each falling on the violet end of the other as they are represented in the oblong figure PTMN

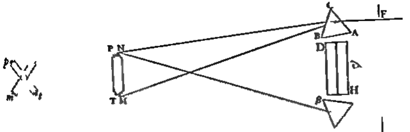


Fig 20

one another by a greater refraction of the violet to p and m than of the red to n and t do differ in degrees of refrangibility

I illuminated al.o a little circular piece of white paper all over with the lights of both prisms intermixed and when it was illuminated with the red of one pectrum and deep violet of the other so as by the mixture of those [illegible] first at a les distance

a red one and a violet one whereof the violet was farthest from the paper and

shew that these two images were nothing else than the lights of the two prisms, which had been intermixed on the purple paper but were parted again by their unequal refraction made in the third prism, through which the paper was viewed. This also was observable that if one of the prisms at the window (suppose that which cast the violet on the paper) was turned about its axis to make all the colours in this order violet, indigo blue green yellow orange red fall successively on the paper from that prism, the violet image changed colour accordingly turning successively to indigo blue green, yellow and red, and in changing colour came nearer and nearer to the red image made by the other prism, until when it was also red both images became fully coincident

I placed also two paper circles very near one another the one in the red light of one prism and the other in the violet light of the other The circles were each of them [illegible] and behind them the wall was dark, that the experiment [illegible] be disturbed by any light coming from thence These circles thus illuminated, I viewed through a prism so held that the refraction might be made towards the red circle and as I went from them they came nearer and nearer [illegible] and afterwards when I went still further from them they parted again [illegible] order the violet by a greater refraction being carried beyond the red.

Exper 8 In summer when the sun's light [illegible] I placed a prism at the hole of the window-shut [illegible] experiment [illegible] so that

its axis might be parallel to the axis of the world and at the opposite wall in the S n s refracted light I placed an open book Then going six feet and two ... the bove-mentioned lens by which the ... th

experiment The boo ... lace where the paper was when the letters of the book illuminated by the ... est red light of the solar image falling upon it did cast their species on that paper most distinctly A ... mot on of the Sun and consequent motion of ... of the blue passed over those letters ... by ... I noted again the place of the paper when they cast ... this last place of the paper was ... two and three quarters ... end of the image by a greater refraction converge and meet ... n the red end But in trying this the chamber was as dark as I could make it For if these colours be diluted and weakened by the mixture of any adventitious light the distance between the places of the paper will not be so great This distance in the second experiment where the colours of natural bodies were made use of was but an inch and a half by reason of the imperfection of those colours Here in the colours of the prism which are manifestly more full intense and lively than those of natural bodies the distance is two inches and three quarters And were the colours still more full I question not but that the distance would be considerably greater For the coloured light of the prism by the interfering of the circles described in the second figure of the fifth experiment (Fig 15) and also by the light of the very bright clouds next the Sun s body intermixing with these colours and by the light scattered by the inequalities in the polish of the prism was so very much compounded that the species which those faint and dark colours the indigo and violet cast upon the paper were not ... ll observed

EXPER ... A prism ... nother and half right ones ... Sun s light let into a dark chamber through ... in the third experiment And turning the prism slowly about its axis ... all the ... h one of its angles and was refracted by it began to be ... of the glass I observed that ...

the rest I conceiv ... were most refrangible did first of all by a total ... in that light than the rest and that afterwards the rest also by a total ... flexion became as copious as the e To try this I made the reflected light pass through another prism and being refracted by it to fall afterwards upon a sheet of white paper placed at some distance behind it and there by that r ... tion to paint the usual colours of the prism And then causing the first ... axis as above I observed that when the ... ray ... and appeared of a blue and violet light on the

d th second pri m received a sensible in which was least refracted and after as green yellow and red began to be ight of those colours on the paper received as great an increa e as the vio et and blue had done before Whence tis manifest that the beam of light reflected by the base of the prism being aug — d by th le s refrang that all such incidence on the base of the prism no man ever doubted allowed that light by such reflexions suffers no alteration in its modifications and properties. I do not here take notice of any refractions made in the ides of the first prism because the light enters it perpendicularly at the first ide and goes out perpendicularly at the econd side and therefore suffers none So then the Sun s incident li ht being of the same temper and con titution with his emergent light and the last being compounded of rays differently refrangible the first mu t be in like manner compounded

ILLUSTRATION In the twenty first Figure ABC is the first prism BC its base B and C its equal angles at the base each of 45 degrees A its rectangular vertex FM a beam of the Sun s light let into a dark room through a hole F one third part of an inch broad, M its incidence on the base of the prism MG a less refracted ray MH a more refracted ray MN the beam of light reflected from the base VXY the second prism by which this beam in passing through it is refracted, Nt the less refracted light of this beam, and Np the more refracted part thereof When the first prism ABC is turned about it axis according to the order of the letters ABC the ray MH emerges more and more obliquely out of that prism, and at length

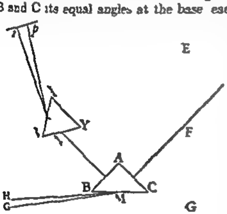


Fig 21

—

ersion of the spectrum base BC causing no alteration therein.

EXPER 10 Two prisms which were like in shape I tied so together that their axes and opposite sides being parallel, they composed a parallelepiped And the sun shining into my dark chamber through a little hole in the window shut, I placed that parallelepiped in his beam at some distance from the hole in such a posture that the axes of the prisms might be perpendicular to the incident rays, and that those rays, being incident upon the first side of one prism, might go on through the two contiguous sides of both prisms and

emerge out of the last side of the second prism This side being parallel to the first side of the first prism caused the emerging light to be parallel to the incident Then beyond these two prisms I placed a third which might refract that emergent light and by that refraction cast the usual colours of the prism upon the opposite wall or upon a sheet of white paper held at a convenient distance behind the prism for that refracted light to fall upon it After this I turned the parallelopiped about its axis and found that when the contiguous sides of the two prisms became so oblique to the incident rays that those rays began all of them to be reflected those rays which in the third prism had suffered the greatest refraction and painted the paper with violet and blue were first of all by a total reflexion taken out of the transmitted light the rest remaining and on the paper painting their colours of green yellow orange and red as before and afterwards by continuing the motion of the two prisms the rest of the rays also by a total reflexion vanished in order according to their degrees of refrangibility The light therefore which emerged out of the two prisms is compounded of rays differently refrangible seeing the more refrangible rays may be taken out of it while the less refrangible remain But this light being trajected only through the parallel superficies of the two prisms if it suffered any change by the refraction of one superficies it lost that impression by the contrary refraction of the other superficies and so being restored to its pristine constitution became of the same nature and condition as at first before its incidence on those prisms and therefore before its incidence was as much compounded of rays differently refrangible as afterwards

ILLUSTRATION In the twenty second Figure ABC and BCD are the two prisms tied together in the form of a parallelepiped their sides BC and CB being contiguous and their sides AB and CD parallel And HJK is the third prism by which the Sun s light propagated through the hole F into the dark chamber and there passing through those sides of the prisms AB BC CB and CD is refracted at O to the white paper PT falling there partly upon P by a greater refraction partly upon T by a less refraction and partly upon R and other intermediate places by intermediate refractions By turning the parallelepiped ACBD about its axis according to the order of the letters A C D B at length when the contiguous planes BC and CB become sufficiently oblique

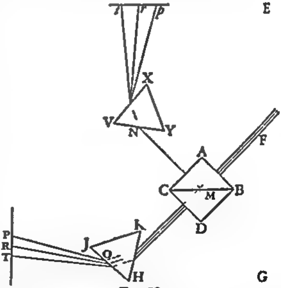


Fig 22

to the rays FM which are incident upon them at M there will vanish totally out of the refracted light OPT first of all the most refracted rays OP (the rest OR and OT remaining as before) then the rays OR and other intermediate ones and lastly the least refracted rays OT For when the plane BC becomes sufficiently oblique to the rays incident upon it those rays will begin to be totally reflected by it towards N and first the most refrangible rays will

be totally reflected (as was explained in the preceding experiment) and by con

hey are in order

at R and T So

the rays are separated out of the emergent light MO agree in colour and in all other properties so far as my observation reaches and therefore are deservedly reputed of the same nature and constitution and by consequence the one is compounded as well as the other But after the most refrangible rays begin to be totally reflected, and thereby separated out of the emergent light MO that light changes its colour from white to a dilute and faint yellow a pretty good orange a very full red successively and then totally vanishes. For after the most refrangible rays which paint the paper at P with a purple colour are by a total reflexion taken out of the beam of light MO the rest of the colours which appear on the paper at R and T being mixed in the light MO compound there a faint yellow and after the blue and part of the green which appear on the paper between P and R are taken away the rest which appear between R and T (that is the yellow orange red and a little green) being mixed in the beam MO compound there an orange and when all the rays are by reflexion taken out of the beam MO except the least refrangible which at T appear of a full red, their colour is the same in that beam MO as afterwards at T the refraction of the prism HJK serving only to separate the differently refrangible rays without making any alteration in their colours as shall be more fully proved hereafter All which confirms as well the first Proposition as the second.

Now if this experiment and the former be conjoined and made one by applying a fourth prism VXY [Fig. 22] to refract the reflected beam MN towards tp the conclusion will be clearer For then the light Np which in the fourth prism is more refracted, will become fuller and stronger when the light OP which in the third prism HJK is more refracted, vanishes at P and afterwards when the less refracted light OT vanishes at T the less refracted light Nt will become increased whilst the more refracted light at p receives no further increase And as the trajected beam MO in vanishing is always of such a colour as ought to result from the mixture of the colours which fall upon the paper PT so is the reflected beam MN always of such a colour as ought to result from the mixture of the colours which fall upon the paper pt. For when the most refrangible rays are by a total reflexion taken out of the beam MO and leave that beam of an orange colour the excess of those rays in the reflected light does not only make the violet, indigo and blue at p more full, but also makes the beam MN change from the yellowish colour of the Sun's light to a pale white inclining to blue and afterward recover its yellowish colour again so soon as all the rest of the transmitted light MOT is reflected.

Now seeing that in all this variety of experiments whether the trial be made in light reflected, and that either from natural bodies as in the first and second experiments or specular as in the ninth, or in light refracted, and that

e ther before the unequally refracted rays are by diverging separated from one hich they have altogether appear ev or after they are eparated nothe and appe sixth seventh and eighth n rficies destroying each which d that

reflexion as in the te uffer un equal refractions and those sorts are more refra r epara tion which were more refracted before it as in the sixth and following experi ment and if the Sun s light be trajected through three or more cro s prisms first pri m are refracted more than others and proportion as appears by in s light is an heterogeneous mixture of rays some of which are co ore refrangible than others as was proposed

PROPOSITION 3 THEOREM 3

The Sun s light consists of rays differing in reflexibility and those rays are mo r flexible than others which are more refrangible

h and tenth experiments for in the ninth ex h n t which in hat ba e

mon base of the two pri

PROPOSITION 4 PROBLEM 1

rays of compound light

noth r by

that separation in those

becomes perfect But in all places between those return e in numerable circles there described which are severally illuminated by homogeneal rays by interfering with one another and being everywhere commixed do render the light sufficiently compound But if these circles whilst their centres keep their distances and positions could be made less in diameter their interfering one with another and by consequence the mixture of the heterog neous rays would be proportionally diminished In the twenty third FM be the circles which so many sorts of experiment illumi g in a continual s oblong image

PT that image is composed as was explained in the fifth experiment And let *cj bh cj dk el fm* be so many less circles lying in a like continual series between two parallel right lines *af* and *gm* with the same distances between their centres and illuminated by the same sorts of rays that is the circle *ag* with the same sort by which the corresponding circle AG was illuminated and the circle *bh* with the same sort by which the corresponding circle BH was illuminated and the rest of the circles *cj dk el fm* respectively [illegible] minated. In the figure PT composed of the greater circles three of those circles AG BH CJ are so

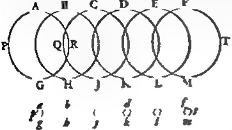


Fig 23

expanded into one another that the three sorts of rays by which those circles are illuminated, together with other innumerable sorts of intermediate rays are mixed at QR in the middle of the circle BH. And the like mixture happens throughout almost the whole length of the figure PT But in the figure *pt* composed of the less circles the three less circles *ag bh, cj* which answer to those three greater do not extend into one another nor are there anywhere mingled so much as any two of the three sorts of rays by which those circles are illuminated, and which in the figure PT are all of them intermingled at BH.

Now he that shall thus consider it will easily understand that the mixture is diminished in the same proportion with the diameters of the circles. If the diameters of the circles whilst their centres remain the same be made three times less than before the mixture will be also three times less if ten times less the mixture will be ten times less and so of other proportions. That is, the mixture of the rays in the greater figure PT will be to their mixture in the less *pt* as the latitude of the greater figure is to the latitude of the less For the latitudes of these figures are equal to the diameters of their circles And hence it easily follows that the mixture of the rays in the refracted spectrum *pt* is to the mixture of the rays in the direct and immediate light of the Sun as the breadth of that spectrum is to the difference between the length and breadth of the same spectrum.

So then if we would diminish the mixture of the rays we are to diminish the diameters of the circles Now these would be diminished if the Sun's diameter to which they answer could be made less than it is or (which comes to the same purpose) if without doors at a great distance from the prism towards the sun, some opake body were placed with a round hole in the middle of it, to intercept all the Sun's light excepting so much as coming from the middle of his body could pass through that hole to the prism For so the circles AG BH and the rest would not any longer answer to the whole disc of the Sun, but only to that part of it which could be seen from the prism through that hole that is to the apparent magnitude of that hole viewed from the prism. But that these circles may answer more distinctly to that hole a lens is to be placed by the prism to cast the image of the hole (that is every one of the circles AG BH &c) distinctly upon the paper at PT after such a manner as by a lens placed at a window the species of objects abroad are cast distinctly upon a paper

within the room and the rectilinear sides of the oblong solar image in the fifth experiment became distinct without any penumbra If this be done it will not be necessary to place that hole very far off no not beyond the window And therefore instead of that hole I used the hole in the window shut as follows

EXPER 11 In the Sun s light let into my darkened chamber through a mall round hole in my window shut at about ten or twelve feet from the window I placed a lens by which the image of the hole might be distinctly cast upon a

same distance from the prism as before moving the paper either towards the prism or from it until I found the just distance where the rectilinear sides of the image became most distinct For in this case the circular images of the hole which compose that image after the same manner that the circles *ag bh cj* &c do the figure *pt* [Fig 23] were terminated most distinctly without any penumbra and therefore extended into one another the least that they could and by consequence the mixture of the heterogeneous rays was now the least of all By this means I used to form an oblong image (such as is *pt*) [Fig 23 and 24] of circular images of the hole (such as are *ag bh cj* &c) and by using a greater or less hole in the window shut I made the circular images *ag bh cj* &c of which it was formed to become greater or less at pleasure and thereby the mixture of the rays in the image *pt* to be as much or as little as I desired

ILLUSTRATION In the twenty fourth Figure F represents the circular hole in the window shut MN the lens whereby the image or species of that hole is

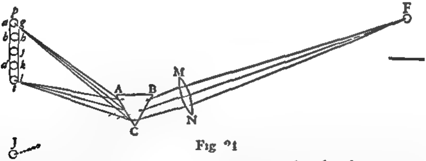


Fig °1

cast distinctly upon a paper at J ABC the prism whereby the rays are at their emerging out of the lens refracted from J towards another paper at *pt* and the round image at J is turned into an oblong image *pt* falling on that other paper This image *pt* consists of circles placed one after another in a rectilinear order as was sufficiently explained in the fifth experiment and these circles are equal

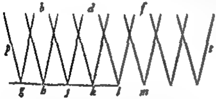
ole F and there-
hed whilst their
dth of the image
than its length
in h and MF

uses to be refracted irregularly by the inequalities of the prism

Yet instead of the circular hole F tis better to substitute an oblong hole

ampler and the image will become much broader and therefore more fit to

mediate ones answering to the triangular hole in shape and bigness and lying one after another in a continual series between two parallel lines af and gm



Fig

These triangles are a little intermingled at their bases but not at their vertices and therefore the light on the brighter side af of the image where the bases of the triangles are is a little compounded but on the

tional to the distances of the places from that obscurer side gm And having a spectrum pt of such a composition we may try experiments either in its stronger and less simple light near the side af or in its weaker and simpler light near the other side gm as it shall seem most convenient

But in making experiments of this kind the chamber ought to be made as dark as can be lest any foreign light mingle itself with the light of the spectrum pt, and render it compound especially if we would try experiments in the more simple light next the side gm of the spectrum which being fainter will have a less proportion to the foreign light and so by the mixture of that light be more troubled, and made more compound The lens also ought to be good such as may serve for optical uses and the prism ought to have a large angle suppose of 65 or 70 degrees and to be well wrought being made of glass free from bubbles and v

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within the room and the rectilinear sides of the oblong
expe... b

EXPER 11 In the Sun s light let into my d ... e hole in the window shut as follows

the lens Fe ... ance of six eight ten ... which I thr ...
I placed a ... ajected light might be refracted either upwards or sideways and thereby the round image which the l ... upon the paper m ... third experiment ... same distance fr ... as before moving the paper either towards the prism or from it until I found the just distance where the rectilinear ... the image be ... hole ... cj &c ... penum ... ended into one another the least that they could and by consequence the mixture of the heterogeneous rays was now the least of all By this means I used to form an oblong image (such as is *pt*) [Fig 23 and 24] of circular images of the hole (such as are *ag* *bh* *cj* &c) and by using a greater or less hole in the window shut I made the circular images *ag* *bh* *cj* &c of which it was formed to become greater or less at pleasure and thereby the mixture of the rays in the image *pt* to be as much or as little as I desired

ILLUSTRATION In the twenty fourth Figure F represents the circular hole in the window shut MN the lens whereby the image or species of that hole is

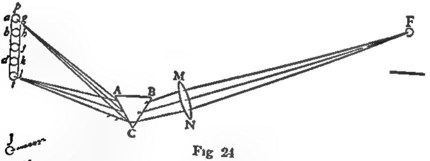

Fig 24

cast distinctly upon a paper at J ABC the prism whereby the rays are at their emerging out of the lens refracted from J tow ... d ... ne ...

... and co ... equently answer in magnitude to the hole F and therefore by diminishing that hole they may be at pleasure diminished whilst their centres remain in their places By this means I made the breadth of the image *pt* to be forty times and sometimes sixty or seventy times less than its length As for instance if the breadth of the hole F be one tenth of an inch and MF

the distance of the lens from the hole be 18 feet and if pB or pM the distance of the image p from the prism or lens be 10 feet and the refracting angle of the prism be 62 degrees the breadth of the image pt will be one-twelfth of an inch and the length about six inches, and therefore the length to the breadth as 72 to 1 ... the light of this image 71 times less compound than

perceived by sense except perhaps in the indigo and violet. For these being dark colours do easily suffer a sensible allay by that little scattering light which uses to be refracted irregularly by the inequalities of the prism

Yet instead of the circular hole F tis better to substitute an oblong hole shaped like a long parallelogram with its length parallel to the prism ABC For if this hole be an inch or two long and but a tenth or twentieth part of an inch broad, or narrower the light of the image pt will be as simple as before or simpler and the image will become much broader and therefore more fit to have experiments tried in its light than before

Instead of this parallelogram hole may be substituted a triangular one of equal sides, whose base for instance is about the tenth part of an inch and its height an inch or more For by this means if the axis of the prism be parallel to the perpendicular of the triangle the image pt [Fig 25] will now be formed of equicrural triangles ag, bh, ci, dk, el, fm &c and innumerable other intermediate ones answering to the triangular hole in shape and bigness and lying one after another in a continual series between two parallel lines af and gm These triangles are a little intermingled at their bases but not at their vertices and therefore the light on the brighter side af of the image where the bases of the triangles are is a little compounded but on the darker side gm is altogether uncompounded and in all places between the sides the composition is proportional to the distances of the places from that obscurer side gm And having a spectrum pt of such a composition we may try experiments either in its stronger and less simple light near the side af or in its weaker and simpler light near the other side gm as it shall seem most convenient

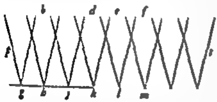

Fig 25

But in making experiments of this kind the chamber ought to be made as dark as can be lest any foreign light mingle itself with the light of the spectrum pt and render it compound especially if we would try experiments in the more simple light next the side gm of the spectrum which being fainter will have a less proportion to the foreign light and so by the mixture of that light be more troubled, and made more compound The lens also ought to be good such as may serve for optical uses and the prism ought to have a large angle suppose of 65 or 70 degrees, and to be well wrought being made of glass free from bubbles and ...

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of very little convex polite risings like waves The edges also of the prism and lens so far as they may make any irregular refraction must be covered with a black paper glued on And all the light of the Sun s beam let into the chamber which is useless and unprofitable to the experiment ought to be intercepted with black paper or other black obstacles For otherwise the useless light bein reflected every way in the chamber will mix with the oblong spectrum and ... lt onth

increase the refraction I sometimes impregnated the ... saccharum saturni

PROPOSITION 5 THEOREM 4

Homogeneal light is refracted regularly without any dilatation splitting or shatter ... on bodies by ... of rays

The first part of this Proposition has been already ... in the ... and will further appear by the experiments which follow ... 5th

from the prism I found that the spectrum ... was not oblong as when 'tis made (in the third experiment) by refracting the Sun s compound light but was (so far as I could judge by my eye) perfectly circular the length being no greater than the breadth Which shews that this ... without any dilatation of the rays

another paper circle of the same bigness ... I viewed both circles through a prism The circle illu...

tinctly defined as when 'tis viewed with ... whole Proposition

EXPER 14 In the homogeneal light I placed flies and such like minute objects and viewing them through a prism I saw their parts as distinctly defined as if I had viewed them with the naked eye The same objects placed in the Sun s unrefracted heterogeneal light which was white I viewed also through a p... that I could not distinguish ... so the letters of a ... print one while in the homog... the heterogeneal ... letters ... confused ... they appeared so

distinct that I could read readily and thought I saw them as distinct as when I viewed them with my naked eye In both cases I viewed the same objects

from me and in the same situ

y which th objects were illu

in the other compound and

e and confused in the latter

ifference of the lights Which

proves the whole Proposition

And in these three experiments it is further very remarkable that the colour of homogeneal light was never changed by the refraction

PROPOSITION 6 THEOREM

The sine of incidence of every ray considered apart is to its sine of refraction in a given ratio

That every ray considered apart is constant to itself in some degree of refrangibility is sufficiently manifest out of what has been said Those rays

... also in

least

... is

... those

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experiments. The refraction therefore of every ray apart is regular and what rule that refraction observes we are now to shew

The late writers in Optics teach that the sines of incidence are in a given proportion to the sines of refraction as was explained in the fifth Axiom and some by instruments fitted for measuring of refractions, or otherwise experimentally examining this proportion do acquaint us that they have found it accurate But whilst they not understanding the different refrangibility of several rays, conceived them all to be refracted according to one and the same proportion, tis to be presumed that they adapted their measures only to the middle of the refracted light so that from their measures we may conclude only that the rays which have a mean degree of refrangibility (that is, those which when separated from the rest appear green) are refracted according to a given proportion of their sines And therefore we are now to shew that the like given proportions obtain in all the rest. That it should be so is very reasonable Nature being ever conformable to herself but an experimental proof is desired And such a proof will be had if we can shew that the sines of refraction of rays differently refrangible are one to another in a given proportion when their sines of incidence are equal. For if the sines of refraction of all the rays are in given proportions to the sine of refractions of a ray which has a mean degree of refrangibility and this sine is in a given proportion to the equal sines of incidence those other sines of refraction will also be in given proportions to the equal sines of incidence Now when the sines of incidence are equal it will appear by the following experiment that the sines of refraction are in a given proportion to one another

EXPER. 1. The Sun shining into a dark chamber through a little round hole

in the window shut let S [Fig 26] represent his round white image painted on the opposite wall by his direct light PT his oblong coloured image made by refracting that light with a prism placed at the window and pt or $2p\,2t$ $3p\,3t$ his oblong coloured image made by refracting again the same light sideways with a second prism placed immediately after the first in a cross position to it as was explained in the fifth experiment that is to say pt when the refraction of the second prism is small $2p\,2t$ when its refraction is greater and $3p\,3t$ when it is greatest For such will be the diversity of the refractions if the refracting angle of the second prism be of various magnitudes suppose of fifteen or twenty degrees to make the image pt of thirty or forty to make the image $2p\,2t$ and of sixty to make the image $3p\,3t$ But for want of solid glass prisms with angles of convenient bignesses there may be vessels made of polished plates of glass cemented together in the form of prisms and filled with water These things being thus ordered I observed that all the solar images or coloured

very nearly converge to the place S on which

painted his white round image when the prisms were taken away The axis of the spectrum PT (that is the line drawn through the middle of it parallel to its rectilinear sides) did when produced pass exactly through the middle of that white round image S And when the refraction of the second prism was equal to the refraction of the first the refracting angles of them both being about 60 degrees the axis of the spectrum $3p\,3t$ made by that refraction did when produced pass also through the middle of the same white round image S But when the refraction of the second prism was less than that of the first the produced axes of the spectrums tp or $2t\,2p$

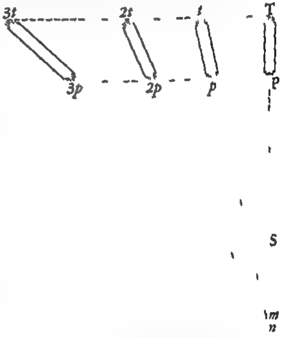

Fig 26

made by that refraction did cut the produced axis of the spectrum TP in the points m and n a little beyond the centre of that white round image S Whence the proportion of the line $3tT$ to the line $3pP$ was a little greater than the proportion of $2tT$ or $2pP$ and this proportion a little greater than that of tT to pP Now when the light of the spectrum PT f

wall

refrac

of the ... the proportions of the sines being derived they come out equal so far as by viewing the spectrums and using some mathematical reasoning I could estimate For I did not make an accurate computation So then the proposition holds true in every ray apart so far as appears by experiment And that it is accurately true may be demonstrated upon this supposition *That bodies refract light by acting upon its rays in lines*

perpendicular to their surfaces But in order to this demonstration I must distinguish the motion of every ray into two motions the one perpendicular to

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that space shall be always equal to the square root of the sum of the squares of the perpendicular velocity of that motion or thing at its incidence on that space and of the square of the perpendicular velocity which that motion or thing would have at its emergence if at its incidence its perpendicular velocity was infinitely little

And the same proposition holds true of any motion or thing perpendicularly retarded in its passage through that space if instead of the sum of the two squares you take their difference The demonstration mathematicians will easily find out and therefore I shall not trouble the reader with it

Suppose now that a ray coming most obliquely in the line MC [Fig 1] be refracted at C by the plane RS into the line CN and if it be required to find the line CE, into which any other ray AC shall be refracted let MC AD be the sines of incidence of the two rays and NG EF their sines of refraction and let the equal motions of the incident rays be represented by the equal lines MC and AC and the motion MC being considered as parallel to the refracting plane let the other motion AC be distinguished into two motions AD and DC one of which AD is parallel and the other DC perpendicular to the refracting surface In like manner let the motions of the emerging rays be distinguished into two whereof the perpendicular ones are $\frac{MC}{NG}$ CG and $\frac{AD}{EF}$ CF And if the force of the refracting plane begins to act upon the rays either in that plane or at a certain distance from it on the one side and ends at a certain distance from it on the other side and in all places between those two limits acts upon the rays in lines perpendicular to that refracting plane and the actions upon the rays at equal distances from the refracting plane be equal and at unequal ones either equal or unequal according to any rate whatever that motion of the ray which

the emerging ray CN you write $\frac{}{NG}$ CG as above then the perpendicular velocity of any other emerging ray CE which was $\frac{AD}{EF}$ CF will be equal to the square root of $CDq - \frac{MCq}{NGq}$ CG And by squaring these equals and adding to them the equals ADq and MCq$-$CDq and dividing the sums by the equals CFq$+$EFq and CGq$+$NGq you will have $\frac{MCq}{NGq}$ equal to $\frac{MCq}{NGq}$ Whence AD the sine of incidence is to EF the sine of refraction as MC to NG that is in a given ratio And this demonstration being general without determining what

light is or by what kind of force it is refracted or assuming any thing further than that the refracting body acts upon the rays in lines perpendicular to its surface I take it to be a very convincing argument of the full truth of this Proposition

So then if the ratio of the sines of incidence and refraction of any sort of rays be found in any one case tis given in all cases, and this may be readily found by the method in the following Proposition

PROPOSITION 7 THEOREM 6

The perfection of telescopes is impeded by the different refrangibility of the rays of light

The imperfection of telescopes is vulgarly attributed to the spherical figures of the glasses and therefore mathematicians have propounded to figure them by the conical sections To shew that they are mistaken I have inserted this proposition the truth of which will appear by the measure of the refractions of the several sorts of rays and these measures I thus determine

In the third experiment of this first part where the refracting angle of the prism was 62½ degrees the half of that angle 31 degrees 15 minutes is the angle of incidence of the rays at their going out of the glass into the air and the sine of this angle is 5 188 the radius being 10 000 When the axis of this prism was parallel to the horizon and the refraction of the rays at their incidence on this prism equal to that at their emergence out of it I observed with a quadrant the angle which the mean refrangible rays (that is those which went to the middle of the Sun s coloured image) made with the horizon and by this angle and the Sun s altitude observed at the same time I found the angle which the emergent rays contained with the incident to be 44 degrees and 40 minutes and the half of this angle added to the angle of incidence 31 degrees 15 minutes makes the angle of refraction which is therefore 53 degrees 35 minutes and its sine 8 047 These are the sines of incidence and refraction of the mean refrangible rays and their proportion in round numbers is 20 to 31 This glass was of a colour inclining to green The last of the prisms mentioned in the third experiment was of clear white glass its refracting angle 63½ degrees the angle which the emergent rays contained with the incident 45 degrees 50 minutes the sine of half the first angle 5 262 the sine of half the sum of the angles 8 157 and their proportion in round numbers 20 to 31 as before

From the length of the image which was about 9¾ or 10 inches subduct its breadth which was 2⅛ inches and the remainder 7¾ inches would be the length of the image were the Sun but a point and therefore subtends the angle which the most and least refrangible rays when incident on the prism in the same lines do contain with one another after their emergence Whence this angle is 2 degrees 0 7 For the distance between the image and the prism where this angle is made was 18½ feet and at that distance the chord 7¾ inches subtends an angle of 2 degrees 0 7 Now half this angle is the angle which these emergent rays contain with the emergent mean refrangible rays and a quarter thereof (that is 30 2) may be accounted the angle which they would contain with the same emergent mean refrangible rays were they coincident to them within the glass and suffered no other refraction than that at their emergence For if two equal refractions the one at the incidence of the rays on the prism the other at their emergence make half the angle 2 degrees

ll make about a quarter of that angle and
efraction of the mean
les of refraction of the
degrees 4 55 whose

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refraction and 8 the remainders and 25 shew that in small refractio is
refract on of the most re-
fractions
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the whole refraction of the mean refrangible rays.

Whence they that are skilled in Optics will easily understand that the breadth of the least circular space into which object-glasses of telescopes can collect all sorts of parallel rays, is about the 27¼th part of half the aperture of the glass, or 55th part of the whole aperture and that the focus of the most refrangible rays is nearer to the object-glass than the focus of the least refrangible ones by about the 27½th part of the distance between the object glass and the focus of the mean refrangible ones.

And if rays of all sorts flowing from any one lucid point in the axis of any convex lens be made by the refraction of the lens to converge to points not too remote from the lens the focus of the most refrangible rays shall be nearer to the lens than the focus of the least refrangible ones by a distance which is to the 27½th part of the distance of the focus of the mean refrangible rays from the lens as the distance between that focus and the lucid point from whence the rays flow is to the distance between that lucid point and the lens very nearly

Now to examine whether the difference between the refractions which the most

Exper. 16 The lens which I used in the second and eighth Experiments being placed six feet and an inch distant from any object collected the species of that object by the mean refrangible rays at the distance of six feet and an inch from the lens on the other side And therefore by the foregoing rule it ought to collect the species of that object by the least refrangible rays at the distance of six feet and 3⅔ inches from the lens and by the most refrangible ones at the distance of five feet and 10⅓ inches from it So that between the two places where these least and most refrangible rays collect the species there may be the distance of about 5⅓ inches For by that rule as six feet and an inch (the distance of the lens from the lucid object) is to twelve feet and two inches (the distance of the lucid object from the focus of the mean refrangible rays) that is as one is to two so is the 27½th part of six feet and an inch (the distance between the lens and the same focus) to the distance between the focus of the most refrangible rays and the focus of the least refrangible ones which is therefore 5¼ inches that is very nearly 5⅓ inches Now to know whether this measure was true I repeated the second and eighth experiment with coloured light which was less compounded than that I there made use of

For I now separated the hete

men from this spectrum to cc
same distance on the othe

ated with indigo and viol
could not read them whereupo ... I found it was full of veins running from the one end of the glass to the other so that the refraction ... not be regular I took another prism the ... instead of the ...
the strokes ... upon these lines in such manner that the ... along the colours from one end of the spectrum to the other I found that the focus where the indigo or confine of this colour and violet cast the species of the black lines most distinctly to be about four inches or 4¼ nearer to the lens than the focus where the deepest red cast the species of the same black lines most distinctly The violet was so faint and dark that I could not discern the species of the lines distinctly by that colour and therefore considering that the prism was made of a dark-coloured glass inclining to green I took another prism of clear white glass but the spectrum of colours which this prism made had long white streams of faint light shooting out from both ends of the colours which made me conclude that something was amiss and viewing the prism I found two or three little bubbles in the glass which refracted the light irregularly Wherefore I covered that part of the glass with black paper and letting the light pass through another part of it which was free from such bubbles the spe ...
those irregular ...
the violet so d
violet and no ... was next the end of the spectrum I su ... therefore that this faint and dark colo ...
layed by that scattering light ...

... yet it being of a white colour ... the sense so strongly as to disturb the phenomena of that weak and dark colour the violet and therefore I tried (as in the 12th 13th and 14th experiments) whether the light of this colour did no ... sensible mixture of heterogene

... and thinness of its light ... the axis of the lens I divided therefore those parallel black lines into equal parts by which I might readily know the distances of the colours in the spectrum from one another and noted the distances of the lens from the foci of such colours as cast the species of the lines distinctly and then considered whether the difference of those distances bear such proportion to

5⅓ inches the greatest difference of the distances which the foci of the deepest red and violet ought to have from the lens as the distance of the observed
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breadth And my observations were a... ion

When I observed and compared the deepest sensible red and the colour in
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But here it is to be noted that I could not see the red to the full end of the spectrum but only to the centre of the semicircle which bounded that end or a little farther and therefore I compared this red not with that colour which was exactly in the middle of the spectrum or confine of green and blue but with that which verged a little more to the blue than to the green And as I reckoned the whole length of the colours not to be the whole length of the spectrum but the length of its rectilinear sides so completing the semicircular ends into circles when either of the observed colours fell within those circles I measured the distance of that colour from the semicircular end of the spectrum and subducting half this distance from the measured distance of the two colours I took the remainder for their corrected distance and in these observations set down this corrected distance for the difference of the distances of their foci from the lens For as the length of the rectilinear sides of the spectrum would be the whole length of all the colours were the circles of which (as we shewed) that spectrum consists contracted and reduced to physical points so in that case this corrected distance would be the real distance of the two observed colours

When, therefore I further observed the deepest sensible red and that blue whose corrected distance from it was 7/12 parts of the length of the rectilinear sides of the spectrum the difference of the distances of their foci from the lens was about 3¼ inches and as 7 to 12 so is 3¼ to 5⁴⁄₇

When I observed the deepest sensible red and that indigo whose corrected distance was 8/12 or ⅔ of the length of the rectilinear sides of the spectrum the difference of the distances of their foci from the lens was about 3⅔ inches and as 2 to 3 so is 3⅔ to 5½

When I observed the deepest sensible red and that deep indigo whose corrected distance from one another was 9/12 or ¾ of the length of the rectilinear sides of the spectrum the difference of the distances of their foci from the lens was about 4 inches and as 3 to 4 so is 4 to 5⅓

When I observed the deepest sensible red and that part of the violet next the indigo whose corrected distance from the red was 10/12 or ⅚ of the length

For I now separated the heterogeneous rays from one another by the method I described in the eleventh experiment so as to make a coloured spectrum about twelve or fifteen times longer than broad Th
book and placing the above mentı
inch from this spectrum to collect t
same distance on the other side I f
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could r [illegible] I found it was full of veins running [illegible] one end of the glass to the other so that the refraction could not be regular I took another prism therefore which was free from veins and instead of the letters I used two or three parallel black lines a little broader than the strokes of the letters and casting the colours upon these lines in such manner that the lines ran along the colours from one end of the spectrum to the other I found that the focus where the indigo or confine of this colour and violet cast the species of the black lines most distinctly to be about four inches or 4¼ nearer to the lens than the focus whe
species of the
that I cou
therefore [illegible] of a dark coloured glass inclining to green I took another prism of clear white glass but the spectrum of colours which this prism made had long white streams of faint light shooting out from both ends of the colours which made me conclude that something was amiss and viewing the prism I found two or three little bubbles in the glass which refracted the light irregularly Wherefore I covered that part of the glass with black paper and letting the light pass through another part of it which was free from such bubbles the spectrum of colo
those irregul
the violet so
violet and i [illegible] it which was next the end of the spectrum I suspected therefore that this faint and dark colour might be allayed by that scattering light which was refracted and reflected irregularly partly by some very small bubbles in the glasses and partly by the inequalities of their polish which light tho it was but little yet it being of a white colour might suffice to affect the sense so strongly as to disturb the phenomena of that weak and dark colour the violet and therefore I tried (as in the 12th 13th and 14th experiments) whether the light of th
sen
frac
as t [illegible] and by consequence out of this viol [illegible] had it been sensibly compounded with white light And therefore I concluded that the reason why I could not see the
ly by this colour was only the darkness of

better with them than it did with me. For I directed the axis always nearly to the middle of the colours, and then the faint ends of the spectrum being remote from the axis, cast their species less distinctly on the paper than they would have done had the axis been successively directed to them.

Now by what has been said, it is certain that the rays which differ in refrangibility do not converge to the same focus; but if they flow from a lucid point, as far from the lens on one side as their foci are on the other, the focus of the most refrangible rays shall be nearer to the lens than that of the least refrangible, by above the fourteenth part of the whole distance; and if they flow from a lucid point so very remote from the lens that before their incidence they may *be accounted parallel, the focus of the most refrangible rays* shall be nearer to the lens than the focus of the least refrangible by about the 27th or 28th part of their whole distance from it. And the diameter of the circle in the middle space between those two foci which they illuminate, when they fall there on any plane, perpendicular to the axis (which circle is the least into which they can all be gathered) is about the 55th part of the diameter of the aperture of the glass. So that it is a wonder that telescopes represent objects so distinct as they do. But were all the rays of light equally refrangible, the error arising only from the sphericalness of the figures of glasses would be many hundred times less. For if the object-glass of a telescope be plano-convex, and the plane side be turned towards the object, and the diameter of the sphere whereof this glass is a segment, be called D, and the semidiameter of the aperture of the glass be called S, and the sine of incidence out of glass into air be to the sine of refraction as I to R; the rays which come parallel to the axis of the glass shall in the place where the image of the object is most distinctly made be scattered all over a little circle whose diameter is $\frac{Rq \times S\,cub}{Iq \times D\,quad}$ very nearly, as I gather by computing the errors of the rays by the method of in-f

be 100 feet or 1200 inches, and S the semidiameter of the aperture be two inches, the diameter of the little circle $\left(\text{that is } \frac{Rq \times S\,cub}{Iq \times D\,quad}\right)$ will be

31×31×8

inches. And therefore the error arising from the spherical figure of the glass is to the error arising from the different refrangibility of the Rays as

I answer, 'tis because the erring rays are not scattered uniformly over all that circular space, but collected infinitely more densely in the centre than in any other part of the circle, and in the way from the centre to the circumference grow continually rarer and rarer, so as at the circumference to become infinitely rare; and by reason of their rarity are not strong

of the rectilinear sides of the spectrum the diffe en e of t foci from the len sometimes the blue an a a the Sun shone clear and I held my eye very near to the paper on which the l n lines I coul violet whic ...en by above half the vio ... ing these experiments I had observed that the species of those colours only appear distinct which were ... so that if the bl and then ... erefore I con trived to ... spectrum of colours shorter than before so that both its ends might be nearer to the axis of the lens And now its length was about $2\frac{1}{2}$ inches and breadth about $\frac{1}{5}$ or $\frac{1}{6}$ of an inch Also instead of the black lines on which the spectrum was cast I made one black line broader than those th t I might see its species more ea ly nd ...

I ... ... of the spectrum and made the o f... observations

When I observed the deepest sensible red and that part of the violet whose corrected distance from it was about $\frac{8}{9}$ parts of the rectilinear sides of the spectrum the difference of the distances of the foci of those colours from the lens was one time $4\frac{2}{3}$ another time $4\frac{3}{4}$ another time $4\frac{7}{8}$ inches and as 8 to 9 so are $4\frac{2}{3}$ $4\frac{3}{4}$ $4\frac{7}{8}$ to $5\frac{1}{4}$ $5\frac{11}{3}$ $5\frac{31}{64}$ respectively

When I observed the deepest sensible red and deepest sensible violet (the corrected distance of which colours when all things were ordered to the best advantage and the Sun hone very clear was about $\frac{11}{1}$ or $\frac{15}{16}$ parts of the length of the rectilinear sides of the coloured spectrum) I found the difference of the distances of their foci from the lens sometimes $4\frac{3}{4}$ sometimes $5\frac{1}{4}$ and for the most part 5 inches or thereabouts and as 11 to 12 or 15 to 16 so is five inches to $5\frac{5}{11}$ or $5\frac{1}{3}$ inches

And by this progression of experiments I satisfied myself that had the light at the very ends of the spectrum been strong enough to make the species of the black lines appear plainly on the paper the focus of the deepest violet would have been found nearer to the lens than the focus of the deepest red by about $5\frac{1}{3}$ inches at least And this is a further evidence that the sines of incidence and refraction of the several sorts of rays hold the same proportion to one another in the smallest refractions which they do in the greatest

My progress in making this nice and troublesome experiment I have set down more at large that they that shall try it after me may be aware of the circumspection requisite to make it su ... annot make it ... he proportion ... the distances ... ould be the success in the more distant colours by a better trial And yet if they u e a broader lens than I did and fix it to a long strait staff by means of which it may be readily and truly directed to the colour whose focus is desired I question not but the experiment will succeed

darker colours than these and much more rarefied may be neglected For the dense and bright light of the circle will obscure the rare and weak light of these ... l m l th m almost insen ble The sens ble

round about it which a spectator will scarce regard And therefore in a

experience for some astronomers have found the diameters of the fixed stars in telescopes of between 20 and 60 feet in length to be about 5 or 6 or at most 8 or 10 seconds But if the eye-glass be tinted faintly with the smoke of a lamp or torch to obscure the li ht of the star the fainter light in the circumference of the star ceases to be visible and the star (if the glass be sufficiently soiled with smoke) appears something more like a mathematical point And for the same reason the enormous part of the light in the circumference of every lucid point ought to be less discernible in shorter telescopes than in longer because the shorter transmit less light to the eye

Now that the fixed stars by reason of their immense distance appear like points unless so far as their light is dilated by refraction may appear from hence that when the moon passes over them and eclipses them their light vanishes not gradually like that of the planet but all at once and in the end of the eclipse it returns into ight all at once or certainly in less time than the second of a minute the refraction of the moon s atmosphere a little protracting the time in which the light of the star first vanishes and afterwards returns into sight

Now if we suppose the sensible image of a lucid point to be even 250 times narrower than the aperture of the glass yet this image would be still much

is not the spherical figures of glasses but the different refrangibility of the rays which hinders the perfection of telescopes

ious lengths magnify with equal distinctness the apertures of the object glasses and the charges or magnifying powers ought to be as the cubes of the square roots of their lengths which doth not answer to experience But the errors of the rays arising from the different refrangibility are as the apertures of the object-glasses and thence to make telescopes of various lengths magnify with equal distinctness their apertures and charges ought

enough to be visible unless in the centre and very nea ... represent one of ... AC and let BFG ... circumference the ... N and by my re ... the light in any place B will be to its density in N as AB to BC and the whole light within the lesser circle BFG will be to the whole light within the greater AED as the excess of the square of AC above the square of AB is to the square of AC As if BC be the fifth part of AC the light will be four times denser in B than in N and the whole light within the less circle will be to the whole light within the greater as nine to twenty five Whence it s evident that the light with in the less circle must strike the sense much more strongly than that faint and dilated light round about between it and the circumference of the greater

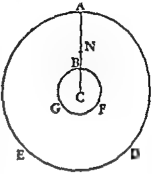

Fig 27

But it s further to be noted that the most luminous of the prismatic colours are the yellow and orange These affect the senses more strongly than all the rest together and next to these in strength are the red and green The blue compared with these is a faint and dark colour and the indigo and violet are much darker and fainter so that these compared with the stronger colours are little to be regarded The images of objects are therefore to be placed not in the focus of the mean refrangible rays which are in the confine of green and blue but in the focus of those rays which are in the middle of the orange and yellow there where the colour is most luminous and fulgent (that is in the brightest yellow that yellow which inclines more to orange th n to gre ... by th ... are ... mea ... place the image of the object in the focus of the ... rays and all the yellow and orange will fall within a circle whose diameter is about the 250th part of the diameter of the apert ... ld the b ...

... colours will fall with ... the ... of b ...

... willow green) about one quarter will fall within this circle and three-quarters without and that which falls without will be spread through about four or five times more space than that which falls within and so in the gross be rarer and if compared with the whole light within it will be about 25 times rarer than all that taken in the gross or rather more than 30 or 40 times rarer becau e the deep red in the end of ...

... affect the sense es pecially since the deep red and willow green of this light are much darker colours than the rest And for the same reason the blue and violet being much

charged Had it magnified but 30 or 25 times it would have made the object appear more brisk and pleasant Two of these I made about 16 years ago and have one of them still by me by which I can prove the truth of what I write Yet it is not so good as at the first For the concave has been divers times tarnished and cleared again by rubbing it with very soft leather When I made

inches in diameter the one convex, the other concave ground very true to one another On the convex I ground the object metal or concave which was to be polished, till it had taken the figure of the convex and was ready for a polish Then I pitched over the convex very thinly by dropping melted pitch upon it and warming it to keep the pitch soft whilst I ground it with the concave copper wetted to make it spread evenly all over the convex. Thus by working it well I made it as thin as a groat and after the convex was cold I ground it

a brisk motion for about two or three minutes of time leaning hard upon it Then I put fresh putty upon the pitch and ground it again till it had done making a noise and afterwards ground the object metal upon it as before And this work I repeated till the metal was polished grinding it the last time with all my strength for a good while together and frequently breathing upon the pitch to keep it moist without laying on any more fresh putty The object metal was two inches broad and about one-third part of an inch thick to keep it from bending. I had two of these metals and when I had polished them both I tried which was best and ground the other again to see if I could make it better than that which I kept And thus by many trials I learned the way of polishing till I made those two reflecting perspectives I spake of above For this art of polishing will be better learned by repeated practice than by my description Before I ground the object metal on the pitch I always ground the putty on it with the concave copper till it had done making a noise because if the particles of the putty were not by this means made to stick fast in the pitch they would by rolling up and down grate and fret the object metal and fill it full of little holes.

But because metal is more difficult to polish than glass and is afterwards very apt to be spoiled by tarnishing and reflects not so much light as glass quick-silvered over does I would propound to use instead of the metal a glass ground concave on the foreside and as much convex on the backside and quick-silvered over on the convex side The glass must be everywhere of the same thickness exactly Otherwise it will make objects look coloured and indistinct By such a glass I tried about five or six years ago to make a reflecting telescope of four feet in length to magnify about 150 times and I satisfied myself that there wants nothing but a good artist to bring the design to perfection. For the glass being wrought by one of our London artists after such a manner as they grind glasses for telescopes though it seemed as well wrought as the object-glasses use to be yet when it was quick-silvered, the reflexion

to be as the square roots of their lengths and this answers to experience as is well known For instance a telescope of 64 feet in length with an aperture of 2⅔ inches magnifies about 120 times with as much distinctness as one of a foot in length with ⅓ of an inch aperture magnifies 15 times

Now were it not for this different refrangibility of rays telescopes might be brought to a greater perfection than we have yet described by composing the object-glass of two glasses with water between them Let ADFC [Fig 28] represent the object glass composed of two glasses ABED and BEFC alike convex on the outsides AGD and CHF and alike concave on the insides BME BNE with water in the concavity BMEN Let the sine of incidence out of glass into air be as I to R and out of water into air as K to R and by consequence out of glass in to water as I to K and let the diameter of the sphere to which the convex sides AGD and CHF are ground be D and the diameter of the sphere to which the concave sides BME and BNE are ground be to D as the cube root of KK—KI to the cube root of RK—RI and the refractions on the concave sides of the glasses will very much correct the errors of the refractions on the convex sides so far as they arise from the sphericalness of the figure And by this means might telescopes be brought to sufficient perfection were it not for the different refrangibility of several sorts of rays But by reason of this different refrangibility I do not yet see any other means of improving telescopes by refractions alone than that of increasing their lengths for which end the late contrivance of Huygens seems well accommodated For very long tubes are cumbersome and scarce to be readily managed and by reason of their length are very apt to bend and shake by bending so as to cause a continual trembling in the objects whereby it becomes difficult to see them distinctly whereas by his contrivance the glasses are readily manageable and the object glass being fixed upon a strong upright pole becomes more steady

Fig 8

Seeing therefore the improvement of telescopes of given lengths by refractions is desperate I contrived heretofore a perspective by reflexion using instead of an object glass a concave metal The diameter of the sphere to which the metal was ground concave was about 25 English inches and by consequence the length of the instrument about six inches and a quarter The eye-gla s was plano-convex and the diameter of the sphere to which the convex side was ground was about ⅕ of an inch or a little less and by consequence it magnified between 30 and 40 times By another way of measuring I found that it magnified about 35 times The concave metal bore an aperture of an inch and a third part but the aperture was limited not by an opaque circle covering the limb of the metal round about but by an opaque circle placed between the

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with a concave eye-glass I could read at a greater distance with my own

tance of this prism from the speculum be such that the rays of the light PQ RS &c which are incid nt upon th speculum in lines parallel to th e axis thereof may enter the prism at th e side LF and be reflected by the side FG and ... to the point T which mu t be the ... vex eye-gla s H ... ays at their com ... through a small round hole or aperture made in a ... las to be covered which ... t enough to pass through ... which tis made intercept ... ces from the verges of the ... feet lo g (reckoning ...

... vement that the speculum be an inch or two broader than the aperture at the least and that the glass of the speculum be thick that it bend not in the working The prism LFG must be no bigger than is necessary and its back side FG must not be quick-silvered over For without quick-silver it will reflect all the light incident on it from the speculum

In this instrument the object will be inverted but may be erected by making the quare sides FF and FG of the pri m LFG not plane but sph rically con vex that the rays may cross as well before they come at it as afterward between it and the eye-glass. If it be desired that the instrument bear a larger aperture that may be also done by composing the speculum of two glasses with water between them

If the theory of making telescopes could at length be fully brought into practice yet there would be certain bounds beyond which telescopes could not perform For the air through which we look upon the stars is in a perpetual tremor as may be seen by the tremulous motion of shadows cast from high ...

apart and by means of their various and sometimes contrary tremors fall at one and the same time upon different points in the bottom of the eye and their trembling motions are too quick and confused to be perceived severally And all these illuminated points constitute one broad lucid po nt composed of those many trembling points confusedly and insensibly mixed with one anotl er by very short and swift tremors and thereby cause the star to appear broader th ...

t mors of the atmosphere The only remedy is a most serene and quiet air such as may pe haps be found on the tops of the highest mountains above the grosser clouds.

discovered innumerable inequalities all over the glass And by reason of these inequalities objects appeared indistinct in this instrument For the errors of reflected rays caused by any inequality of the glass are about six times greater than the errors of refracted rays cau ed b b

... and no sensible prejudice to it and by consequence that nothing is wanting to perfect these telescopes but good workmen who can grind and polish glasses truly spherical An object glass of a fourteen foot telescope made by an artificer at London I once m n d siderably by grinding
grinding lest
enough for pol ses I have not yet tried But he that shall try either this or any other way of polishing which he may think better may do well to make his glasses ready for polishing by grinding them without that violence wherewith our London workmen press their glasses in grinding For by such violent pressure glasses are apt to bend a little in the grinding and such bending will certainly spoil their figure To recommend therefore the consideration of these reflecting glasses to such artists as are curious in figuring glasses I shall describe this optical instrument in the following Proposition

PROPOSITION 8 PROBLEM 2

... the foreside AB and as much convex on the backside CD so that it be every where of an equal thickness Let it not be thicker on one side than on the other lest it make objects appear coloured and indistinct and let it be very truly wrought and quick silvered over on the backside and set in the tube VXYZ which must be very black within Let EFG represent a prism of glass or crystal placed near the other end of the tube in the middle of it by means of a handle of brass or iron FGK to the end of which made flat it is cemented Let this prism be rectangular at E and let the other two angles at F and G be accurately equal to each other and by consequence equal to half right ones and let the plane sides FE and GE be square and by consequence the third side FG a rectangular parallelogram whose length is to its breadth in a subduplicate proportion of two to one Let it be so placed in the tube that the axis of the speculum may pass through the middle of the square side EF perpendicularly and by consequence through the middle of the side FG at an angle of 45 degrees and let the side FF be turned towards the speculum and the dis-

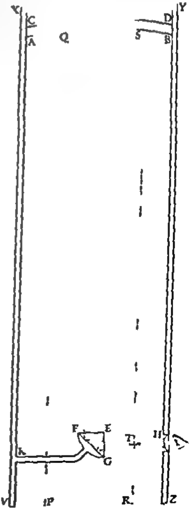


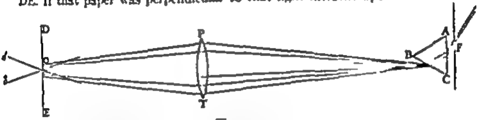
Fig 29

— h hen of . d'm led I it that the
lass large
the vessel
with salt

water or clear oil

Exper. 2 The Sun's light let into a dark chamber through the round hole F
— — I C ol and he

DE. If that paper was perpendicular to that light incident upon it as tis



Fig

represented in the posture DF all the colours upon it at O appeared white But if the paper being turned about an axis parallel to the prism became very

the prism in all these cases remained the same

Exper. 3 Such another experiment may be more easily tried as follows Let a broad beam of h S

Fig 3

posture de or into blue and violet as in the posture And if the light before it fall upon the paper be t ice refracted the same way by t o parallel prisms these colours will become the more conspicuous Here all the middle parts of the broad beam of white light which fell upon the paper did without any confine of shadow to modify it become coloured all over with one uniform colour the colour being always the same in the middle of the paper as at the edges and the colour

Part II

PROPOSITION 1 THEOREM 1

THE PHENOMENA OF COLOURS IN REFRACTED OR REFLECTED LIGHT ARE NOT CAUSED BY NEW MODIFICATIONS OF THE LIGHT VARIOUSLY IMPRESSED ACCORDING TO THE VARIOUS TERMINATIONS OF THE LIGHT AND SHADOW

The Proof by Experiments

ough an oblong
inch or some-
ry large prism
ABC distant about 20 feet from t e ho e then (with its white part) through an oblong hole H, whose breadth is about the fortieth or sixtieth part of an inch and which is made in a black opaque body GI and

mitted through the hole H fall afterwards upon a white paper p that hole H at the distance of three or four feet from it and there paint the

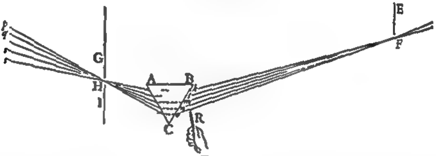


Fig 1

usual colours of the prism (suppose red at t yellow at s green at r blue at q and violet at p) *you may with an iron wire or any such like slender opaque* body whose breadth is about the tenth part of an inch by intercepting the rays at k l m n or o take away any one of the colours at t s r q or p whilst the other colours remain upon the paper as before or with an obstacle something bigger *you may take away any two or three or four colours together* the rest remaining So that any one of the colours as well as violet may become outmost in the confine of the shadow towards p and any one of them as well as red may become outmost in the confine of the shadow towards t and any one of them may *also border upon the shadow made within the colours* by the obstacle R intercepting some intermediate part of the light and lastly any one of left alone may border upon the shadow on either hand All the

confines of shadow whereby light

the opinion of philosophers In trying these things tis to be observed that by how much the holes F and H are narrower and the intervals between them and the prism greater and the chamber darker by so much the better doth the

prisms about their common axis all the colours were made to vanish but the red the light which makes that red being left alone appeared of the very

refraction or
an modifica
rs This un
changeableness of colour I am now to describe in the following Proposition

Propo ition 2 Theorem 2

All homogeneal light has its proper colour answering to its degree of refrangibility and that colour cannot be changed by reflexions and refractions

In the experiments of the fourth Proposition of the first part of this first

knew by refracting with a prism sometimes one very little part of this light sometimes another very little part as is described in the twelfth experiment of the first part of this book. For by this refraction the colour of the light was never changed in the least If any part of the red light was refracted it remained totally of the same red colour as before No orange no yellow no green or blue no other new colour was produced by that refraction Neither did the colour any way change by repeated refractions but continued always the same red entirely as at first The like constancy and immutability I found also in the blue green and other colours So also if I looked through a prism upon any body illuminated with any part of this homogeneal light as in the fourteenth experiment of the first part of this book is described I could not perceive any new colour generated this way All bodies illuminated with compound light appear through prisms confused (as was said above) and tinged with various new colours, but those illuminated with homogeneal light appeared through prisms neither less distinct nor otherwise coloured than when viewed with the naked eyes Their colours were not in the least changed by the refraction of the interposed prism I speak here of a sensible change of colour for the light which

Exper.
were they
bodies as
grass blu
cock's fea

changed according to the —
change in the refractions (
And therefore these colou
new modifications of light by refractions and shadows ... om some other cause than the

If it be asked What then is their cause? I an
ture *de* be
gible ones
therefore
wherever t
may in some measure app ... position of the first part of this book and will more fully appear hereafter And the contrary happens in the posture of the paper *δε* the more refrangible rays being then predominant which always tinge light with blues and violets

EXPER 4 The colours of bubble

t ... uation even whilst the eye
any light or cast any shadow rem... unmoved And therefore their colours arise from some regular cau e which depends not on any confine of shadow What this cause is will be shewed in the next book

To these experiments may be added the tenth experiment of the first part of this first book where the Sun s light in a dark room being trajected through the parallel superficies of two prisms tied together in the form of a parallel epiped became totally of one uniform yellow or red colour at its emerging out of the prisms Here in the production of these colours the confine of shadow can have nothing to do For the light changes from white to yellow orange and red successively without any alteration of the confine of shadow And at both edges of the emerging light where the contrary confines of shadow ought to produce different effects the colour is one and the same whether it be white yellow orange or red And in the middle of the emerging light where there is no confine of shadow at all the colour is the very same as at the edges the whole light at its very first emergence being of one uniform colour whether white yellow orange or red and going on thence perpetually without any change of colour such as the confine of shadow is vulgarly ... refracted light afte
new r
from
and also because the refractions ... the refractions remain the same which destroy one another s effects They arise not therefore from any modi fications of light made by refractions and shadows but have some other cause What that cause is we shewed above in this tenth experiment and need not here repeat it

There is yet another material circ
or this
owards
ellow
prism
... the light before its incidence on that prism And yet in that experiment we found that when by turning the two first

those rectilinear sides AF and GM And therefore in those rectilinear sides when distinctly defined there is no new colour generated by refraction I ... between the two outmost circles TMF and ... oth ends to fall ... ne and the sam

whilst an assistant whose eyes for distinguishing colours ... I ... the than mi ... the confines ... the yellow ...

lines divided after the manner of a musical chord Let GM be produced to X that MX may be equal to GM and conceive GX λX ιX ηX εX γX αX MX to be in proportion to one another as the numbers 1 8/9 5/6 3/4 2/3 3/5 9/16 ...

least refrangible rays out of glass into air was (by a method described above) found in proportion to their sines of refraction as 50 to 77 and 78 divide the difference between the sines of refraction 77 and 78 as the line GM is divided by those intervals and you will have 77 77⅛ 77⅕ 77⅓ 77½ 77⅔ 77⅞ 78 the sines of refraction of those rays out of glass into air their common sine of incidence being 50 So then the sines of the incidences of all the red making rays out of glass into air were to the sines of their refractions not greater than 50 to 77

77⅛ unto that of 50 to 77⅛ And by the like limits above-mentioned were the refractions of the rays belonging to the rest of the colours defined the sines of the red making rays extending from 77 to 77⅛ those of the orange-making from 77⅛ to 77⅕ ...

made out of air into glass are easily derived

Exper. 8 I found moreover that when light goes out of air through several contiguous refracting mediums as through water and glass and thence goes out

homogeneal light appeared totally red in blue light totally blue in green light totally green and so of other colours In the homogeneal light of any colour they all appeared totally of that same colour with this only difference that some of them reflected that light more strongly others more faintly I never yet found any body which by reflecting homogeneal light could sensibly change its colour

From all which it is manifest that if the Sun s light consisted of but one sort of rays there would be but one colour in the whole world nor would it be possible to produce any new colour by reflexions and refractions and by consequence that the variety of colours depends upon the composition of light.

DEFINITION

The homogeneal light and rays which appear red or rather make objects appear so I call rubrific or red making those which make objects appear yellow green blue and violet I call yellow making green making blue-making violet-making and so of the rest And if at any time I speak of light and rays as coloured or endued with colours I would be understood to speak not philosophically and properly but grossly and accordingly to h

cent on

t

I

a

tl

sc ou ut object are noth ng h

that sort of rays more copiously

their dispositions to propagate t

the sensorium they are sensations of those motions under the forms of colours

PROPOSITION 3 PROBLEM 1

To define the refrangibility of the several sorts of homogeneal light answering to the several colours

For determining this Problem I made the following experiment

EXPER 7 When I had caused the rectilinear side AF GM (F

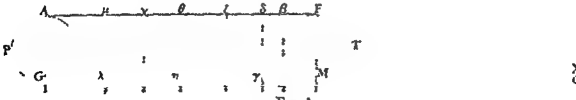


Fig 4

par which the spectrum of compound light PT is composed and which in the middle parts of the pectrum interfere and are intermixed with one another are not intermixed in their outmost parts where they touch

those rectilinear sides AF and GM And therefore in those rectilinear sides when distinctly defined there is no new colour generated by refraction I observed also that if anywhere bet veen the two outmost circles TMF and PGA a right line as γδ was cross to the spectrum so as both ends to fall perpendicularly upon its rectilinear side there appeared one and the same ... from one end of this lin to the other I delineated ... CMT and in trying ... he paper so that the ... gree with it exactly

whilst an assistant whose eyes for distinguishing co ours were more critical than mine did by right lines aβ γδ εζ &c drawn cross the spectrum note the confines of the colours (that is of the red MaβF of the orange aγδβ of the yellow γ ... δ of the green ... of the blue ... of the indigo λμ ... and of the ... same ... h with ... d cross

lines divided after the manner of a musical chord Let GM be produced to X that MX may be equal to GM and conceive GX λX ... X ... X γX aX, MX to be in proportion to one another as the numbers 1 8/9 5/6 3/4 2/3 3/5 9/16 1/2 and so to represent the chords of the key and of a tone a third minor a fourth a fifth a sixth major a seventh and an eighth above that key And the intervals Ma aγ γε εη ... λ and λG will be the spaces which the several colours (red, orange yellow green blue indigo violet) take up

Now these intervals or spaces subtending the differences of the refractions of the rays going to the limits of those colours (that is to the Points M a γ ε ... λ, G) may without any sensible error be accounted proportional to the ...

found in proportion to their sines of refraction as 50 to 77 and 78 divide the difference between the sines of refraction 77 and 78 as the line GM is divided by those intervals and you will have 77 77 1/8 77 1/5 77 1/3 77 1/2 77 2/3 77 7/9 78

... nor less than 50 to 77 1/2 but they varied from one another according to all intermediate proportions. And the sines of the incidences of the green making rays were to the sines of their refractions in all proportions from that of 50 to 77 1/3 unto that of 50 to 77 1/2 And by the like limits above-mentioned were the refractions of the rays belonging to the rest of the colours defined the sines of ...

There ...

contiguous refracting mediums as through water and glass and thence goes out

again into air whether the refracting superficies be parallel or incl d to another that light as often as by con
ow

prisms of glas v of water Now those colours argue a diverging and separation of the heterogeneous rays f om on by means of their unequal ref
And on the contrary the p
the rays there is no such se
no inequality of their whe o gather the two following theorems
1 The excesses of the sines of of
common sine of incidence when t
mediums immediately into one ar ater medium (suppose of air) are to one another in a given proportion
2 The proportion of the sine of incidence to the ine of
the same sort of
portion
into any
sine of re into the second medium
By the first theorem the refractions of the rays of every sort made out of any medium into air are known by having the refraction of the rays of any one sort As for instance if the refractions of the rays of every sort out of rain water into air be desired let the common

t ast of the exce
the common sine of ater into air to which sine (if you add all the above mentioned excesses) you will have the desired sines of the refractions 108 108⅛ 108⅕ 108⅓ 108½ 108⅔ 108⅘ 109
By the latter theorem the refraction out of one medium into another is gathered as often as you have the refractions out of them both into any third medium As if the sine of incidence of any ray out of glass into air be to its sine of refraction as 20 to 31 and the sine of incidence of the s me
water be to its sine of
of glass into water w
that

is less necessary to the argument

and by the successes I met with in the trial I dare promise that to him who shall argue truly and then try all things with good glasses and sufficient circumspection the expected event will not be wanting But he is first to know what colours will arise from any others mixed in any assigned proportion

PROPOSITION 4 THEOREM 3

Colours may be produced by composition which shall be like to the colours of any of the colours of homogeneal Light

For a mixture of homogeneal red and yellow compounds an orange like in appearance of colour to that orange which in the series of unmixed prismatic colours lies between them but the light of one orange is homogeneal as to refrangibility and that of the other is heterogeneal and the colour of the one if viewed through a prism remains unchanged that of the other is changed and resolved into its component colours red and yellow And after the same manner other neighbouring homogeneal colours may compound new colours like the intermediate homogeneal ones as yellow and green the colour between them both and afterwards if blue be added there will be made a green the middle colour of the three which enter the composition For the yellow and blue on either hand if they are equal in quantity they draw the intermediate green equally towards themselves in composition and so keep it as it were in equilibrium that it verge not more to the yellow on the one hand and to the blue on the other but by their mixed actions remain still a middle colour To this mixed green there may be further added some red and violet and yet the green will not presently cease but only grow less full and vivid and by increasing the red and violet it will grow more and more dilute until by the prevalence of the added colours it be overcome and turned into whiteness or some other colour So if to the colour of any homogeneal Light the Sun s white light composed of all sorts of rays be added that colour will not vanish or change its species but be diluted and by adding more and more white it will be diluted more and more perpetually Lastly if red and violet be mingled there will be generated according to their various proportions various purples such as are not like in appearance to the colour of any homogeneal Light and of these purples mixed with yellow and blue may be made other new colours

PROPOSITION 5 THEOREM 4

Whiteness and all grey colours between white and black may be compounded of colours and the whiteness of the Sun s Light is compounded of all the primary colours mixed in a due proportion

The Proof by Experiments

EXPERIMENT The Sun shining into a dark chamber through a little round hole in the window shut and his Light being there refracted by a prism to cast his coloured image PT (Fig. 5) upon the opposite wall I held a white paper V to that image in such manner that it might be illuminated by the coloured Light

reflected from thence and yet not intercept any part of that light in its passage from the prism to the spectrum And I found that when the paper was held nearer to any colour than to the rest it appeared of that colour to which it approached nearest but when it was equally or almost equally distant from all the colours so that it might be equally illuminated by them all it appeared white And in this last situation of the paper if some colours were intercepted the paper lost its white colour and appeared of the colour of the rest of the light which was not intercepted So then the paper was illuminated with lights of various colours (namely red yellow, green blue and violet) and every part

Fig 5

of the light retained its proper colour until it was incident on the paper and became reflected thence to the eye so that if it had been either alone (the rest of the light being intercepted) or if it had abounded most and been predominant in the light reflected from the paper it would have tinged the paper with its own colour and yet being mixed with the rest of the colours in a due proportion it made the paper look white and therefore by a composition with the rest produced that colour The several parts of the coloured light reflected from the spectrum whilst they are propagated from thence through the air do perpetually retain their proper colours because wherever they fall upon the eyes of any spectator they make the several parts of the spectrum to appear under their proper colours They retain therefore their proper colours when they fall upon the Paper V and so by the confusion and perfect mixture of those colours compound the whiteness of the light reflected from thence

EXPER 10 Let that spectrum or solar image PT [Fig 6] fall now upon the lens MN above four inches broad and about six feet distant from the prism ABC and so figured that it may cause the coloured light which divergeth from the prism to converge and meet again at its focus G about six or eight feet distant from the lens and there to fall perpendicularly upon a white paper DE And if you move this paper to and fro you will perceive that near the lens as at *de* the whole solar image (suppose at *pt*) will appear upon it intensely

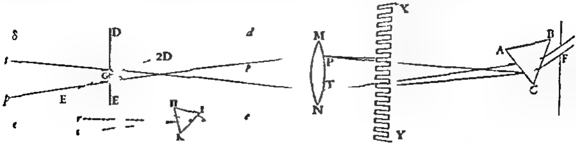


Fig 6

t ... from the lens

and thereby make the colours to appear again but yet in a ... suppose at k where the red t is now above which before was below and the violet p is below which before was above

... focus G where the light appears totally

the whiteness will cease and degenerate into that colour which arises from ... composition of the other colours which are not intercepted And then if the intercepted colours be let pass and fall upon that compound colour they mix with it and by their mixture restore the whiteness So if the violet blue and green be intercepted the remaining yellow orange and red will compound upon the paper an orange and then if the intercepted colours be let pass they will fall upon this compounded orange and together with it decompound a white So also if the red and violet be intercepted the remaining yellow green and blue will compound a green upon the paper and then the red and violet being let pass will fall upon this green and together with it decompound a white And that in this composition of white the several rays do not suffer any change in their colorific qualities by acting upon one another but are only mixed and by a mixture of their colours produce white may further appear by these arguments.

If the paper be placed beyond the focus G suppose at k and then the red colour at the lens be alternately intercepted and let pass again the violet ...

letting pass the violet which crosseth it

And if the paper be placed at the focus G and the white round image at G be viewed through the prism HIK and by the refraction of that prism be translated to the place rv and there appear tinged with various colours (namely the violet at v and red at r and others between) and then the red colours at the lens be often stopped and let pass by turns the red at r will accordingly disappear and return as often but the violet at v will not thereby suffer any change And so by stopping and letting pass alternately the blue at the lens, the blue at v will accordingly disappear and return without any change made in the red at r The red therefore depends on one sort of rays and the blue on another sort which in the focus G where they are commixed do not act on one another And there is the same reason of the other colours.

I considered further that when the most refrangible rays Pp and the least ...

be tinged in that focus with the colour of the predominant rays provided those

rays severally retained their colours or colorific qualities in the composition of white made by them in that focus But if they did not retain them in that white but became all of them severally endued there with a disposition to strike the sen e with the perception of white then they could never lose their whiteness by such reflexions I inclined therefore the paper to the rays very obliquely as in the econd experiment of this second part of the first book that the most refrangible rays might be more copiously reflected than the rest and the whiteness at length changed successively into blue indigo and violet Then I inclined it the contrary way that the least refrangible rays might be more copious in the reflected light than the rest and the whiteness turned ucces sively to yellow orange and red

Lastly I made an instrument XY in fashion of a comb whose teeth be
number sixteen ~
teeth about two i
instrument near
tooth whilst the o them went on through the interval of the teeth to the paper DE and there painted a round solar image But the paper I had first placed so that the image might appear white as often as the comb was taken away and then the Comb being as was said interposed the whiteness by rea son of the intercepted part of the colours at the lens did always change into the colour compounded of those colours which were not intercept d nd th t colour was by
of every tooth
purple) did al ... one another I caused therefore all the teeth to pass successively over the lens and when the motion was slow there appeared a perpetual succession of the colours upon the paper but if I so much acceler ated the motion that the colours by reason of their quick succession could not be distinguished from one another the appearance of the single colours ceased There was no red no yellow no green no blue nor purple to be seen any longer but from a confusion of them all there arose one uniform white colour Of the light which now by he ~
part really white C
blue a fifth purple ... proper colour till it strikes the sensorium If the impressions follow one another slowly so that they may be severally perceived there is made a distinct sensation of ll the colo rs on

... indifferently to them all and this is a sensation of white ness By the quickness of the successions the impressions of the several colours are confounded in the sensorium and out of that confusion ari eth a mixed sensation If a burning coal be nimbly moved round in a circle with gyrations continually repeated the whole circle will apear like fire the reason of which is that the sensation of the coal in the everal places of that circle remains im pressed on the sensorium until the coal return again to the same place And o in a quick con ecution of the colours the impression of every colour remains in the sensorium until a revolution of all the colours be completed and that first colour return again The impressions therefore of all the successive colours are at once in the ensorium and jointly stir up a sen ation of them all and so it is

manifest by this experiment that the commixed impressions of all the colours do stir up and beget a sensation of white, that is, that whiteness is compounded of all the colours.

And if the comb be now taken away, that all the colours may at once pass from the lens to the paper, and be there intermixed, and together reflected thence to the spectator's eyes; their impressions on the sensorium being now more subtilly and perfectly commixed there, ought much more to stir up a sensation of whiteness.

You may instead of the lens use two prisms HIK and LMN, which by refracting the coloured light the contrary way to that of the first refraction, may make the diverging rays converge and meet again in G, as you see represented in the seventh Figure. For where they meet and mix, they will compose a white light, as when a lens is used.

Exper. 11. Let the Sun's coloured image PT [Fig. 8] fall upon the wall of a dark chamber, as in the third experiment of the first book, and let the same be viewed through a prism abc, held parallel to the prism ABC, by whose refraction that image was made, and let it now appear lower than before, suppose in the place S over-against the red colour T. And if you go near to the image PT,

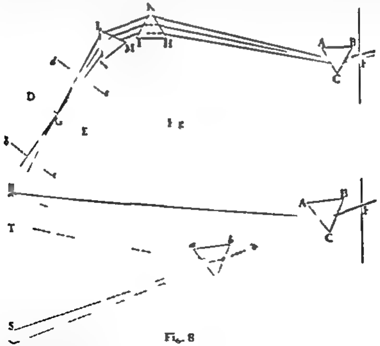


Fig. 8

the spectrum S will appear oblong and coloured like the image PT; but if you recede from it, the colours of the spectrum S will be contracted more and more, and at length vanish, that spectrum S becoming perfectly round and white; and if you recede yet farther, the colours will emerge again, but in a contrary order. Now that spectrum S appears white in that case, when the rays of several sorts which converge from the several parts of the image PT, to the prism abc

are so refracted unequally by it that in their passage from the prism to th they may diverge from one and the — —

f —

urther if the comb be here made use of by whose teeth the colours at the image PT may be successively intercepted the spectrum S when the comb is moved slowly will be perpetually tinged with successive colours But when by accelerating the motion of the comb the succession of the colours is so quick that they cannot be severally seen that spectrum S by a confused and mixed sensation of them all will appear white

EXPER 12 The S n l — — —
XY pl
intersti

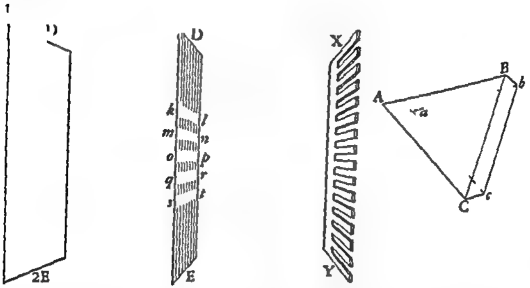


Fig 9

distant from the comb the light which passed through its several interstices painted so many ranges of colours *kl mn op qr* &c which were parallel to one another and contiguous and without any mixture of white And these ranges of colours if the comb was moved continually up and down with a reciprocal motion ascended and descended in the paper and when the motion of the comb was so quick that the colours could not be distinguished from one another the whole paper by their confusion and mixture in the sensorium appeared white

Let the comb now rest and let the paper be removed farther from the prism and the several ranges of colours will be dilated and expanded into one another more and more and by mixing their colours will dilute one another and at length when the distance of the paper from the comb is about a foot or a little more (suppose in the place 2D 2F) they will so far dilute one another as to become white

With any obstacle let all the light be now stopped which passes through any one interval of the teeth so that the range of colours which comes from thence may be taken away and you will see the light of the rest of the ranges to be

extended into the place of the range taken away and there to be coloured Let the intercepted range pass on as before and its colours falling upon the colours of the other ranges and mixing with them will restore the whiteness

Let the paper DE be now very much inclined to the rays so that the most refrangible rays may be more copiously reflected than the rest and the white colour of the paper through the excess of those rays will be changed into blue and violet. Let the paper be as much inclined the contrary way that the least refrangible rays may be now more copiously reflected than the rest and by their excess the whiteness will be changed into yellow and red The several rays therefore in that white light do retain their colorific qualities by which those of any sort whenever they become more copious than the rest do by their excess and predominance cause their proper colour to appear

And by the same way of arguing applied to the third experiment of this second part of the first book, it may be concluded that the white colour of all refracted light at its very first emergence where it appears as white as before its incidence is compounded of various colours.

Exper. 13 In the foregoing experiment the several intervals of the teeth of the comb do the office of so many prisms every interval producing the phenomenon of one prism Whence instead of those intervals using several prisms, I tried to compound whiteness by mixing their colours and did it by using only three prisms as also by using only two as follows Let two prisms ABC and abc [Fig 10] whose refracting angles B and b are equal be so placed parallel to one

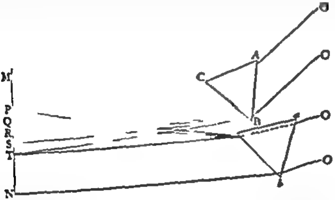


Fig 10

another that the refracting angle B of the one may touch the angle c at the base of the other and their planes CB and cb at which the rays emerge may lie in directum Then let the light trajected through them fall upon the paper MN distant about 8 or 12 inches from the prisms And the colours generated by the interior limits B and c of the two prisms will be mingled at PT and there compound white For if either prism be taken away the colours made by the other will appear in that place PT and when the prism is restored to its place again, so that its colours may there fall upon the colours of the other the mixture of them both will restore the whiteness.

This experiment succeeds also as I have tried when the angle b of the lower prism is a little greater than the angle B of the upper and between the interior

angles B and ∊ there intercedes some n ~ P
and the
anot
than
place most refrangible rays coming from the superior prism take up [illegible] upon the paper in the
all the space from M to P the rays of the same sort which come from the in
ferior prism ought to begin at P and take up all the rest of the space from
thence towards N If the least refrangible rays coming from the superior prism
take up the space MT the rays of the same kind which come from the oth
prism ought to begin at T and take n h
the rays which have intermediate dc
superior prism be extended through
rays through the space MR and a tl ~ them through the space MS
the same sorts of rays coming from the lo
remain
of all t
uniforr
mixed t same
And there
MP and T
reason of the composition by which whiteness the interior space PT This is the
and by what other way soever I made the like composition the result was
whiteness

Lastly if with the teeth of a comb of a due size the coloured lights of the two
prisms which fall upon the space PT be alternately intercepted that space PT
when the motion of the comb is slow will always appear coloured but by
accelerating the motion of the comb so much that the successive colours cannot
be distinguished from one another it will apear white

EXPER 14 Hitherto I have produced whiteness by mixing the colours of
prisms If now the colours of natural bodies are to be mingled let water a
little thickened with soap be agitated to raise a froth and after that froth has
stood a little there will appear to one that shall view it
every where in the
far off that he c
will grow white

EXPER 15 Lastly in attempting

which they
their own
they do not reflect the light of their own colours more sparingly and yet
If red lead for instance and a white paper be placed in the red light of the
coloured spectrum made in a dark ch mb

exceed the light reflected by the red
lead in a much greater proportion And the like happens in powders of other
colours And therefore by mixing such powders we are not to expect a strong

and full white, such as is that of paper, but some dusky obscure one, such as might arise from a mixture of light and darkness, or from white and black, that is, a grey, or dun, or russet brown, such as are the colours of a man's nail, of a mouse, of ashes, of ordinary stones, of mortar, of dust and dirt in highways

of orpiment
tincture and

became perfectly dun. But the experiment succeeded best without minium thus. To orpiment I added by little and little a certain full bright purple, which painters use, until the orpiment ceased to be yellow, and became of a pale red. Then I diluted that red by adding a little *viride aeris*, and a little more blue bice than *viride aeris*, until it became of such a grey or pale white as verged to no one of the colours more than to another. For thus it became of a colour equal in whiteness to that of ashes, or of wood newly cut, or of a man's skin. The

of powders of the same kind. Accordingly as the colour of any powder is more or less full and luminous, it ought to be used in a less or greater proportion.

Now considering that these grey and dun colours may be also produced by mixing whites and blacks, and by consequence differ from perfect whites, not in species of colours, but only in degree of luminousness, it is manifest that there is nothing more requisite to make them perfectly white than to increase their light sufficiently; and, on the contrary, if by increasing their light they can be brought to perfect whiteness, it will thence also follow that they are of the same species of colour with the best whites, and differ from them only in the quantity of light. And this I tried as follows. I took the third of the above-mentioned grey mixtures (that which was compounded of orpiment, purple, bice, and *viride aeris*) and rubbed it thickly upon the floor of my chamber, where the Sun shone upon it through the opened casement; and by it, in the shadow, I laid a piece of white paper of the same bigness. Then going from them to the distance of 12 or 18 feet, so that I could not discern the unevenness of the surface of the powder, nor the little shadows let fall from the gritty particles thereof; the powder appeared intensely white, so as to transcend even the paper itself in whiteness, especially if the paper were a little shaded from the light of the clouds, and then the paper compared with the powder appeared of such a grey colour as the powder had done before. But by laying the paper where the Sun shines through the glass of the window, or by shutting the window that the Sun might shine through the glass upon the powder, and by such other fit means of increasing or decreasing the lights wherewith the powder and paper were illuminated, the light wherewith the powder is illuminated may be made stronger

mina

tryin

told him what the colours were or what I w do no I
two white b l
distance
that he co
if you cor
of the colours which the component powe in the sunshine
æris) have in the same sunshine you mu
well as by the former that perfect white s may be compounded of colours

From what has been said it is also evident
light comp
of th
from
For those colours (by *Prop* II *Part* 2) are unchangeable and whenever all those rays with those their colours are mixed again they reproduce the same white light as before

PROPOSITION 6 PROBLEM 2

In a mixture of primary colours the quantity and quality of each being given to know the colour of the compound

With the centre O [Fig 11] and radius OD describe a circle ADF and distinguish its circumference into seven parts DE EF FG GA AB BC CD proportional to the seven musical tones or intervals of the eight sounds *Sol la fa sol la mi fa sol* contained in an eight that is proportional to the number $\frac{1}{9}$ $\frac{1}{16}$ $\frac{1}{10}$ $\frac{1}{9}$ $\frac{1}{10}$ $\frac{1}{16}$ $\frac{1}{9}$ Let the first part DE represent a red colour the second EF orange the third FG yellow the fourth GA green the fifth AB blue the sixth BC indigo and the seventh CD
are all the
as they
ing the
other so that from D to E be all degrees of red at E the mean colour between red and orange from E to F all degrees of orange at F the mean between orange and yellow from F to G all degrees of yellow and so on Let *p* be the centre of gravity of the arch DE and *q r s t u x* the centres of gravity of the arches EF FG GA AB BC and CD respectively and about those centres of gravity let circles proportional to the number of rays of each colour in the given mixture be described that is the circle *p* proportional to the number of the red making rays in the mixture the circle *q* proportional to the number of the orange making rays in the mixture and so of the rest Find the common centre of gravity of all those circles *p q r s t u x* Let that centre be Z and from the centre of the circle ADF through Z to the circumference drawing the right line OY the place of the point Y in the circumference shall shew the colour aris-

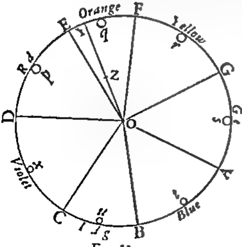


Fig 11

an I the line
r that to
and C the
e middle to-
wards F or G the compound colour shall accordingly be a yellow verging to-
ll be
the
such
a colour as would be made by diluting the intensest yellow with an equal quantity of whiteness and if it fall upon the centre O the colour shall have lost all its intenseness and become a white But it is to be noted that if the point Z fall in or near the line OD the main ingredients being the red and violet the colour

in the circle are opposite to one another be mixed in an equal proportion the point Z shall fall upon the centre O and yet the colour compounded of those two shall not be perfectly white but some faint anonymous colour For I could never yet by mixing only two primary colours produce a perfect white Whether it may be compounded of a mixture of three taken at equal distances in the circumference I do not know but of four or five I do not much question but it may But these are curiosities of little or no moment to the understanding the phenomena of Nature For in all whites produced by Nature there uses to be a mixture of all sorts of rays and by consequence a composition of all colours.

To give an instance of this rule suppose a colour is compounded of these homogeneal colours of violet one part of indigo one part of blue two parts of green three parts, of yellow five parts of orange six parts and of red ten parts Proportional to these parts describe the circles x v t s r q p respectively that is so that if the circle x be one the circle v may be one the circle t two the circle s three and the circles r q and p five six and ten Then I find Z the common centre of gravity of these circles and through Z drawing the line OY the point Y falls upon the circumference between E and F something nearer to E than to F and thence I conclude that the colour compounded of these ingredients will be an orange verging a little more to red than to yellow Also I find that OZ is a little less than one half of OY and thence I conclude that this orange hath a little less than half the fulness or intenseness of an uncompounded orange that is to say that it is such an orange as may be made by mixing an homogeneal orange with a good white in the proportion of the Line OZ to the Line ZY this proportion being not of the quantities of mixed orange and white powders but of the quantities of the lights reflected from them.

This rule I conceive accurate enough for practice though not mathematically accurate and the truth of it may be sufficiently proved to sense by stopping any of the colours at the lens in the tenth experiment of this book For the rest of the colours which are not stopped but pass on to the focus of the lens will there compound either accurately or very nearly such a colour as by this rule ought to result from their mixture

PROPOSITION 7 THEOREM 5

All the colours in the universe which a[...] power of [...] these and [...] problem

For it [...] n proved (Prop 1 Part 2) that the changes of colours made by refractions do not arise from any new modifications of the ra[...] those refractions and by the [...]

[...] refrangibility cannot be changed by [...] Prop [...] Part 2) and that their [...] 2 Part 1) and by consequence that those their colours are likewise immutable It has also been proved directly by refracting and reflecting homogeneal lights apart that their colours cannot be changed (Prop 2 Part 2) It has been proved also that when the several sorts of rays are mixed and in crossing pa[...]s through the same space they do not act on [...] other s[...] sensori[...] a sensa[...] when by [...] concourse and mixtures of all sorts of rays a white colour is produced the white is a mixture of all the colours which the rays would have apart (Prop 5 Part 2) The rays in that mixture do not lo[...]e or alter their several colorific qualities but by all their various kinds of actions mixed in the sensorium beget a sensation of a middling colour between all their colours which [...] whiteness For whiteness is a mean between all colours having itself indifferently to them all so as with equal facility to be tinged with any of them A red powder mixed with a little blue or a blue with [...] presently lose its colo[...] tinged with th[...] whatev[...] of rays [...] rays hav[...] beginning their several colorific qualities as well as their several refrangibilities and retaining them perpetually unchanged notwithstanding any refractions or reflexions they may at any time suffer and that whenever any sort of the Sun s rays is by any means (as by reflexion in [...] 9 and 10 Part 1 or by refraction [...] the rest they then manifest [...] proved and the sum of all this [...] to the propo[...]ition here to be proved For if the Sun s light is mixed of several sorts of ra[...] originally their several [...] ing their refract[...] keep those their [...] then all the colo[...] [...] as constantly ought to arise from the original colorific qualities of the rays whereof the lights consist by which tho[...]e colours are seen And therefore if the reason of any colour whatever be required we have nothing else to do than to con[...]ider how the rays in the Sun s light have by reflexions or refractions or other cause been parted from one

another or mixed together, or otherwise to find out what sorts of rays are in the light by which that colour is made, and in what proportion, and then by the last problem to learn the colour which ought to arise by mixing those rays (or their colours) in that proportion. I speak here of colours so far as they arise from light. For they appear sometimes by other causes, as when by the power of phantasy we see colours in a dream, or a madman sees things before him which are not there, or when we see fire by striking the eye, or see colours like the eye of a peacock's feather by pressing our eyes in either corner whilst we ... Where these and such like causes interpose not, the colour ... as I ... often ... this in

PROPOSITION 8 PROBLEM 3

By the discovered properties of light to explain the colours made by prisms

Let ABC [Fig. 1] represent a prism refracting the light of the Sun which comes into a dark chamber through a hole Fφ almost as broad as the prism, and let MN represent a white paper on which the refracted light is cast, and suppose the most refrangible or deepest violet-making rays fall upon the space Pπ, the least refrangible or deepest red-making rays upon the space Tτ, the middle sort between the indigo-making and blue-making rays upon the space Qχ, the middle sort of the green-making rays upon the space Rρ, the middle sort between the yellow-making and orange-making rays upon the space Sσ, and

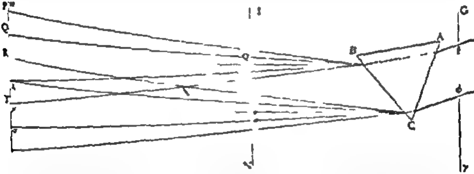


Fig 1

other intermediate sorts upon intermediate spaces. For so the spaces upon which the several sorts adequately fall will by reason of the different refrangibility of those sorts be one lower than another. Now if the paper MN be so near the prism that the spaces PT and πτ do not interfere with one another, the distance between them Tπ will be illuminated by all the sorts of rays in that proportion to one another which they have at their very first coming out of the prism, and consequently be white. But the spaces PT and πτ on either hand will not be illuminated by them all, and therefore will appear coloured. And particularly at P, where the outmost violet-making rays fall alone, the colour must be the deepest violet. At Q where the violet-making and indigo-making rays are mixed, it must be a violet inclining much to indigo. At R where the

violet-making and ...
rays are mixed ...
compound a mid...
mixed ...
same ru...
the prog...
till at T ... all the col...

So again on the other side of the white at τ where the least refrangible or utmost red making rays are alone the colour must be the deepest red At σ the mixture of red and orange will compound a red inclining to orange At ρ the mixture of red orange yellow and one half of the green must compound a middle colour between orange and yellow At χ the mixture of all colours but violet and indigo will compound a faint yellow verging more to green than to orange And this yellow will grow more faint and dilute continually in its progress from χ to π where by a mixture of all sorts of rays it will become white

These colours ought to appear were the Sun s light perfectly white but because it inclines to yellow the ... eby tis ... T will ... colours in order from P to ... indigo blue very f... by the c... will find

These are the colours on both side the ...

... become less compounded and by ... more intense then before And this also agrees with experience

And if one look through a prism upon a white object encompassed with blackness or darkness the reason of the colours arising on the edges is much the same as will appear to one that shall a little consider it If a black object be encompassed with a white one the colours which appear through the prism are to be derived from the light of the white one spreading into the regions of the black and therefore they appear in a contrary order to that when a white object is surrounded with black And the same is to be understood when an object is viewed whose parts are some of them less luminous than others For in the borders of the more and less luminous parts colours ought always by the same principles to arise from the excess of the light of the more luminous and to be of the same kind as if the darker parts were black but yet to be more faint and dilute

What is said of colours made by prisms may be easily applied to colours made by the glasses of telescopes or microscopes or by the humours of the eye For if the object glass of a telescope be thicker on one side than on the other or if one-half of the glass or one-half of the pupil of the eye be covered with any opaque substance the object-glass or that part of it or of the eye which is not covered may be considered as a wedge with crooked sides and every wedge of glass or other pellucid substance has the effect of a prism in refracting the light which passes through it

H— ... colours in the ninth and tenth experiments of the first part arise ... d B t t

sufices only to bring that yellow to a ju ...

tinge it with a manifestly blue colour To obtain therefore a better blue I used instead of the yellow light of the Sun the white light of the clouds, by varying a littl the experiment as follows

Exper. 16 Le HFC [Fig 13] represent a prism in the open air and S the eye of the spectator viewing the clouds by their light coming into the prism at the plane side FIGh and refected in it by its base HFIC and thence going out through its plane side HEFh to the eye And when the prism and eye are conveniently placed so that the angles of incidence and reflexion at the base may be about 40 degrees, the spectator will see a bow MN of a blue colour ...

nothing else than by reflexion of a specular superficies seems so odd a ... enon, and so difficult to be explained by the vulgar hypothesis of philosophers ...

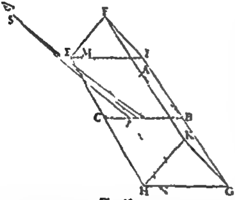


Fig 13

plane cuts the base ... in the angles Spc 40 degrees $\frac{1}{8}$, and Stc 40 degrees $\frac{1}{28}$, and the point p will be the limit beyond which none of the most refrangible rays can pass through the base of the prism and be refracted whose incidence is such that they may be reflected to the eye and the point t will be the like limit for the least refrangible rays (that is, beyond which none of them can pass through the base) whose incidence is such that by reflexion they may come to the eye And the point r taken in the middle way between p and t will be the like limit for the meanly refrangible rays And therefore all the least refrangible rays which fall upon the base beyond t (that is between t and B) and can come from thence to the eye will be reflected thither but on this side t (that is between t and c) many of these rays will be transmitted through the base And all the most refrangible rays which fall upon the base beyond p (that is between p and B) and can by reflexion come from thence to the eye will be reflected thither but everywhere between p and c many of these rays will get through the base and be refracted and the same is to be understood of the meanly refrangible rays on either side of the point Whence it follows that the base of the prism must everywhere between t and B by a *total reflexion of all sorts of rays to the eye* look white and bright and everywhere between p and C by reason of the transmission

of many rays of every sort look more pale obscure and dark But at r and in other places between p and t where all the more refrangible rays are reflected to the eye and many of the less refrangible ... cess of the most refrangible in the reflecte colour which is violet and blue And thi anywhere between the ends of the prism ...

PROPOSITION II PROBLEM 4

By the discovered properties of light to expl...

Th... h...

... upon these drops certainly causes ... o appear to a spectator standing in a due position to the rain and Sun And hence it is now agreed upon that this bow is made by refraction of the Sun's light in drops of falling rain This was understood by ... and of late more ... Domini by his fi... ... itten above 20 years before ... es there how the interior bow is made in round drops of rain by two refractions of the Sun's light and one reflexion between them and the exterior by two refractions and two sorts of reflexions between them in each drop of water and proves his explications by experiments made with a phial full of water and with globes of glass filled with water and placed in the Sun to make the colours of the two bows appear in them The same explication Descartes hath pursued in his *Meteors* and mended that of the exterior bow But whilst they understood not the true origin of colours it's necessary to pursue it here a little farther For understanding therefore how the bow is made let

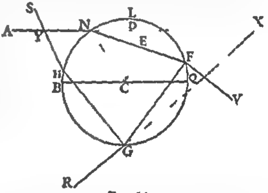


Fig 14

a drop of rain or any other spherical transparent body be represented by the sphere BNFG [Fig 14] described with the centre C and semi diameter CN And let AN be one of the Sun's rays incident upon it at N and thence refracted to F where let it either go out of th... V or be ref... flecte... dent r... ... t ey meet in X ... NF let fall the pe... circumference at let the sine of inc... ... to the sine of refraction as I to R Now if you suppose the point of incidence N to move from the point B continually till it come to L the arch QF will first increase and then decrease and so will the angle AXR which the rays AN and GR co... n and the arch QF ... RR to $\sqrt{3RR}$ which the rays in

AN and HS contain will first decrease and then increase and grow least when AD is to CA as $\sqrt{II-RR}$ to $\sqrt{8RR}$ in which case NI will be to ND as 3R to I And so the angle which the next emergent ray (that is the emergent ray after three reflexions) contains with the incident ray AN will come to its limit when AD is to CA as $\sqrt{II-RR}$ to $\sqrt{15RR}$ in which case NI will be to ND as 4R I ...

... of all this mathematicians will easily examine

Now it is to be observed that as when the sun comes to his tropics days increase and decrease but a very little for a great while together so when by increasing the distance CD these angles come to their limits th ...

... angles of emergence and by consequence according to their different degrees of refrangibility emerge most copiously in different angles and being separated from one another appear each in their proper colours And what those angles are may be easily gathered from the foregoing theorem by computation.

For in the least refrangible rays the sines I and R (as was found above) are 108 and 81 and th...
42 degrees and ...
And in the most ...
by computation ...
minutes and the ... degrees and ... minutes

Suppose now that O [Fig. 15] is the spectator's eye and OP a line drawn parallel to the Sun's rays and let POE POF POG POH be angles of 40 degrees 1 minutes 42 degrees 2 minutes 50 degrees 57 minutes and 54 degrees 7 minutes respectively and these angles turned about their common side OP shall with their other sides OE OF OG OH describe the verges of two rain

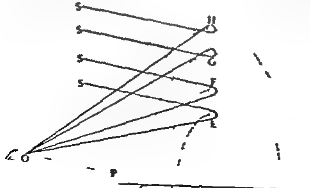


Fig. 15

bows AF BE and CHDG For if E F G H be drops placed any where in the conical superficies described by OE OF OG OH and be illuminated by the Sun s rays SE SF SG SH the angle SEO being equal to the angle POE or 40 degrees 17 minutes shall be the greatest angle in which the most refrangible rays can after one reflexion be ref

in the l OE

anc e

A s

n that region

l can emerge out of the which the least refrangible , or 42 degrees

mo

l

r

p illuminate by the s mo

c

p

v

the white light of the

Again the angle SGO being equal to the angle POG or 50 51 shall be the least angle in which the least refrangible rays can after two reflexions emerge out of the drops and therefore the least refrangible rays shall come most copiously to the eye from the drops in the line OG and strike the sense with the deepest red in that region And the angle SHO being equal to the angle POH or 54 degrees 7 minutes shall be the least angle in which the most refrangible rays after two reflexions can emerge out of the drops and therefore those rays shall come most con

and

n the line OH

t by the same

colours in the order which their degrees of refrangibility require that is in the progress from G to H or from the inside of the bow to the outside in this order red orange yellow green blue indigo violet And since these four lines OE OF OG OH may be situated anywhere in the mentioned conical superficies wh

l ne

e

o

t the red of both t

between the bows The breadth o bow EOF measured across the colours shall be 1 degree 45 minutes and the breadth of the exterior GOH shall be 3 degrees 10 minutes and the d

8 degre

angle

outerm

bows 56 degrees 57 minutes diameter of the

that of the exterior 3 degrees 40 minutes their di tance 8 degrees 25 minutes will be 2 degrees 15 minutes the greatest semi diameter of the interior bow 42 degrees 17 minutes and the

h are the dimensions of the
heir colours appear strong
I measured the greates
nd the breadth of the red
des the outmost faint red
e may allow 3 or 4 minutes
es more besides the violet
louds that I could not
lue and violet togetl er
whole breadth of tl is
e between this in and
the exterior iris was about 8 degrees and so ta n The exterior iris was broader than the interior but so faint especially on the blue side that I coul l not measure its breadth distinctly At another time when both bows appeared more distinct I measured the breadth of the interior iris 2 degrees 10 minutes and the breadth of the red yellow and green in the exterior iris was to the breadth of the same colours in the interior as 3 to 2

This explication of the rainbow is yet further confirmed by th known experiment (made by Antonius de Dominis and Descartes) of hanging up any where in the sunshine a glass gl be filled with water and viewing it in such a posture that the rays which come from the globe to the eye may contain with

become less (suppose by depressing the globe to F) there will ap ar ot er colours yellow green and blue successive in the same side of the globe But if th an le be made about 50 degrees (suppose by liftin up the globe to G) there will appear a red colour in that side of the globe towards the Sun and if the angle be made greater (suppose by lifting up the globe to H) tl e red will turn successively to the other colours yellow green and blue The same thing I have tried by letting a globe rest and raising or depressing the eye or otherwise moving it to make the an le of a just magnitude

this were certain the colours of the globe and rainbow ought to appear in a contrary order to what we find But the colours of the candle being very faint the mistake seems to arise from tl e difficulty of discerning what colours fall on the eye For on tl e contrary I have sometimes had occasion to observe in the Sun s light refracted by a prism that the spectator always sees that colour in the prism which falls upon his eye And the same I have found true also in candle-light For when the prism is moved slowly from the line which is drawn directly from the candle to tl e eye the red appears first in the prism and tl en the blue and therefore each of them is seen when it falls upon the eye For the red passes over the eye first and then the blue

The light which comes through drops of rain by two refractions without any reflexion ought to appear strongest at the distance of about 26 degrees from tl e Sun and to decay gradually both ways as the distance from him increases and decreases. And the same is to be understood of light transmitted through

spherical hailstones And if the hail be a little flatted as it oft t
transmitted may grow so strong at
to f

it to intercept the light
uygens has observed) and make the inside thereof more
distinctly defined than it would otherwise be For such h i
spherical by terminating the l t
nd

ws
v u eitest lines
b hich passes through a l on l
three or more e
those c e parhelia it may perhaps
be sensi

PROPOSITION 10 PROBLEM 5

By the discovered properties of light to explain the permanent colo f l bodies

Th
ray
refr ence appears red Violets
refle tangible most copiously and thence have their colo d
so of other bodies Every body reflects th
th n
i

ot in the homogeneal lights obtained by the solution of the
problem proposed in the fourth Proposition of the first part of this book you
nl will find as I have done that every body
the light of i

less resplenc
most resplen l

i
g
t
as have t u st and most viv
compared together Thus for inst nabar and ultra marine blue or
some other full blue be held together in the red homogeneal light they will both
appear red but the cinnabar will appear of a strongly luminous and resplendent
red and the ultra marine blue of a faint obscure and dark red and if they be
held together in the blue homogeneal light they will both appe
ultra marine will appear of
cinnabar of a faint and da
bar reflects the red light m n the ultra marine doth and
the ultra marine reflects the blue light much more copiously than the cinnabar
doth The same experiment may be tried successfully with red lead and indigo

or with any other two coloured bodies if due allowance be made for the different strength or weakness of their colour and light

... of the colours of natural bodies is evident by these experi...

thence it is certain that some bodies ...

refrangible rays more copiously

And that this is not only a true reason of these colours but even the only reason may appear further from this consideration that the colour of homogeneal light cannot be changed by the reflexion of natural bodies

For if bodies by reflexion cannot in the least change the colour of any one sort of rays they cannot appear coloured by any other means than by reflecting those which either are of their own colour or which by mixture must produce it

But in trying experiments of this kind care must be had that the light be sufficiently homogeneal For if bodies be illuminated by the ordinary prismatic colours they will appear neither of their own daylight colours nor of the colour ...

... of the ... llow

orange-making and yellow-making rays these rays in the reflected light will be more in proportion to the light than they were in the incident green light and thereby will draw the reflected light from green towards their colour And therefore the red lead will appear neither red nor green but of a colour between both

In transparently coloured liquors tis observable that their colour uses to vary with their thickness Thus for instance a red liquor in a conical glass held between the light and the eye looks of a pale and dilute yellow at the bottom where tis thin and a little higher where tis thicker grows orange and where tis still thicker becomes red and where tis thickest the red is deepest and darkest For it is to be conceived that such a liquor stops the indigo-making and violet-making rays most easily the blue-making rays more difficultly the green-making rays still more difficultly and the red-making most difficultly and that if the thickness of the liquor be only so much as suffices to stop a competent number of the violet-making and indigo-making rays without diminishing much the number of the rest the rest must (by Prop 6 Part 2)

darker as the yellow making and orange making rays are more and more stopped by increasing the thickness of the liquor so that few rays besides the red making can get through

Of this kind is an experiment lately related to me by Mr Halley who in diving deep into the sea in a diving vessel found in a clear sunshine day that when he was sunk many fathoms deep into the water the upper part of his hand on which the Sun shone directly through the water and through a small glass window in the vessel appeared of a red colour like that of a damask rose and the water below and the under part of his hand illuminated by light reflected from the water below looked green For thence it may be gathered that the sea water reflects back the violet and blue making rays most easily and lets the red making rays pass most freely and copiously to great depths For thereby the Sun s direct light at all great depths by reason of the predominating red making rays must appear red and the greater the depth is the fuller and inten

pen

refl

a green

Now if there be two liquors of full colours (suppose a red and blue) and both of them so thick as suffices to make their colours sufficiently full though either liquor be sufficiently transparent apart yet will you not be able to see through both togethe

liquor and only the b

both This Mr Hook

liquors and was surprised at the unexpected event the reason of it being then unknown which makes me trust the more to his experiment though I have not tried it myself But he that would repeat it must take care the liquors be of very good and full colours

Now whilst bodies become coloured by reflecting or transmitting this or that sort of rays more copiously than the rest it is to be conceived that they stop and stifle in themselves the rays which they do not reflect or transmit For if gold be foliated and held between your eye and the light the light looks of a greenish blue and therefore massy gold lets into its body the blue-making rays to be reflected to and fro within it till they be stopped and stifled whilst it

one sort of light most copiously and reflect another sort and thereby look of

the same colour in all positions of the eye though this I cannot yet affirm by experience For all coloured bodies so far as my observation reaches may be seen through if made sufficiently thin and therefore are in some measure transparent and differ only in degrees of transparency from tinged transparent liquors these liquors as well as those bodies by a sufficient thickness becoming opaque A transparent body which looks of any colour by transmitted light may also look of the same colour by reflected light the light of that colour being

reflected by the farther surface of the body or by the air beyond it And then the reflected colour will be diminished and perhaps cease by making the body very thick, and pitching it on the backside to diminish the reflexion of its ... h ... ht reflected from the tinging particles may pre-

...

ist dispute

that bodies have such properties ... d ...

Propo ition 11 Problem 6

By mixing coloured l hts to compound a beam of light of the same colour and ... experience the truth of

... Sun s l ht let into a ... ards the lens MN and ... et blue green yellow ... this lens converge again ... compo n l a

lens be equal o that the rays wh ch converged from the lens to ... without refraction would there have crossed and diverged again may by the refract n of th second prism be reduced into parall lism and diverge no more

perfectly white to the very edges of the l ght and at all distances from ... continue perfectly and totally white like a beam of the Sun s ligl t. For till this happens the position of the prisms and lens to one ano her mu t be corrected and then if by the help of a lon beam of wood as is represented in the Figure ... that purpose ... be ... ne experiments in this ... n the Sun s direct ligl t,

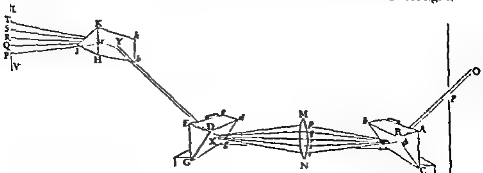


Fig 16

For this compounded beam of light has the ~
with all the same propert e -
c

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t ey ente
arise not f
but fr
with th

So for insta ... a lens 4¼ inches broad and two prisms on either hand 6¼ feet distant from the lens made such a beam of compounded light to examine the reason of the colours made by prisms I refracted this compounded beam of light XY with another prism HIK kh and thereby cast the usual prismatic colours PQRST upon the paper LV placed behind And then by stopping any of the colours p q r s t at the lens I found that the same colour would vanish at the paper So if the purple p was stopped at the lens the purple P upon the paper would vanish and the r ... s would remain unaltered unles ...
in it ...
by ir ... l
vani
XY ... at the lens o the colou ... out of it by new refractions are no other than those of which its whiteness was compounded The refraction of the prism HIK kh generates the colours PQRST upon the paper not by changing the colorific qualities of the rays but by separating the rays which had the very same colorific qualities before they entered the composition of the refracted beam of white light XY For otherwise the rays which were of one colour at the lens might be of another upon the paper contrary to what we find

So again to examine the reason of the colours of natural bodies I placed such bodies in the beam of light XY and found that they all appeared there of those their own colours which they have in daylight and that those colours depend upon the rays which had the same colours at the lens before they entered the composition of that beam Thus fo ...
by this beam appears of the same red colour ...
you intercept the green mak ...
mo

k ... ppear white like silver
(... shews that its yellowness arises from the ... of the
intercepted rays tinging that whiten... t
n ... t

l

l ... blue colour whil t its
t ... remains perfect and by the loss of some blue-making rays
wherewith it was allayed becomes more intense and full And on the con

and red, but only transmit[illegible]
before and reflect those most copiously which were blue-making before. And
after the same manner may the reasons of other [illegible] be examined by
try[illegible] them in this artificial beam of light. [illegible]

BOOK TWO

Part I

Observations concerning the reflexions, refractions, and colours of thin transparent bodies.

It has been observed by others, that transparent substances (as glass, water, air, &c.) when made very thin by being blown into bubbles, or otherwise formed into plates, do exhibit various colours according to their various thinness, altho' at a greater thickness they appear very clear and colourless. In the former book I forbore to treat of these colours, because they seemed of a more difficult consideration, and were not necessary for establishing the properties of light there discoursed of. But because they may conduce to further discoveries for completing the theory of light, especially as to the constitution of the parts of natural bodies, on which their colours or transparency depend; I have here set down an account of them. To render this discourse short and distinct, I have first described the principal of my observations, and then consider'd and made use of them. The observations are these.

Observation 1. Compressing two prisms hard together that their sides (which by chance were a very little convex) might somewhere touch one another: I found the place in which they touched to become absolutely transparent, as if they had there been one continued piece of glass. For when the light fell so obliquely on the air, which in other places was between them, as to be all reflected; it seemed in that place of contact to be wholly transmitted, insomuch that when looked upon, it appeared like a black or dark spot, by reason that little or no sensible light was reflected from thence, as from other places; and when looked through it seemed (as it were) a hole in that air which was formed into a thin plate, by being compress'd between the glasses. And through this hole objects that were beyond might be seen distinctly, which could not at all be seen through other parts of the glasses where the air was interjacent. Although the glasses were a little convex, yet this transparent spot was of a considerable breadth, which breadth seemed principally to proceed from the yielding inwards of the parts of the glasses, by reason of their mutual pressure. For by pressing them very hard together it would become much broader than otherwise.

Obs. 2. When the plate of air, by turning the prisms about their common axis, became so little inclined to the incident rays, that some of them began to be transmitted, there arose in it many slender arcs of colours which at first were shaped almost like the conchoid, as you see them delineated in the first Figure. And by continuing the

Fig. 1

motion of the prisms these arcs increased and bended more and more about the said transparent spot till they were compl ted encom passing it and

These arcs uau colour d between them s or circle motion of yellow anc these colou black red ow white blue viol much fainter than the blue and violet

The motion of the prism

the

s before

ne rings or some parts of them appeared only black and white they were very distinct and well defined and the blackness seemed as intense as that of the central spot Also in the borders of the rings where the colours began to emerge out of the whiteness they were pretty distinct which made them vi ible to a very great multitude I have sometimes numbered above thirty successions (reckoning every black and white ring for one succession) and seen more of them which by reason of their smallness I could not number But in other positions of the prisms at which the rings appeared of many colours I could not distinguish above eight or nine of them and the exterior of tho e were very confused and dilute

In these two Observations to see the rings distinct and without any other colour than black and white I found it necessary to hold my eye at a good distance from them For by approaching nearer although in the same inclina tion of my eye to the plane of the rings there emerged a bluish colour out of the white which by dilating itself more and more into the black rendered the circles less distinct and left the white a little tinged with red and yellow I found also by looking through a lit or oblong hole which was narrower than the pupil of my eye and held clo e to it parallel to the pri ms I could see the circles much distincter and vi ible to a far greater number than otherwise

OBS 4 To observe more nicely the order of the colours which aro e out of the white circles as the rays became less and le s inclined to the plate of air I took two object gla es (the one a plano-convex for a fourteen foot tele c pe and the other a large double convex fo

m one of the circles and then y lifted the upper glass from the lower to make them succe sively vani h again in the ame place The colour which by pre sing the glasses together emerged last in the middle of the other colours would upon its first appearance look like a circle of a colour almost uniform from the circumference to the centre and by compressing the glasses still more grow continually broader until a new colour emerged in its centre and thereby it became a ring en

compassi.. that new colour And by compressing the glasses still more the diame er of this ring would increase and the breadth of it orbit or perim t r decrease until ano her new colour emerged in the centre of the la t And so n

 fo .th a fifth and other following new colours successively

 colo th la t

 per gla

 ra ht of

d.. .nguish their species than before

succession and quantity to be as followeth

Next to the pellucid central spot made by the contact of the glasses succeeded blue white yellow and red The blue was so littl in quantity that I coul l not d

 h circles made by the prism nor could I well d tinguish any

 seemed about a

 lu The

 r vic et

 l vivid

 ach more

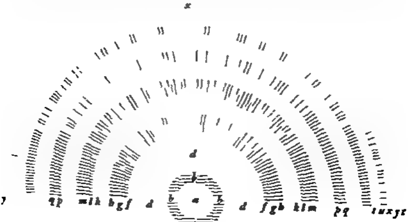
 the

 rcuit or

purple seemed more

 en was mu h more

co..picuou.. being as brisk and copious as any of t o r colours except the yellow but the red began to be a little faded inclining very much to purple After this succeeded the fourth circuit of green and red The green was very copiou.. and lively inclining on the one side to blue and on the other side to yellow But in this fourth circuit there was neither violet blue nor yellow and the red was very imperfect and dirty Also the succeeding colours became more and more imperfect and dilute till after three or four revolutions they ended



Fig

in perfect whiteness Their form when the glasses were most compressed so as to make the black spot appear in the centre is delineated in the second figure where *a b c d e f g h i k l m n o p q r s t v x y* denote the colours reckoned in order from the center black blue white yellow red violet blue green yellow red purple blue green yellow red green red greenish blue red greenish blue pale red greenish blue reddish white

OBS 5 To determine the interval of the glasses or thickness of the interjacent air by which each colour was produced I measured the diameters of the first six rings at the most lucid part of their orbits and squaring them I found their squares to be in the arithmetical progression of the odd numbers 1 3 5 7 9 11 And since one of these glasses was plane and the other spherical their intervals at those rings must be in the same progression I measured also the diameters of the dark or faint rings between the more lucid colours and found their squares to be in the arithmetical progression of the even numbers 2 4 6 8 10 12 And it being very nice and difficult to take these measures exactly I repeated them divers times at divers parts of the glasses that by their agreement I might be confirmed in them And the same method I used in determining some others of the following observations

OBS 6 [illegible]

of the air or aereal interval of the glasses at that ring But some time after suspecting that in making this observation I had not determined the diameter of the sphere with sufficient accurateness and being uncertain whether the plano-convex glass was truly plane and not something concave or convex on that side which I accounted plane and whether I had not pressed the glasses together as I often did to make them touch (for by pressing such glasses together their parts easily yield inwards and the rings thereby become sensibly *broader than they would be did the glasses keep their figures)* I repeated the experiment and found the diameter of the sixth lucid ring about $\frac{8}{100}$ parts of an inch I repeated the experiment also with such an object glass of another telescope as I had at hand This was a double convex ground on both sides to one and the same sphere and its focus was distant from it $83\frac{2}{5}$ inches And thence if the sines of incidence and refraction of the bright yellow light be assumed in proportion as 11 to 17 the diameter of the sphere to which the glass was figured will by computation be found 182 inches This glass I laid upon a flat one so that the black spot appeared in the middle of the rings of colour without any other pressure than that of the weight of the glass And now measuring the diameter of the fifth dark circle as accurately as I could I found it the fifth part of an inch precisely This measure was taken with the points of a pair of compasses on the upper surface on the upper glass and my eye was about eight or nine inches distance from the glass almost perpendicularly over it and the glass was $\frac{1}{6}$ of an inch thick and thence it is easy to [illegible] ter than

there-

[illegible] e semi

diameter equal to $\frac{5}{79}$ parts Now as the diameter of the sphere (182 inches) is to the semidiameter of this fifth dark ring ($\frac{5}{79}$ parts of an inch) so is this semidiameter to the thickness of the air at this fifth dark ring which is therefore

—11— or —111— parts of an inch and the fifth part thereof (viz. th —1—th part of an inch) is the thickness of the air at the first of these dark rings.

The same experiment I repeated with another double convex object-glass ground on both sides to one and the same sphere. Its focus was distant from it ... of that sphere was 184 inches. Th...

dark

ng the

h part

first

of the rings as above

I tried the same thing by laying these object-glasses upon flat pieces of a broken looking-glass and found the same measures of the rings which makes me rely upon them till they can be determined more accurately by glasses ground to larger spheres though in such glasses greater care must be taken of a true plane

These dimensions were taken when my eye was placed almost perpendicularly over the glasses being about an inch or an inch and a quarter distant from the incident rays and eight inches distant from the glass so that the rays were inclined to the glass in an angle of about four degrees. Whence by the following Observation you will understand that had the rays been perpendicular to the glasses the thickness of the air at these rings would have been ... to the secant of four degrees (that is of ... his ...

ness multiplied by the progression 1 3 5 ...

the air at the most luminous parts of all the brightest rings viz. ... &c. their arithmetical means ... &c. being its thicknesses at the darkest parts of all the dark ones

Obs. ... The rings were least when my eye was placed perpendicularly over the glasses in the axis of the rings and when I viewed them obliquely they became bigger continually swelling as I removed my eye farther from the axis ... the diameter of the same circle at several obliquities

...portion expressed in the following Tab...

Angle of incidence on the ai		Angle of refraction into the air		Diamete of the ring	Thic ness of the ai
D g	M				
00	00	00	00	10	10
06	26	10	00	10 1/13	10 /13
12	45	20	00	10 1/3	10 /3
18	49	30	00	10 3/4	11 1/2
24	30	40	00	11 /5	13
29	37	50	00	12 1/	15 1/
33	58	60	00	14	20
35	47	65	00	15 1/4	23 1/4
37	19	70	00	16 1/	28 1/4
38	33	75	00	19 1/4	37
39	27	80	00	22 6/7	52 1/4
40	00	85	00	29	84 1/1
40	11	90	00	35	122 1/

In the two first columns are expre ed the obliquities of the incident and emergent rays to the plate of the air that is their angles of incidence and refraction In the third column the diameter of any coloured ring obliquities is expres ed in parts of ...

the rays ...

the circu ... uch also ten ...

its thick ... ays are perpen ...

And from ...

air is prop ... ean pro-

...

bigge ... e refraction is made out of the ... ate of air or from the sine of incidence when the refraction is made out of the plate of air into the glass

OBS 8 The dark spot in the middle of the rings increa ed al o by the obliquation of the eye although almost insensibly But if instead of the object gla es the prisms were made use of its increase was more manifest when viewed so obliquely that no colours appeared about it It was least when the rays were incident most obliquely on the ...

decreased it increa ed more ...

then decreased again but not ... ed before ...

evident that the transparency ...

...

not

... at the perimeter of that spot

... perpendicularly was about a fifth or sixth part of their interval at the circumference of the said red

OBS 9 By looking through the two contiguous object-glasses I found that

— light a l y rd r of the violet blue

colours were yellowish red black violet green yellow red &c. But these colours were very faint and dilute unless when the light was trajected very obliquely through the glasses for by that means they became pretty vivid Only the first yellowish red like the blue in the fourth Observation was so little and faint as scarcely to be discerned Comparing the coloured rings made by reflexion with these made by transmission of the light I found that white was opposite to black red to blue of red and violet That is those which when looked upon which in one case exhibited of the other colours The where AB CD are the

manner you have represented in surfaces of the glasses contiguous at E and the black lines between them are their distances in arithmetical progression and the colours written above are those below by light transmitted

sequently the intervals of the glasses a diums (water and air) are as about three to four Perhaps it may be a general rule that if any other medium more or less dense than water be compressed at the rings caused thereby will be to their ines are which measure the refraction

I pressed the upper

n of the ambient water into that place

times seen more than twenty of them whereas in the open air I could not discern above eight or nine

Obs 13 Appointing an assistant to move the prism to and fro about its axis, that all the colours might successively fall on that part of the paper which I saw by reflexion from that part of the glasses where the circles appeared so that all the colours might be successively reflected from the circles to my eye

whilst I held it immovable I found the circles which the red light made to be manifestly bigger than those which were made by the blue and violet And it was very pleasant to see them gradually swell or contract accordingly as the colour of the light was changed The interval of the glasses at any of the rings when they were made by the utmost red light was to their interval at the same ring when made by the utmost violet greater than as 3 to 2 and less than as 13 to 8 By the ... Observation ... very nearl ... unless when two ... were made use of instead of the object-glasses For then at a certain great obliquity of my eye the rings made by the several colours seemed equal and at a greater obliquity those made by the violet would be greater than the same rings made by the red the refraction of the prism in this case causing the most refrangible rays to fall more obliquely on that plate of the air than the least refrangible ones Thus the experiment succeeded in the coloured light which was sufficiently strong and copious to make the rings sensible And thence it may be gathered that if the most refrangible and least refrangible rays had been copious enough to make the ... without the mixture ... here was 14 to 9 woul ... $14\frac{1}{4}$ or $14\frac{1}{3}$ to 9

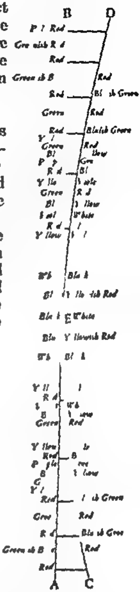


Fig 3

OBS 14 Whilst the prism was turned about its axis with a uniform motion to make all the several colours fall successively upon the object-glasses and thereby to make the rings contract and dilate the contraction or dilatation of each ring thus made by the variation of its colour was swiftest in the red and slowest in the violet and in the ... intermediate degrees of col... of contraction and dilatati... degrees of each colour I found that it was greatest in the red less in the yellow still less in the blue and least in the violet And to make as just an estimation as I could of the proportions of their contractions or dilatations I observed that the whole contraction or dilatation of the diameter of any ring made by all the degrees of red was to that of the diameter of the same ring made by all the degrees of violet as about four to three or five to four and that when the light was of the middle colour between yellow and green the diameter of the ring was very nearly an arithmetical mean between the greatest diameter of the same ring made by the outmost red and the least diameter thereof made by the outmost violet—contrary to what happens in the colours of the oblong spectrum made by the refraction of a prism where the red is most contracted the violet most expanded and in the midst of all the colours is the confine of green and blue And hence I seem to collect that the thicknesses of the air between the glasses there where the ring is successively made by the limits of the five

green blue violet) in order (that is by the
the
blue
o one
in a
ler
are the
orange
rings are successively made by
n blue violet in order) are to one another as the cube
the notes in an
quares of the

d in the open

Obs 15 These rings were not o
our only with which they were
colours immediately upon the
ark spaces which were between
glasses without any variation

g 4

of colour For on a white paper placed behind it would paint rings of the same

Obs. 16 The squares of the diameters of these rings made by any prismatic colour were in arithmetical progression as in the fifth Observation And the diameter of the sixth circle when made by the citrine yellow and viewed almost perpendicularly was about [illegible] parts of an inch or a little less agreeable to the sixth Observation

The precedent Observations were made with a rarer thin medium terminated by a denser such as was air or water compressed between two glasses In those that follow are set down the appearances of a denser medium thinned within a rarer such as are plates of Muscovy glass bubbles of water and some other thin substances terminated on all sides with air

Obs 17 If a bubble be blown with water first made tenacious by dissolving a little soap in it tis a common observation that after a while it will appear

tinged with a great variety of colours To defend these bubbl[illegible] from being agitated by the external air (whereby [illegible] oved one among anoth[illegible] as soon as I had blo[illegible] its colours er[illegible]

[illegible] y and oversp[illegible] order to the [illegible] successively In the [illegible] there grew in the [illegible] first Observation [illegible] than ½ or ¾ of a [illegible] At first I thought there had been no [illegible] from the water in that place but observing it more curiously I saw within it several smaller round spots which appeared much blacker and darker than the rest whereby I knew that there was some reflexion at the other places which were not so dark as those spots And by further trial I found that I could see the images of some things (as of a candle or the Sun) very faintly reflected not only from the gre[illegible] from the little darker spot[illegible]

Be[illegible] col[illegible] sides of the bubble [illegible] rea[illegible] the subsiding of th[illegible] black spots generated [illegible] the top of the bubbl[illegible]

OBS 18 Be[illegible] than those c[illegible] distinguished [illegible] were observed in [illegible] whilst a black subs[illegible] And they were the c[illegible] red blue red blue [illegible] red green red yellow green blue purple red yellow green blue violet red yellow white blue black

The three first successions of red and blue were very dilute and dirty especially the first where the red seemed [illegible] there was [illegible] (and [illegible]

The [illegible] after [illegible] incline[illegible] brisk and good willow green [illegible] changed to a bluish colour but there succeeded neither blue nor violet

The fifth red at first inclined very much to purple and afterwards became more bright and brisk but yet not very pure This was [illegible] bright and intense yellow wh[illegible]

[illegible] in quantity than th[illegible]

[illegible] to red [illegible]

br[illegible] best of all the reds Then

[illegible] orange followed an intense bright and copious yellow which was [illegible] to a greenish yellow and [illegible] and the blue was [illegible] a green The blue [illegible] colour but yet which succeeded became very good [illegible] something inferior to the former blue and the violet was intense and deep with little or no redness in it and less in quantity than the blue

In the last red appeared a tincture of scarlet next to violet which soon changed to a brighter colour inclining to an orange and the yellow which followed was at first pretty good and lively but afterward it grew more dilute until by degrees it ended in perfect whiteness And this whiteness if the water was very tenacious and well tempered would slowly spread and dilate itself over the greater part of the bubble continually growing paler at the top where at length it would crack in many places and those cracks as they dilated would appear of a pretty good but yet obscure and dark sky colour the white between the blue spots diminishing until it resembled the threads of an irregular network and soon after vanished and left all the upper part of the bubble [illegible] colour And this colour after the aforesaid manner [illegible] whole bubble [illegible] bottom and [illegible] than the rest) [illegible] spots [illegible] nd

If the water was not very tenacious the black spot [illegible] the white without any sensible intervention of the blue And sometimes they would break forth within the precedent yellow or red or perhaps within the blue of the second order before the intermediate colours had time to display themselves.

By this description you may perceive how great an affinity these colours have with those of air described in the fourth Observation although set down in a contrary order by reason that they begin to appear when the bubble is thickest and are most conveniently reckoned from the lowest and thickest part of the bubble upwards

OBS. 19 Viewing in several oblique positions of my eye the rings of colours [illegible] were sensibly dilated by [illegible] as those made by thinned [illegible] dilated so much as when viewed most obliquely to arrive at a part of the plate more than twelve times thicker than that where they appeared when viewed perpendicularly whereas in this case the thickness of the water at which they arrived when viewed most obliquely was to that thickness which exhibited them by perpendicular rays something less than as 8 to 5 By the best of my observations it was between 15 [illegible]

same appearance of colours in all positions of the eye And then the colours which were seen at its apparent circumference by the obliquest rays would be different from those that were seen in other places by rays less oblique to it.

And divers spectators might see the same part of it of differing colours by viewing it at very differing obliquities Now observing how much the colours at the same places of the bubble or at divers places of equal thickness were varied by the several obliquities of the rays by the assistance of the 4th 14th 16th and 18th Observations as they are hereafter explained I collect the thickness of the water requisite to exhibit any one and the same colour at several obliquities to be very nearly in the proportion expressed in this Table

Incidence on the water		*Refraction into the water*		*Thickness of the water*
Deg	*Min*	*Deg*	*Min*	
00	00	00	00	10
15	00	11	11	$10\frac{1}{4}$
30	00	22	1	$10\frac{4}{5}$
45	00	32	2	$11\frac{4}{5}$
60	00	40	30	13
75	00	46	25	$14\frac{1}{2}$
90	00	48	35	$15\frac{1}{5}$

In the two first columns are expressed the obliquities of the rays to the superficies of the water (that is their angles of incidence and refraction) where I suppose that the sines which measure them are in round numbers as 3 to 4 though probably the dissolution of soap in the water may a little alter its refractive virtue In the third column the thickness of the bubble at which any one colour is exhibited in those several obliquities is expressed in parts of which ten constitute its thickness when the rays are perpendicular And the rule found by the seventh Observation agrees well with these measures if duly applied namely that the thickness of a plate of water requisite to exhibit one and the same colour at several obliquities of the eye is proportional to the secant of an angle whose sine is the first of a hundred and six arithmetical mean proportionals between the sines of incidence and refraction counted from the lesser sine that is from the sine of refraction when the refraction is made out of air into water otherwise from the sine of incidence

I have sometimes observed that the colours which arise on polished steel by heating it or on bell metal and some other metalline substances when melted and poured on the ground where they may cool in the open air have like the colours of water bubbles been a little changed by viewing them at divers obliquities and particularly that a deep blue or violet when viewed very obliquely hath been changed to a deep red But the changes of these colours are not so great and sensible as of those made by water For the scoria or vitrified part of the metal which most metals when heated or melted do continually protrude and send out to their surface and which by covering the

light appeared of a contrary colour to that which it exhibited by reflexion

would be blue And on th contrary when b reflected light it appeared b ue

any variation of their species ... prod ce an colour depends only on the density of the plate and not on that of the amb ent medium. And hence by the 10th and 16th Observations, may be known the thickness which bubbles of water or plates of Muscovy glass or other substances have at any colour produced by th m

Obs. 22 A thin transparent body which is denser than it amb ent medium exhib more brisk and vivid colours than that which is so much rarer as I have particularly observed in the air and glass Fo lowing glass very thin at a Lamp furnace those plates encompassed with air d' l exh bit colours much more vivid th n those of air made thin between two glasses

... several rings I ...

at distant times in th first of which the whiteness ap ... all th colours and in the other the whiteness which preceded them all

Obs. 24 When the two object-glasses were laid upon one anothe so as to make the rings of the colours appear though with my naked eye I could no discern above eigh or nine of those rings yet by viewin them through a prism I h e seen a far great mul tude insomuch that I could number more than fort besides man others that were so very small and close toge her that I could not keep my eye steady on them severally so as to number them but by their extent I have sometimes estimated them to be more than a hundred. And

But it wa but one side of these rings (name that towards hich the refraction was made) wh h by that refraction wa rend red d tinct and the o her side became more confused than when viewed

F. 5

with my naked eye And their segments o arcs which on the other side appeared so numerous for the most part exceeded not the third part of a circl If the refract on wa very great o th prism very d ant from the object-glasses th middle part of those arcs b came also confused, so as to disappea and co st tute an even whiteness whilst on eith side their end as lso the whole arcs farthest from the centre became distin te than before appearing in the form as you see them designed in the fifth Figure

The arcs where they seemed distinctest were only white and black successively without any other colours intermixed But in other places there appeared colours whose order was inverted by the refraction in such manner that if I first held the prism very near the object glasses and then gradually removed it farther off towards my eye the colours of the 2d 3d 4th and following rings shrunk towards the white that emerged between them until they wholly vanished into it at the middle of the arcs and afterwards emerged again in a contrary order But at the ends of the arcs they retained their order unchanged

I have sometimes so laid one object glass upon the other that to the naked eye they have all over seemed uniformly white without the least appearance of any of the coloured rings and yet by viewing them through a prism great multitudes of those rings have discovered themselves And in like manner plates of Muscovy glass and bubbles of glass blown at a lamp-furnace which were not so thin as to exhibit any colours to the naked eye have through the prism exhibited a great variety of them ranged irregularly up and down in the form of waves And so bubbles of water before they began to ... colours to the n... about with ... necessary t ... or very nearly parallel to the horizon and to dispose it so that the rays might be refracted upwards

Part II

Remarks upon the foregoing Observations

HAVING given my O... m to unfold the causes c... the simplest of them (s... 24th and 24th) I first explain the more compounded And first to shew how the colours in the fourth and eighteenth Observations are produced let there be taken in any right line from the point Y (Fig 6) the lengths YA YB YC YD YE YF YG YH in proportion to one another as the cube roots of the squares of the numbers 1/2 9/16 3/5 2/3 3/4 5/6 8/9 1 whereby the length of ... l to sound ... the numb... points A ... perpendiculars Aα Bβ &c be erected by whose intervals the extent of the several colours set underneath against them is to be represented Then divide the line Aα in such proportion as the numbers 1 2 3 5 6 7 9 10 11 &c set at the points of division denote And through those d... 6N 7O &c ... ss of any thin transparent l... reflected in the first ring ... by the 10th Observation HK will represent its thickness at which the utmost red is most copiously reflected in the same series Also by the 5th and 16th Observations A6 and HN will denote the thicknesses at which those extreme colours are most copiously reflected in the second series and A10 and HQ the thicknesses at which they are most copiously reflected in the third series and so on And the thickness at which any of the intermediate

colours are reflected most copiously will according to the 14th Observation be defined by the distance of the line AH from the intermediate parts of the lines 2K, 6N, 10Q, &c. against which the names of those colours are written below.

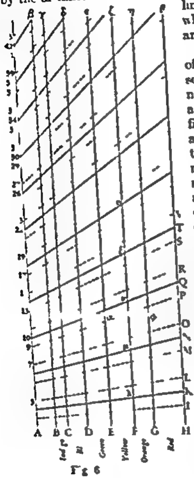


Fig 6

But farther to define the latitude of these colours in each ring or series let A1 design the least thickness, and A3 the greatest thickness, at which the extreme violet in the first series is reflected, and let HI and HL design the like limits for the extreme red, and let the intermediate colours be limited by the intermediate parts of the lines 1I and 3L against which the names of those colours are written, and so on but yet with this caution that the reflexions be supposed strongest at the intermediate spaces 2K, 6N, 10Q, &c. and from thence to decrease gradually towards these limits 1I, 3L, 5M, 7O, &c. on either side, where you must not conceive them to be precisely limited but to decay indefinitely. And whereas I have assigned the same latitude to every series I did it because although the colours in the first series seem to be a little broader than the rest by reason of a stronger reflexion there, yet that inequality is so insensible as scarcely to be determined by observation.

Now according to this description

original colours of which the colour exhibited in the open air is compounded. Thus if the constitution of the green in the third series of colours be desired apply the ruler as you see at πρσ, and by its passing through some of the blue

The arcs where they seemed distinctest were only wh sively without any other colo ne

r

ring ... alus m ... es and then ...

var

cor

I have sometim eye they have all ... ued uniformly white without ... that to the naked of any of the coloured rings and yet by viewing them through ... great multitudes of those rings have ... plates of Muscovy glass and ... were not so thin as to exhibit ... manner

... rays might be refracted ... parallel to the horizon and ... refracted upwards

Part II

Remarks upon ...

Ha... un... sim... m to I fi... the ... more compounded And first ... 12th 18th 20th and 24th) fourth and eighteenth Observations are produced let there be taken in ... to shew how the colours in the right line from the point Y (Fig 6) the ... YG YH in ... numbers ...

is to be represented ... the several colours ... be erected by 1 2 3 5 6 7 9 10 those divisions from ...

Now if A2 be ... body at ... or series ... ness at w... red is most copiously reflected ... represent its thick Al o by the 5th and 16th Observations A6 and HN will denote the thicknesses at which those extreme colours are most copiously reflected in the same series and A10 and HQ the thicknesses at which ... reflected in the second series

any colour will be $\frac{3}{4}$ of the thickness of air producing the same colour. And so [illegible] 21 t Observation, the thickness of a plate of

[illegible]

hundred thousand equal parts.

The thickness of coloured plates and particles

		of Air	Water	Glass
Their colours of the first order	Very black	[illegible]	[illegible]	[illegible]
	Black	1	[illegible]	[illegible]
	Beginning of black	[illegible]	1[illegible]	1[illegible]
	Blue	[illegible]	1[illegible]	1[illegible]
	White	5[illegible]	3[illegible]	3[illegible]
	Yellow	[illegible]	[illegible]	4[illegible]
	Orange	8	6	5[illegible]
	Red	9	6[illegible]	[illegible]
Of the second order	Violet	11[illegible]	8[illegible]	[illegible]
	Indigo	1[illegible]	9[illegible]	8[illegible]
	Blue	14	10[illegible]	9
	Green	15[illegible]	11[illegible]	9[illegible]
	Yellow	16[illegible]	1[illegible]	10[illegible]
	Orange	1[illegible]	13	11[illegible]
	Bright red	18[illegible]	13[illegible]	11[illegible]
	Scarlet	19[illegible]	14[illegible]	1[illegible]
Of the third order	Purple	1[illegible]	15[illegible]	13[illegible]
	Indigo	22[illegible]	16[illegible]	14[illegible]
	[illegible]	23[illegible]	17[illegible]	15[illegible]
	Green	[illegible]	18[illegible]	16
	Yellow	[illegible]	[illegible]	1[illegible]
	Red	[illegible]	1[illegible]	18[illegible]
	Bluish red	32	[illegible]	[illegible]
Of the fourth order	Bluish-green	34	[illegible]	
	Green	35[illegible]	[illegible]	[illegible]
	Yellowish-green	3[illegible]	[illegible]	23[illegible]
	Red	40[illegible]	[illegible]	[illegible]
Of the fifth order	Greenish-blue	46	34[illegible]	[illegible]
	Red	[illegible]	39	34
Of the sixth order	Greenish blue	58[illegible]	44	38
	Red	65	48[illegible]	4[illegible]
Of the seventh order	Greenish blue	71	53[illegible]	45[illegible]
	Ruddy white	[illegible]	5[illegible]	4[illegible]

Observations For if you move the ruler gradually from AH through all distances having passed over the first space which denotes little or no reflexion to be made by thinnest substances it will first arrive at 1 the violet and then very quickly at the blue and green which together with that violet compound blue and then at the yellow and red by whose further addition that blue is converted into whiteness which whiteness continues during the tr
ruler from I to 3 and after that by
colours turns first to com
cea eth at L Ther
order during the tr
than before and for the same rea on
instead of the there intercedes between the bl
mixture of orange yellow green
exh

of the second order
to a reddish purple then the blue and green which are less mixed with other colours and consequently more lively than before pecially the green then follows the yello
is dist
red is arious
et and
there

verges with the colours of the fifth series ose mixture the succeeding yellow and red are very much diluted and made dirty especially the yellow which being the weaker colour is carce able to shew itself After this the several series inte
their colours become
revolutions (in wh
are in all places ound an even whiteness

And since by the 19th Observation the rays endued with one colour are transmitted where those of another colour are reflected the rea on of the colours made by the transmitted light in the 9th and 20th Observations is from hence evident

If not only the order and species of thos
ness of the plate or thin body
of an inch that may be also ob
tions For according to those
which between two glasses exh the most luminous parts of the f
rings were

to determine what thick
represented by G or by any other distance of the r from
But further since by the 10th O
thickness of water
as 4 to 3 and by the of thin bodies are not varied
by varying the ambient medium the thickne s of a bubble of water exhibiting

— { the violet by
most of all ex
ne and become
ars belonging to
main to interfere
...ble to so great numbers
use of the rings which
they exhibit become ... obliquity of the eye by
reason that the rays in their passage through that air which intercedes the glasses are very nearly parallel to those lines in which they were first incident ... and consequently the rays endued with several colours are not ... h prisms
...mudered and
...tance appear
...r at hand but
also yield a violet colour at both the edges ... And the reason is that the rays which enter the eye at several parts of the pupil have several obliquities to the glasses and those which are most oblique if considered apart would represent the rings bigger than those which are the least oblique. Whence the breadth of the perimeter of every white ring is expanded outward by the obliquest rays and inwards by the least oblique. And this expansion is so much the greater by how much the greater is the difference of the obliquity that is by how much the pupil is wider or the eye nearer to the glasses. And the breadth of the violet must be most expanded because the rays apt to excite a sensation of that colour are most oblique to a second or farther superficies of the thinned air at which they are reflected and have also the greatest variation of obliquity which makes that colour soonest emerge out of the edges of the white ...

which to the naked eye seem of an even and uniform transparentness ... without any terminations of shadows, the refraction of a prism should make rings of colours appear whereas it usually makes objects appear coloured only ...

breadth of their circumferences they so much interfere and are blended to-

on the other side more complicated and contracted And where by a due refraction they are so much contracted that the several rings become narrower than

unless it be further desired to delineate the manner how the colours appear when the two object-glasses are laid upon one another To do which let there be described a large arc of a circle and a straight line wh h —
arc and parallel to th + —
as the numb
and its tang ... es of the glasses terminating the interjacent air and the places where the occult lines cut the arc will show at what distances from the centre or point of contact each colour is reflected

There are also other uses of this Table For by its assistance the thickness of the bubble in the 19th Observation was determined by the colours which it exhibited And so the bigness of the parts of natural bodies may be conjectured by their colours as shall be hereafter shewn Also if two or more very thin plates be laid one upon another so a to com
n h

u uscov
yellow of th
it accordin
of the secon ... thickness exhibiting the purple o ... third order

To explain in the next place the circumstances of the 2d and 3d Observations that is how the rings of the colours may (by turning the prisms about their common axis the contrary way to that expressed in those observations) be converted into white and black rings and afterwards into rings of colours again the colours of each ring lying now in an inverted order it must be remembered that those rings of colours are dilated by the obliquation of the rays to the air which intercedes the glasses and that according to the Table in the 7th Observation their dilatation or increase of their diameter is most manifest and speedy when they are obliquest Now the rays of yellow being more refracted by the first superficies of the said air than those of red are thereby made more oblique to the second ... f

become of equal extent wi
the green blue and viole ... much dilated by the still greater obliquity of their rays as to become all very nearly of equal extent with the red that is equally distant from the centre of the rings And then all the colours of the same ring must be coincident and by their mixture exhibit a white ring And these white rings must have black and dark rings between them because they do not spread and interfere with one another as before And for that reason al o they must become distincter and visible to far greater numbers But yet the violet being obliquest will be something more dilated in proportion to its extent than the other colours and so very apt to appear at the exterior verges of the white

Afterwards by a greater obliquity of the rays the violet and blue become more sensibly dilated than the red and yellow and so being farther removed from the centre of the rings the colours must emerge out of the white in an order contrary to that which they had before the violet and blue at the exterior

the position of the circles made successively by the several colours will be found such in respect of one another as I have described in the Figures abrr or abrv o a,,T And by the same method the truth of the explications of

... of glass ... further ... ut their ... postures ... es in one ... that it them and ... The reason is that the superficies of such glass ... any cavities and swellings, which how shallow soever do a little vary the thickness of the plat For at the several sides of those cavities for the reasons newly described there ought to be produced waves in several postures of the prism. Now though it be but some very small and narrower parts of the glass by which these waves for the most part are caused yet they may seem to extend themselves over the whole ... of those parts there are colours of several ...

dispersed to several places, so as to ... were divers orders of colours promiscuously reflected from that part of the glass

These are the principal phenomena of thin plates or bubbles whose explications depend on the properties of light which I have heretofore delivered And these you see do necessarily follow from them and agree with them even to their very least circumstances and not only so but do very much tend to their proof Thus, by the 24th Observation it appears that the rays of several colours made as well by thin plates or bubbles as by refractions of a prism have several degrees of refrangibility whereby those of each order which at the reflexion from the plate or bubble are intermixed with those of other orders, are separated from them by refraction and associated together so as to become visible by themselves like arcs of circles For if the rays were all alike refrangible tis impossible that the whiteness which to the naked sense appears uniform should by *refraction have its parts transposed and ranged into those* black and white arcs.

It appears also that the unequal refractions of difform rays proceed not from any contingent irregularities such as are veins an uneven polish or fortuitous position of the pores of glass unequal and casual motions in the air or ether the spreading, breaking or dividing the same ray into many diverging parts or the like For admitting any such irregularities it would be impossible for refractions to *render those rings* so very distinct and well defined as they do in the 24th Observation It is necessary therefore that every ray have its proper and constant degree of refrangibility connate with it according to which its refraction is ever justly and regularly performed and that several rays have several of those degrees.

And what is said of their refrangibility may be also undersood of their reflexibility that is of their dispositions to be reflected, some at a greater and others at a less thickness of thin plates or bubbles namely that those dispositions are also connate with the rays and immutable as may appear by the

to interfere with one another they must appear distinct and also white if the constituent colours be so much contracted as to be wholly coincident But on the other side where the orbit of every ring is made broader by the farther unfolding of its colours it must interfere more with other rings than before and so become less distinct

To explain this a little further suppose the concentric circles AV, and BX [Fig 7] represent the red and violet of any order which together with the intermediate colours constitute any one of these rings Now these being viewed through a prism the violet circle BX will by a greater refraction be farther translated from its place th n t 1
on h

ay be translated to bx so as to ap
p ua nearer to it at x than before and if the red be farther translated to
the violet may be so much farther
and if the red be yet farther transl
farther translated to $\beta\xi$ as to pass be u at ξ and convene with it at e and f And this being understood not only of the red and violet but of all the other intermediate colours and also of every revolution of those colours you will easily perceive how those of the same revolution or order by their nearness at

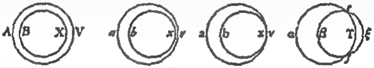


Fig 7

at and Tξ and the
tinct arcs of cir
severally at x nd again
appear severally but yet in a contrary order to that which they had before and still retain beyond e and f But on the other side at ab ab or $a\beta$ the e colours must become much more confused by being dilated and spread so as to interfere with those of other orders And the same confusion will happen at Tξ between e and f if the refraction be very great or the pri m very distant from the object-gla es in which case no parts of the rings will be seen save only two little arcs at e and f whose distance from one another will be aug mented by removing the prism still farther from the object glasses And these little arcs must be distinctest and whitest at their middle and at their ends where they begin to grow confused they must be coloured And the colours at one end of every arc must be in a contrary order to those at the other end by reason that they cross in the intermediate white namely their ends which verge towards Tξ will be red and yellow on that side next the centre and blue and violet on the other side But their other ends which verge from Tξ will on the contrary be blue and violet on that side towards the centre and on the other ide red and yellow

Now as all the e th
way of reasoning
in a dark room by rough a pri m by reflexion of the several prismatic colours which an a sistant causes to move to and fro upon a wall or paper from whence they are reflected whilst the spectator s eye the pri m and the object glasses (as in the 13th Observation) are placed steady

k passeth obliquely out of one medium into ano her which re-
of their refractive
e a total reflexion.
ne of incidence at
r the sines are which measure
of he circle and consequently
greatest difference of the
to air where the refraction
al reflexion begins when the
In passing out of glass into
f the sines 20 to 31 th total
reflexion begins when the angle of incidence legrees 10 minutes and so in passing out of crystal or more strongly refracting mediums into air there is still a less obliquity requisite to cause a total reflexion Superficies therefore
ident on them and

observing that in the superficies interceding two transparent mediums as are air water oil common glass crystal metallin glasses island glasses white transparent arsenic diamond &c.) the reflexion is stronger or weaker accordingly as the superficies hath a greater or less refracting power For in the confine of air and sal-gem tis stronger than in the confine of air and water and still stronger in the confine of air and common glass or crystal and stronger in the confine of

still
well-
rectified oil of vitriol or spirit of turpentine If water be distinguished into two parts by any imaginary surface the reflexion in the confine of those two parts

accordingly as those mediums differ more or less in their refracting powers. Hence in the confine of common glass and crystal there ought to be a weak reflexion and a stronger reflexion in the confine of common and metalline glass though I have not yet tried this. But in the confine of two glasses of equal density there is not any sensible reflexion, as was shewn in the first Observation. And the same may be understood of the superficies separating two crystals or two liquors or any other substances in which no refraction is caused. So then, the reason why uniform pellucid mediums (such as water glass or crystal) have no sensible reflexion but in their external superficies where they are adjacent to other mediums of a different density is because all their contiguous parts have one and the same degree of density

PROPOSITION 2

The least parts of almost all natural bodies are in some measure transparent And the opacity of those bodies ariseth from the multitude of reflexions caused in their internal parts

That this is so has been observed by others and will easily be granted by them that have been conversant with microscopes And it may be also tried by applying any substance to a hole through which some light is immitted

13th 14th and 15th Observations comm

By the

mixture of

colours F

and 24th

n the 3d 12th

test that although in the 4th and 18th

Observations there appear no more than eight or nine of

are really a far greater number

another as after those eight or n

and constitute an even and sen

that

there is a constant

and refrangibility the most refrang

violet the least refrangible red

to be the same

and reflexibility the violet being in

cumstances reflected at least thickness of

greatest th

Whence

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by consequence that all the

pearances of colours in the

Part III

Of the

I AM n

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they appear of divers

as they are disposed to reflect most copiously the rays originally endued with those colours But their constitutions whereby they reflect some rays more copiously than others remain to be discovered and these I shall endeavour to manifest in the following Propositions

PROPOSITION I

Those superficies of transparent bodies reflect the greatest quantity of light which have the greatest refracting power that is which intercede mediums that differ most in their refractive densities And in the confines of equally refracting mediums there is no reflexion

The analogy between reflexion and refraction will appear by considering

...seth obliquely out of one medium into another which re-
... of their refractive
... a total reflexion.
... sine of incidence at
... the radius of the circle and consequently
... re there is the greatest difference of the
... out of water into air where the refraction
... 3 to 4 the total reflexion begins when the
... es 35 minutes. In passing out of glass into
... by the ratio of the sines 20 to 31 the total
... also in
... re is
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... and

... observing that in
... s are air water oil
common glass crystal metalline glasses ... ite transparent arsenic diamond, &c.) the reflexion is stronger or weaker accordingly as the superficies hath a greater or less refracting power. For in the confine of air and sal-gem 'tis stronger than in the confine of air and water, and still stronger in the confine of air and common glass or crystal, and stronger in the confine of air and a diamond. If any of these, and such like transparent solids, be immerged in water, its reflexion becomes much weaker than before; and still weaker if they be immerged in the more strongly refracting liquors of well-rectified oil of vitriol or spirit of turpentine. If water be distinguished into two parts by any imaginary surface, the reflexion in the confine of those two parts is none at all. In the confine of water and ice 'tis very little; in that of water and oil 'tis something greater; in that of water and sal-gem still greater; and in that of water and glass, *or crystal or other denser substances still greater*, accordingly as those mediums differ more or less in their refracting powers. Hence in the confine of common glass and crystal there ought to be a weak reflexion, and a stronger reflexion in the confine of common and metalline glass, though I have not yet tried this. But in the confine of two glasses of equal density there is not any sensible reflexion, as was shewn in the first Observation. And the same may be understood of the superficies separating two crystals, or two liquors, or any other substances in which no refraction is caused. So then the reason why uniform pellucid mediums (such as water, glass, or crystal) have no sensible reflexion but in their external superficies, where they are adjacent to other mediums of a different density, is because all their contiguous parts have one and the same degree of density.

Proposition 2

The least parts of almost all natural bodies are in some measure transparent: And the opacity of those bodies ariseth from the multitude of reflexions caused in their internal parts.

That this is so has been observed by others, and will easily be granted by them that have been conversant with microscopes. And it may be also tried by applying any substance to a hole through which some light is immitted

nto d

ꞁ ti usparent if it be of a suffi ꞁ u ess Only white metalline bodies must be excepted which by reason of their excessive density seem to reflect almost all the light incident on their first superficies unless by solution in menstruums they be reduced into very small particles and then they become transparent

PROPOSITION 3

Between the parts of opaque and colou d bod
eml

pu roid of both air and water but p laps not wholly void of all substance between the pa t of l

The truth of th
secon
bodie
bodie u istices between them because reflexions are caused only in superficies which separate mediums of a differing density (Prop 1)

But further that this discontinuity of parts is the princ p l opacity of bodie w ll
trans
densi upp u in water or oil the *Oculus mundi* stone steeped in water linen cloth oiled or varnished and many other substances soaked in such liquors as will intimately pervade their little pores become by that means more transparent than others he contrary the most separating their par

g formed into m ny small bubbles either alone n the ith oil of turpen tine or which it will not perfectl u to the increase of the opacity of these bodies it conduces something that by the 23d Observation the reflexions of very thin transparent substances are considerably stronger than those made by the same substances of a greater thickness

PROPOSITION 4

The parts of bodies and their interstices must not be less than of some definite bigness to render them opaque and coloured

For the opaquest bodies if their parts be subtly divided (as metals by being dissolved in acid menstruums &c) become perfectly tran parent And you may also remember that in the eighth Observation there was no sensible reflexion at the superficies of the object-glasses where they were very near one another though they did not absolutely touch And in the 17th Observation the reflexion of the water bubble where it became thinnest was almost insen ible so as to cause very black spots to appear on the top of the bubble by the want of reflected light

a hile
e
t

as full of pores or interstices between their ... their parts and interstices to be too small to cause reflexions in their common surfaces

PROPOSITION 5

The transparent parts of bodies according to their several sizes reflect rays of one colour and transmit those of another on the same grounds that thin plates or bubbles do reflect or transmit those rays And this I take to be the ground of all their colours

... being of an even thickness appears

natural bodies being like so many fragments of a plate must on the same

nity of their properties The
ilarly those of peacock tail
of several colours in several
positions of the eye after the very same manner that thin plates were found

grosser lateral branches or fibres of those feathers And to the same purpose it is that the webs of some spiders by being spun very fine have appeared coloured as some have observed and that the coloured fibres of some silks by varying the position of the eye do vary their colour Also the colours of silks cloths and other substances which water or oil can intimately penetrate become more faint and obscure by being immerged in those liquors and recover their vigour again by being dried much after the manner declared of thin bodies in the 10th and 21st Observations Leaf gold some sorts of painted glass the infusion of *Lignum nephriticum* and some other substances reflect one colour and transmit another like thin bodies in the 9th and 20th Observations. And some of those coloured powders which painters use may have their colours a little changed by being very elaborately and finely ground Where I see not what can be justly pretended for those changes besides the breaking of their parts into less parts by that contrition after the same manner that the colour of a thin plate is changed by varying its thickness For which reason also it is that the coloured flowers of plants and vegetables by being bruised usually become more transparent than before or at least in some degree or other change their colours. Nor is it much less to my purpose that by mixing divers liquors very odd and remarkable productions and changes of colours may be effected of which no cause can be more obvious and rational than that the saline corpuscles of one liquor do variously act upon or unite with the tinging corpuscles of another so as to make them swell or shrink (whereby not only their bulk but their density also may be changed) or to divide them

into smaller corpuscles (whereby a coloured liquor may become transparent) or to make many of them associate into one cluster whereby two transparent liquors may compose a coloured one For we see how apt those saline menstruums are to penetrate and dissolve substances to which they are applied and some of them to precipitate what others dissolve In like manner if we consider the various phenomena of the atmosphere we may observe that when vapours are first raised they hinder not the transparency of the air being divided into parts too small to cause any reflexion in their superficies But when in order to compose drops of rain they ... see and constitute globules of all ... y become of con ... ve ... constitute clouds of ... And I see not what can be rationally co ... in so transparent a substance as water for the production of these colours besides the various sizes of its fluid and globular parcels

PROPOSITION 6

The parts of bodies on which their colours depend are denser than the medium which pervades their interstices

This will appear by considering that the colour of a body depend ... on the rays which are incident ...

t

t ...

of colours in so great a ...

fusedly ...

than an ... very imperfect and dirty colour ... the thin body or small particle be much denser than the ambient medium the colours according to the 19th Observation are so little changed by the variation of obliquity that the rays which are reflected least obliquely may predominate over the rest so much as to cause a heap of ... h particles to appear very intensely of the ...

It cond ...

cordir ...

within ... by the rarer within the denser

PROPOSITION 7

The bigness of the component parts of natural bodies may be conjectured by their colours

For since the parts of these bodies (by Prop 5) do most probably exhibit the same colours with a plate of equal thickness provided they have the same refractive density and since their parts seem for the most part to have much the same density with water or glass as by many circumstances is obvious to collect to determine the sizes of those parts you need only have recourse to the precedent Tables in which the thickness of water or glass exhibiting any colour is expressed Thus if it be desired to know the ... er of a corpuscle which being of equa ... the third order the number 16¼ sh

The greatest difficulty is here to know of what order the colour of any body is. And for this end we must have recourse to the 4th and 18th Observations from whence may be collected these particulars

... be more and intense are ... l order al ... may ... l the orange and ... e

There may be good *greens* of the fourth order ... are of the third And of this order the green of all vegetables seems to be partly by reason of the intenseness of their colours, and partly because when they wither some of them turn to a greenish yellow and others to a more perfect yellow or orange or perhaps to red passing first through all the aforesaid intermediate colours ... may

doubt is of the same order with those colours into which it changeth because the changes are gradual and those colours though usually not very full yet are often too full and lively to be of the fourth order

Blues and *purples* may be either of the second or third order but the best are of the third. Thus the colour of violets seems to be of that order because their syrup by acid liquors turns red and by urinous and alkalizate turns ... d of

to a green of the second order which red and green especially the green seem too imperfect to be the colours produced by these changes But if the said purple be supposed of the third order its change to red of the second and green of the third may without any inconvenience be allowed

If there be found any body of a deeper and less reddish purple than that of the violets its colour most probably is of the second order But yet there being no body commonly known whose colour is constantly more deep than theirs I have made use of their name to denote the deepest and least reddish purples such [redacted] manifestly transcend their colour in purity

The *bl e* of the first order though very faint and little may possibly be the colour of some substances and particularly the azure colour of the skies seems to be of this order For all vapours when they begin to condense and coalesce into small parcels become first of that bigness whereby such an azure must be reflected before they can constitute clouds of other colours. And so this being the first colour which vapours begin to reflect it ought to be the colour of the finest and most transparent skies in which vapours are not arrived to that grossness requisite to reflect other colours as we find it [redacted] by experience

Wh teness if most intense and luminous is that of the first order if less ...

gold, if foliated, is transparent and all metal become transparent if dissolved in menstruums or vitrified the opacity of white metals ariseth not from their density alone They being less dense than gold would be more transparent

than it did not some other cause concur with their density to make them opaque And this cause I take to be such a bigness of their particles as fits them to reflect the white of the first order For if they be of other thicknesses they may reflect other colours as is manifest by the colours which appear upon hot steel in tempering it and sometimes upon the surface of melted metals in the skin or scoria which arises upon them in their cooling And as the white of the first order is the strongest which can be made by plates of transparent substances so it ought to be stronger in the denser substances of metals than in the rarer of air water and glass Nor do I see but that metallic substances of such a thickness as may fit them to reflect the white of the first order may by reason of their great density (according to the tenor of the first of these Propositions) reflect all the light incident upon them and so be as opaque and splendent as it s possible for any body to be Gold or copper mixed with less than half their weight of silver or tin or regulus of antimony in fusion or amalgamed with a very little mercury become white which shews both that the particles of white metals have much more superficies and so are smaller

are so opaque as not to suffer
them Now it is scarce to be
r are of the second and third

order and therefore the particles of white metals cannot be much bigger than is requisite to make them reflect the white of the first order The volatility of mercury argues that they are not much bigger nor may they be much less lest they lose their opacity and become either transparent as they do when attenuated by vitrification or by solution in menstruums or black as they do when ground smaller by rubbing silver or tin or lead upon other substances to draw black lines The first and only colour which white metals take by grinding their particles smaller is black and therefore their white ought to be that which borders upon the black spot in the centre of the rings of colours that is the white of the first order But if you would hence gather the bigness of metallic particles you must allow for their density For were mercury transparent its density is such that the sine of incidence upon it (by my computation) would be to the sine of its refraction as 71 to 20 or 7 to 2 And therefore the thickness of its particles that they may exhibit the same colours with those of bubbles of water ought to be less than the thickness of the skin of those bubbles in the proportion of 2 to 7 Whence it s possible that the particles of mercury may be as little as the particles of some transparent and volatile fluids and yet reflect the white of the first order

Lastly for the production of *black* the corpuscles must be less than any of those which exhibit colours For at all greater sizes there is too much light reflected to constitute this colour But if they be supposed a little less than is requisite to reflect the white and very faint blue of the first order they will according to the 4th 8th 17th and 18th Observations reflect so very little light as to appear intensely black and yet may perhaps variously refract it to and fro within themselves so long until it happen to be stifled and lost by which means they will appear black in all positions of the eye without any transparency And from hence may be understood why fire and the more subtle dissolver putrefaction by dividing the particles of substances turn them to black why small quantities of black substances impart their colour very freely and intensely to other substances to which they are applied the

minute particles of these by reason of their very great number evenly over spreading the gross particles of others why glass ground very elaborately with sand on a copper plate till it be well polished makes the sand together with what is worn off from the glass and copper become very black why black ... do soonest of all others become hot in the Sun's light and burn

possible but that microscopes may at length be improved to ... of the particles of bodies on which their colours depend if they are not already

be doubted of excepting this position That transparent corpuscles of the same thickness and density with a plate do exhibit the same colour And this I would

covered with microscopes which if we shall at length attain to I fear it will be the utmost improvement of this sense For it seems impossible to see the more secret and noble works of Nature within the corpuscles by reason of their transparency

PROPOSITION 8

The cause of reflexion is not the impinging of light on the solid or impervious parts

have more strongly reflecting parts than water or glass But if that should possibl...
st on
inver

when it light in its passage out of glass into r
be angle
refl
be imagined
enough in tl out of it and at another degree of
obliquity should meet with nothing but parts to reflect it wholly especially
considering that in its passage out of air into glass how oblique
incidence it finds pores enough
I
t
s instead of a
rays
is adj transmitted where the
water to it which argues that th
pends on the
striking of th
a prism place

sh colour pretty copiously trans
m the reflexion be caused by the
why at the
parts
measu

little and almost
but reflected from it

Observation
were by turns transmitted at one thickness and
reflected at another thickness for an indeterminate number of successions
And yet in the superficies of the thinned body where it is of any one thickness
there are as many parts for the rays to impinge on as where it is of any other
thickness Sixthly if reflexion were caused by the parts of reflecting bodies it
would be impossible for thin plates or bubbles at one and the same place to
reflect the rays of one colour and transmit those of another as they do according
to the 13th and 15th Observations For it is not to be imagined that at one
place the rays which for instance

the solid parts of bodies, their reflexions from polished bodies could not be so

... For in polishing glass with sand, putty or tripoli it is not

... glass

be truly plane or truly spherical ...

compose one even surface The smaller the particles of those substances are the smaller will be the scratches by which they continually fret and wear away the glass until it be polished but be they never so small they can wear away the glass no otherwise than by grating and scratching it and breaking the protuberances and, therefore polish it no otherwise than by bringing its roughness to a very fine grain so that the scratches and frettings of the surface ... effected by impinging ... much by the most ... problem how glass polished by fretting substances can reflect light so ... y as it does And this problem is scarce otherwise to be solved than by saying that the reflexion of a ray is effected, not by a single point of the reflecting body but by some power of the body which is evenly diffused all over its surface and by which it acts upon the ray without immediate contact. For that the parts of bodies do act upon light at a distance shall be shewn hereafter

... the solid parts of bodies but ... on ... For

otherwise we must allow two sorts of reflexions. Should all the rays be reflected which impinge on the internal parts of clear water or crystal those substances would rather have a cloudy colour than a clear transparency To make bodies look black, it is necessary that many rays be stopped retained and lost in them and it seems not probable that any rays can be stopped and stifled in them which do not impinge on their parts.

And hence we may understand that bodies are much more rare and porous than is commonly believed. Water is nineteen times lighter and by consequence nineteen times rarer than gold and gold is so rare as very readily and without the least opposition to transmit the magnetic effluvia, and easily to admit quick-silver into its pores and to let water pass through it. For a concave sphere of gold filled with water and soldered up has upon pressing the sphere with great force let the water squeeze through it and stand all over its outside in multitudes of small drops like dew without bursting or cracking the body of the gold, as I have been informed by an eye-witness From all which we may conclude that gold has more pores than solid parts, and by consequence that water has above forty times more pores than parts. And he that shall find out ... he ... of ...

passage through transparent substances

The magnet acts upon iron through all dense bodies not magnetic nor red hot without any diminution of its virtue as for instance through gold silver lead, glass, water The gravitating power of the Sun is transmitted through the vast bodies of the planets without any diminution so as to act upon all their parts to their very centres with the same force and according to the same laws

as if the part upon which it acts were not surrounded with the body of th planet The rays of light whether they be very small bodies projected or only motion or force propagated are moved in right lines and whenever a ray of light is by any obstacle turned out of its rectilinear way it will never return into the same rectilinear way unless perhaps by very great accident And yet light is transmitted through pellucid solid bodies in right lines to ver t distances How bodies can have a suff
the e effect
For the c
reflect the
bod
bet
ma
sp
th
sm of empty spaces between
the till you come to solid particles such as have no pores or empty spaces within them and if in any gross body there be for instance three such degrees of particles the least of which are solid th will have seven times more pores th
degrees of
more po have one
and thirt parts If six degrees the body will have sixty and three times more pores than solid parts And so on perpetually And there are other ways of conceiving how bodies may be exceeding porous But what is really their inward frame is not yet known to us

PROPOSITION 9

Bod
tar

veral considerations First because when light goes out of glass into air as obliquely as it can possibly do if its incidence be ma more oblique it becomes totally refle
has refr
more
con

For here
t determines whether that power by which glass acts upon light shall cause it to be reflected or suffer it to be tran thirdly because those surfaces of tran

PROPOSITION 10

If light be swifter in bodies than in vacuo in the p measure the refract
are very nearly same density
unctuous and su

Let AB repre facing plane surface of any body and IC a ray

incident very obliquely upon the body in C so that the angle ACI may be infinitely little and let CI be the refracted ray From a given point B perpendicular to the refracting surface erect BR meeting with the refracting ray CR in R and if CR represent the motion of the

Fig 8

pendicular to it CB shall represent the motion of the incident ray and BR the motion generated by the refraction as opticians have of late explained.

before its incidence on the first plane had no motion towards it but an infinitely little one and if the forces in all parts of that space between the planes be at equal distances from the planes equal to one another but at several distances be bigger or less in any given proportion the motion generated by the forces in the whole passage of the body or thing through that space shall be in a subduplicate proportion of the forces as mathematicians will easily understand. And, therefore if the space of activity of the refracting superficies of the body be considered as such a space the motion of the ray generated by the refracting force of the body during its passage through that space (that is the motion BR) must be in subduplicate proportion of that refracting force I say therefore that the square of the line BR and by consequence the refract

refractive power in respect of their densities are set down in several columns.

The refraction of the air in this Table is determined by that of the atmosphere observed by astronomers. For if light pass through many refracting substances or mediums gradually denser and denser and terminated with parallel surfaces the sum of all the refractions will be equal to the single refraction which it would have suffered in passing immediately out of the first medium into the last And this holds true though the number of the refracting substances be

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particularly air, which is 3500 times rarer than the pseudo-topaz, and 4400 times rarer than glass of antimony, and 2000 times rarer than the selenitis, glass vulgar, or crystal of the rock, has notwithstanding its rarity the same refractive

The refracting bodies	*The proportion of the sines of incidence and refraction of yellow light*	*The square of BR, to which the refracting force of the body is proportionate*	*The density and specific gravity of the body*	*The refractive power of the body in respect of its density*
A pseudo-topazius, being a natural pellucid, brittle, hairy stone, of a yellow colour	23 to 14	1.699	4.27	3979
Air	3201 to 3200	0.000625	0.0012	5208
Glass of antimony	17 to 9	2.568	5.28	4864
A selenitis	61 to 41	1.213	2.252	5386
Glass vulgar	31 to 20	1.4025	2.58	5436
Crystal of the rock	25 to 16	1.445	2.65	5450
Island crystal	5 to 3	1.778	2.72	6536
Sal gemmæ	17 to 11	1.388	2.143	6477
Alum	35 to 24	1.1267	1.714	6570
Borax	22 to 15	1.1511	1.714	6716
Nitre	32 to 21	1.345	1.9	7079
Danzig vitriol	303 to 200	1.295	1.715	7551
Oil of vitriol	10 to 7	1.041	1.7	6124
Rain water	529 to 396	0.7845	1	7845
Gum arabic	31 to 21	1.179	1.375	8574
Spirit of wine well rectified	100 to 73	0.8765	0.866	10121
Camphor	3 to 2	1.25	0.996	12551
Olive oil	22 to 15	1.1511	0.913	12607
Linseed oil	40 to 27	1.1948	0.932	12819
Spirit of turpentine	25 to 17	1.1626	0.874	13222
Amber	14 to 9	1.42	1.04	13654
A diamond	100 to 41	4.949	3.4	14556

power in respect of its density which those very dense substances have in respect of theirs, excepting so far as those differ from one another.

Again, the refraction of camphor, olive oil, linseed oil, spirit of turpentine and amber, which are fat sulphureous unctuous bodies, and a diamond, which probably is an unctuous substance coagulated, have their refractive powers in proportion to one another as their densities without any considerable variation. But the refractive powers of these unctuous substances are two or three times greater in respect of their densities than the refractive powers of the former substances in respect of theirs.

th m goes into water and a great part remains behind in the form of a dry fixed earth capable of vitrification

Spirit of wine has a refractive power in a middle d gree between those of water and oily substances and accordingly seems to be composed of both

oils by fermentation converted into spirit They find also that oils ix po d in a small quantity upon fermentating vegetables they distil over after fermentation in the form of spirits

So then by the foregoing Table all bodies seem to have their refractive powers proportional to their den ities (or very nearly) excepting so far as they partake more or less of sulphureous oily particles and thereby have their refractive power made greater or less Whence it seems rational to attribute the refractive power of all bodies chiefly if not wholly to the sulphureous parts with which they abound For it s probable that all bodies abound more or less with sulphurs And as light congregated by a burning-glass acts most upon

by the action of the refracted or reflected light

I have hitherto explained the po er of bodies to reflect and refract and shewed that thin transparent plates fibres and particles do according to their several thicknesses and densities reflect several sorts of rays and thereby appear of several colours and by consequence that nothing more is requisite for producing all the colours of natural bodies than the several sizes and densities of their transparent particles But whence it is that these plates fibres and

nature of bodies these the nature of light for both must be understood before the reason of their actions upon one another can be kno n And because the last Proposition depended upon the velocity of light I will begin with a Proposition of that kind

PROPOSITION 11

Light is propagated from lum ous bodies in time and spends about seven or eight m

and Jup ter happen about seven or eight minutes sooner than they ought to do by the Tables and when the Earth is beyond the Sun they happen about seven or e ght minutes later than they ought to do the reason being that the light of

the satellites has farther to go in the latter case th
ameter of the E
tricities of the c
and at all time
mean motions

the Earth and in the three
its is insensible as I find by computation from the theory of
their gravity

PROPOSITION 12

F
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t
next refra

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servations For by those
rays at equal angles of
transparent plate is alternately reflected and transmitted for many successions accordingly as the thickness of the plate increases in arithmetical progression of the numbers 0 1 2 3 4 5 6 7 8 &c so that if the first reflexion (that which makes the first or innermost of the rings of colours there described) be made at the thickness 1 the rays shall be transmitted at the thicknesses 0 2 4 6 8 10 12 &c and thereby make the central spot and rings of light which appear by
thickness 1 3 5 7 9 11 &c and there
reflexion And this alternate reflexion and
Observation continues for above a hundr
tions in the next part of this book for many thousands being propagated from one surface of a glass plate to the other though the thickness of the plate be a quarter of an inch or above so that this alternation seems to be propagated from every refracting surface to all distance

Th
thin p

and therefore it depends on
them both

It is therefore performed at the second surface for if it were performed at the first before the rays arrive at the second it would not depend on the second

It is al influenced by some action or disposition propagated from the first to the second because otherwise at the second it would not depend on the first And this action or disposition in its propagation intermits and returns by equal intervals because in all its progress it inclines the ray at one distance from the first surface to be reflected by the second at another to be transmitted by it and that by equal intervals for innumerable vicissitudes And becau the ray is disposed to reflexion at the distances 1 3 5 7 9 &c and to transmission at the distances 0 2 4 6 8 10 &c (fo

is to be accounted a return of the

same disposition which the ray first had at the di tance 0 that is at its transm..sion through the first refracting surface All which is the thing I would prove

What kind of action or di position this i whether it consi ts in a circulating or a vibrating motion of the ray or of th e medium or something else I do not here enquire Those that are averse from assenting to any new discoveries but such as they can explain by an hypothesis may for the present suppose that as stones by falling upon water put the water into an undulating motion and all bodies by percussion excite vibrations in the air so the rays of light by impinging ... reflecting surface excite vibrations in the refracting or
them agitate the sol d parts of
g them cause the body to grow
e propagated in the refracting
he manner that vibrations are

propagated in the air for causing sound and mo e fa ter than the rays so as to overtake them and that when any ray is in that part of the vibration which conspires with its motion it easily breaks through a refracting surface but when it is in the contrary part of the vibration which impedes its motion it is easily reflected and by consequence that every ray is successively disposed to be easily reflected or easily transmitted by every vibration which overtakes it But whether this hypothesis be true or false I do not here con ider I content myself with the bare discovery that the rays of light are by some cause or other alternately disposed to be reflected or refracted for many vicissitudes.

DEFINITION

The returns of the disposition of any ray to be reflected I will call its fits of easy re exion and those of its d sposit on to be transmitted its fits of easy transmis ion and the space it passes between every return a d the next return, the i erval of its fits

Proposition 13

The reason why the surfaces of all thick transparent bod es reflect part of the li ht i cident on them and efract the est is that some rays at their incidence a e in fits ... m ssion

ht reflected
venly white
all over the plate did through a prism appear waved with many succes ion of

And hence light is in fits of easy reflexion and easy tran mission before its incidence on transparent bodies And probably it is put into such fits at its first

reflexion and transmi sion of the ra s, the body loseth its reflecting power For if the rays, which at their entering into the body are put into fits of easy transmission, arrive at the farthest surface of the body before they be out of those

fits they must be

their reflecting po ... why all opaque bodies when reduced into ... small parts become tran parent

Proposition 14

Those surfaces of transparent bodies which if the ray be in a fit of refraction do refract it most strongly if the ray be in a fit of reflexion do reflect it most easily

For we shewed above in Prop 8 that the cause of reflexion is not the impinging of light on the solid impervious parts of bodies but some other power by which those solid parts act on light at a distance We shewed al o in Prop 9 that bodies reflect and refract light by one and the same po ... variously exercised in various circ ... strongly refracting sur ... evince and rarify both ... ou

Proposition 15

In any one and the same sort of rays emerging in any angle out of a ... surface into one and the same ... refl ...

refraction

This is r ... by the ... th and 19th Observations

Proposition 16

In several sorts of rays emerging in equal a ... the sam ... missi ... lengt ... with c ... intermediate deg ...

... ations

Proposition 17

If ... fits ... an ... action when the rays pass out of the firs ... two mediums into the second

This is manifest by the 10th Observation

Proposition 18

If the rays which paint the colour in th ... a ... are the intervals of their fits ... easy ... transmission

Th ... the fits of easy ref ... any sort of rays refracted in any angle into any medium and thence to know whether the rays shall be reflected or trans-

mitted at their subsequent incidence upon any other pellucid medium Which thing, being useful for understanding the next part of this book was here to be set down. And for the same reason I add the two following Propositions

PROPOSITION 19

If any sort of rays falling on the polite surface of any pellucid medium be reflected back the fits of easy reflexion which they have at the point of reflexion shall still continue to return and the returns shall be at distances from the point of reflexion in the arithmetical progression of the numbers 2 4 6 8 10 12 &c and between these fits the rays shall be in fits of easy transmission

... are of a returning

... from ... t begin from 0 and

to what happens when the fits are propagated from points of refraction.

PROPOSITION 20

The intervals of the fits of easy reflexion and easy transmission propagated from points of reflexion into any medium are equal to the intervals of the like fits which the same rays would have if refracted into the same medium in angles of refraction equal to their angles of reflexion

For when light is reflected by the second surface of thin plates it goes out afterwards freely at the first surface to make the rings of colours which appear ...

fits within the plate after reflexion were not equal both in length and number to their intervals before it. And this confirms also the proportions set down in the former Proposition For if the rays both in going in and out at the first surface be in fits of easy transmission and the intervals and numbers of those fits between the first and second surface before and after reflexion be equal the distances of the fits of easy transmission from either surface must be in the same progression after reflexion as before that is from the first surface which ...

Part IV

Observations concerning the reflexions and colours of thick transparent polished plates

THERE is no glass or speculum how well soever polished but besides the light which it refracts or reflects regularly scatters every way irregularly a faint light by means of which the polished surface when illuminated in a dark room by a beam of the Sun s light may be easily seen in all positions of the eye There are certain phenomena of this scattered light which when I first observed them seemed very strange and surprising to me My Observations were as follows

OBS 1 The Sun shining into my darkened chamber through a hole one-third of an inch wide I let the intromitted beam of light fall perpendicularly upon a glass speculum ground concave on one side and convex on the other to a sphere of five feet and eleven inches radius and quick silvered over on the convex side And holding a white opaque chart or a quire of paper at the centre of the spheres to which the spe l m ~
five feet and ele
of light might p
speculum and tl u back to the ame hole) I observed upon the chart four or five concentric irises or rings of colo ~ l
ing the hole much after the manner that
Observation of the fir ~

L u c u e ll sometimes when the Sun shone very clear there appeared faint lineaments of a sixth and seventh If the distance of the chart from the speculum was much greater or much less than that of six feet the rings became dilute and vanished And if the distance of the speculum f om the ~ l 1
m l gre k

u a y at the rings upon the chart And this posture in always to be understood in the following Ob ervations where no other in expres ed

OBS 2 The colours of the e rainbows succeeded one another from the centre outward in the same form and order with tho e which were made in the ninth Ob ervation of the first part of this book by light not reflected but tran mitted through the two object gla ses For first there was in their common centre a white round spot of faint light something broader than the reflected beam of light which beam sometimes fell upon the middle of the spot and sometimes by a little inclination of the speculum receded from the middle and left the spot white to the centre

This white pot was immediately encompas ed with a dark grey or ru et and that dark grey with the colours of the first iris which colours on the in ide

next the dark grey were a little violet and indigo and next to that a blue which on the outside grew pale and then succeeded a little greenish yellow and after that a brighter yellow and then on the outward edge of the iris a red which on the outside inclined to purple

This iris was immediately encompassed with a second whose colours were in order from the inside outwards purple blue green yellow light red a red [illegible] ... in order [illegible] ... t than that of the former iris.

The fourth and fifth iris seemed of a bluish-green within and red without but so faintly that it was difficult to discern the colours

OBS. 3 Measuring the diameters of these rings upon the chart as accurately as I could, I found them also in the same proportion to one another with the rings made by light transmitted through the two object-glasses. For the diameters of the four first of the bright rings measured between the brightest parts of their orbits, at the distance of six feet from the speculum were $1\frac{11}{16}$, $2\frac{3}{8}$, $2\frac{11}{12}$, $3\frac{3}{8}$ inches, whose squares are in arithmetical progression of the numbers 1, 2, 3, 4. If the white circular spot in the middle be reckoned amongst the rings and its central light where it seems to be most luminous be put equipollent to an infinitely little ring, the squares of the diameters of the rings will be in the progression 0, 1, 2, 3, 4, &c. I measured also the diameters of the dark circles between these luminous ones and found their squares in the progression of the numbers $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, &c. the diameters of the first four at the distance of six feet from the speculum being $1\frac{3}{16}$, $2\frac{1}{16}$, $2\frac{2}{3}$, $3\frac{3}{20}$ inches. If the distance of the chart from the speculum was increased or diminished, the diameters of the circles were increased or diminished proportionally

OBS. 4. By the analogy between these rings and those described in the Observations of the first part of this book, I suspected that there were many more of them which spread into one another and by interfering mixed their colours and diluted one another so that they could not be seen apart. I viewed them, therefore through a prism, as I did those in the 24th Observation of the first part of this book. And when the prism was so placed as by refracting the light of their mixed colours to separate them, and distinguish the rings from one another, as it did those in that Observation, I could then see them distincter than before, and easily number eight or nine of them, and sometimes twelve or thirteen. And had not their light been so very faint, I question not but that I might have seen many more.

OBS. 5. Placing a prism at the window to refract the intromitted beam of light and cast the oblong spectrum of colours on the speculum, I covered the speculum with a black paper which had in the middle of it a hole to let any one [illegible]

[illegible] And when they were illuminated with any one colour the squares of their diameters measured between their most luminous parts were in the arithmetical progression of the numbers, 0, 1, 2, 3, 4 and the squares of the diameters of their dark intervals in the progression of the intermediate numbers

$\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$. But if the colour was varied ... the red they ... colours (yell... answering to ... in green, and greater in green than in blue. And hence I knew, that when the speculum was illuminated with white light, the red and yellow on the outside of the rings were produced by the least refrangible rays, and the blue and violet by the most refrangible, and that the colours of each ring spread into the colours of the neighbo... rings on either side, after the manne... this book, and by mixing diluted o... tinguished, unless near the centre ...

... how much the colours of the several rings spread into one another, I me... th...

... to be seen. To carry on the computation, let us therefore suppose that the differences of the diameters of circles made by the outmost red, the confine of red and orange, the confine of orange and yellow, the confine of yellow and green, the confi... blue, the confine of b... most violet, a... which sound t... ... sol, la, mi, fa, sol, that is, as the numbers $\frac{1}{9}$, $\frac{1}{18}$, $\frac{1}{12}$, $\frac{1}{12}$, $\frac{2}{27}$, $\frac{1}{27}$, $\frac{1}{18}$. And ... the confi...

... (that is, as $\frac{8}{27}$ to $\frac{5}{54}$, or as 16 to 5). And therefore ... $\frac{5}{16}A$. Add the first to $9A$, and subduct the la... diameters of the circles made by the least an... ... rays ... A and $\frac{81\frac{11}{16}}{8}A$. These diameters are therefore to one another as 75 to $61\frac{1}{2}$, or 50 to 41, and their squares as 2500 to 1681, that is, as 3 to 2 very nearly. Which ... differs not much from the ... by the outmost red and outmo... part of this book.

OBS. 6. Placing my eye where these rings appeared plainest, I saw the speculum tinged all over with waves of colours (red, yellow, green, blue) like those which in the Observations of the first part of this book appeared between the object glasses, and upon bubbles of water, but much larger. And after the manner of those, they were of various magnitudes in vario... ...

over against th centre of the concavity of the speculum (that is o feet and — h specu l um) their common centre was in a right line

of the clouds propagated to the speculum ti r by

wh n the Sun shone throu,h that hol upon tl e speculum his light upon it was of the colour of tl ring wl ereon it fell but by its sp len lor obscured the rings made by the li,ht of tl cloud unless when tl speculum was removed to a great distance from tl e wind ow so that l is light upon it might be broad and faint. By varying tl position of my eye an l moving it nearer to or farther from the direct beam of tl S in s light tl colour of tl S in s reflected light constantly varied upon th speculum as it did upon my eye tl same c l our always appearing to a l ystan l r upon my eye which to me appeared upon tl speculum. And tl ence I knew that the rings of colours upon the chart were made by these reflected c lours propa,at l tl itl r from the s eculum in several an,les and that tl ir produ tion depended not upon tl e terminati on of light and shad w

Obs. By th analogy of all tl ese pl en mena with those of tl like rings of colours described in the first part of tl is bo k it seemed to me that these c l urs were prod iced by tl is tl ick plate of gla much aft r tl manner that those were prod iced by very thin plates For up n trial I found that if tl e quick-silver were rubbed off from the backsid of tl specul im tl gla s alone would cause th s m rings of colours but much m re faint than b f re and theref re th ph nomenon depends not upon tl quick-silver unless s far as the qu k-silver by in reasing tl reflexi n of tl e backsid of tl e gl in creases the ligl t of th rings of c lours I found also that a speculum of m tal without glass made som years since for optical uses and very well wr ugl t prod iced n ne of th se rings and th nce I underst od that tl ese rings arise n t from n specular surface al n but depend upon the two surfaces of the

l k of lieu of an th r when more of liq i of anoth r wh n still more obliq i

col urs. And as the reason wl y a tl in pl te appeared of several colours in several obliquities of th rays was that the rays of one and th sam s rt are reflected by the thin plate at one obliquity and transmitted at another and those of other sorts transmitted wh re these are reflected and reflected wl ere these are tran mitted so the reason why th thick plate of gla s wl ereof the sper lum wa m de did appear of various colours in various obliquities and

surface of the gl ss and accordingly as th obliquity became greater and greater emerged and were reflected alternately for many successions and that in one and the same obliquity the rays of one sort were reflected and those of

½ 1½ 2½ 3½ But if the colou w

... by the most refrangible ... the colours of each ring spread into the colours of the neighbouring rings on either side after the manner ... this book and by mixing diluted one ... tinguished unless near the centre ...

... much the colours of the several rings spread into one another I measured the diameters of the second and third rings and found them ... to be to the same diameters wh... to 8 or thereabouts For it wa... Also the circles made successive... from one another than those m... For the circle made by the vio... computation let us therefore su... differences of the diameters of circles made by the outmost red the confine of red and orange ... orange and yello ... the conf... blue ... most ...

... the confine ... to the dif ... the con ...

... (that is as $^5/_7$ to $^5/_{14}$ or as 16 to 5) And therefore these differences will be $^3/_8A$ and $^5/_{16}A$ Add the first to 9A and subduct the last from 9A and ... e diameters of ... $^{41}/_8A$ These ... 11 and their ... that is as 3 to 2 very nearly Which proportion differs not much from the proportion of the diameters of the circles made by the outmost red and outmost violet in the 13th Observation of the first part of this book

OBS 6 Placing my eye where the rings appeared plainest I saw the speculum tinged all over with waves of colours (red yellow green blue) like those which in the Observations of the first part of this book appeared between the object glasses and upon bubbles of water but much larger And after the manner of those they were of various magnitudes in vario...

These seem to be the rea ns of these rings in general and this put me upon
— h h lnes of the glas and con idering whether the dimen ions

the —1— th part of an inch and by the ... ...
thin plate of glass tran mits the same light of the same ring when its thickness
— — 1 — (that i
sing the
he same
l and so
mis ion

and six arithmetical means between the sines of incidence and refraction counted f om the ine of incidence when the refraction is made out of any
— — — — re

34 38((the number of fits of the perpendicular rays in going through the glass towards the white spot in the centre of the rings) hath to 34,385 34,384 34,383
d — — — — — hen h l

f

the radius bein 100 000 000 and the sines of these an les are 60 1 0 0 1,3 1 and 1 5 and the proportional sines of refraction 1 1 2 1 659 2 031 and 34 the radius being 100 000 For since the sines of incidence out of glass into air are to the sines of refraction as 11 to 17 and to the above-mentioned
—

rays to th — — h — — — h h h —
in pa
2 031
of th
3

another transmitted This is manifest by the fifth Observation of this part of this book For in that Observation when the speculum was illuminated by any one of the prismatic colours that light made many rings of the same colour upon the chart with dark intervals and therefore at its emergence out of the speculum was alternately transmitted and not transmitted from the speculum to the chart for many successions according to the various obliquities of its emergence And when the colour cast on the speculum by the prism was varied the rings became of the colour cast on it and varied their bigness with their colour and therefore the light was now alternately transmitted and not transmitted from the speculum to the chart at other obliquities than before It seemed to me therefore that these rings were of one and the same original with those of thin plates but yet with this difference that those of thin plates are made by the alternate reflexions and transmissions of the rays at the second surface of the plate after one passage through it, but here the rays go twice through the plate before they are alternately reflected and transmitted First they go through it from the first surface to the quick-silver and then return through it from the quick silver to the first surface and there are either transmitted to the chart or reflected back to the quick-silver accordingly as they are in their fits of easy reflexion or transmission when they arrive at that surface For the intervals of the fits of the rays which fall perpendicularly on the speculum

therefore since all the rays that enter through the first surface are in their fits of easy transmission at their entrance and as many of these as are reflected by the second are in their fits of easy reflexion there all these must be again in *their fits of easy transmission at their return to the first* and by consequence there go out of the glass to the chart and form upon it the white spot of light in the centre of the rings For the reason holds good in all sorts of rays and therefore all sorts must go out promiscuously to that spot and by their mixture cause it to be white But the intervals of the fits of those rays which are reflected more obliquely than they enter must be greater after reflexion than before by the 15th and 20th Propositions And thence it may happen that the rays at their return to the first surface may in certain obliquities be in fits of easy reflexion and return back to the quick-silver and in other intermediate

and less and more numerous in the more refrangible therefore the rings re-

really found to be in the fifth Observation And therefore

colours encompassing the white spot of light shall be red without any violet within and *yellow and green* and blue in the middle as it was found in the second Observation and these colours in the second ring and those that follow shall be more expanded till they spread into one another and blend one another by *interfering*

These seem to be the reasons of these rings in general and this put me upon ... the thickness of the glass and considering whether the dimensions ... from it by computation ... is concavo-convex plate of ... ich precisely Now by the ... n plate of air transmits the ... ellow) when its thickness is ... rvation of the same part a ... ame ring when its thickness

... glass transmits ... (that is ... osing the ... the same ... nd and so

one-quarter of an inch it transmits the same bright ... Suppose th■ be the bright yellow light transmitted perpendicularly from the ■ ecting convex sid of the glass throu h the concave side to the white pot in ... th 7th and 19th ... 20 h Propos ... the glass the ... of h same ... secant ... ndred

and six arithmetical means between the sines of incidence and r fraction counted from the ine of incidence when the refraction is made out of any plaed body into any medium encompassing it that is in this case out of glass into air Now if the thickness of the glass be increased by degrees so as to bear ... h proport on which

2 the radius being 100 000 000 and the sines of these angles are 762 1 079 1.3 1 and 1 5..3 and the proportional sines of refraction 1 172 1 659 2 031 and 2 34 the radius being 100 000 For since the sines of incidence out of glass into air are to the sines of refraction as 11 to 1 and to the above-mentioned secant ■ 11 to the first of 106 arithmetical means between 11 and 17 (that is

34 383 54 82 respectively And therefore if the thickness in all these cases be one-quarte of an inch (as it is in the glass of which the speculum was made)

the brigh
1
1
of propagated f gles of refraction t
paint rings
light c
the an the speculum and by
conseq s shall be to the distance of the chart from the
speculum as those sines of refraction doubled are to the radius (that is as 1 17
1 659 2 031 and 2 345 doubled are to 100 000)
of the chart from th
the third of these
light upon the ch
eters are to six fe to the radius
Now these diame computation
are the very same with those fo observations by
measuring them viz with 1 11/16 and 3⅜ inches
the theory of deriving these ring f

other unn putting them to the diam
eters of th ones in a subduplicate proportion of the intervals of
the fits of the rays of those colours when equally inclined to the refracting or
reflecting surface which caused those fits that is by putting the
the rings made by the rays in the
(red orange yellow green blue in
of the numbers 1 8/9 5/6 3/4 2/3 3/
monochord sounding the
the rings of th
one another
And thus I
nal with those

but yet to
added the fol
OBS 6 If the rings thus
diameters at equal distance made of such concavo-
convex plates of glass as are ground on the same sphere ought to be recip
in a subduplicate proportion of the

convex plate of glass ground
to the same sphere with the former plate Its thickness was
parts of an inch and the diameters of the three first bright rings measured
between the brightest parts of their orbits at the distance of six feet from the
glass were 3 4⅙ 5⅛ inches Now the thickness of the other glass being one-

1⁄ to ⅓ that is a 31
e numbers are 1 60
s to the second an the
y the thinner gla 3
hird of these Observa
hameters of the rings
nesses of the plates of
glass

So, then in plates of glass which are alike concave on one side and alike
ground on the convex sides and
ings are reciprocally
tes And this lews
sufficiently that the rings depend on both the surface glass They depend
on the convex surface because they are more luminous when that surface is
quick-silvered over than when it is without quick-silver They depend also
how h surface a speculum makes them
se
of
of
the surf ces of those plates because the bigness of the ri g and ro-
portion to one another and the variation of their bigness arising from the
variation of the thickness of the glass and the orders of their colours is such as
ought to result from the Propositions in the end of the third part of this book
derived from the phenomena of the colours of thin plates set down in the first
part.

There are yet other phenomena of these rings of colours but such as follow
from the same Propositions and therefore confirm both the truth of those
Propositions and the analogy between these rings and the rings of colours
made by very thin plates. I hall subjoin some of them

Obs 10 When the beam of the Sun s light was reflected back from the specu
lum, not directly to the hole in the window but to a place a little d tant from
it, the common centre of that spot and of all the rings of colours fell in the
middle way between the beam of the incident light and the beam of the re-
flected light and by consequence in the centre of the spherical concavity of the
speculum whenever the chart on which the rings of colours fell was placed at
that centre And as the beam of reflected light by inclining the speculum re-
ceded more and more from the beam of incident light and from the common

consequence to their angles of refraction at their entrance into the glass but

the bright light of the 34 38...
1 172 and that of the
1 659 2 031 and 2 345
of these rings shall be
paint rings about the white central round spot of light which we said was the ... from the speculum to the chart and there ... of refraction the light
light of the 34 386th ring And the semidiameters of these rings
the angles of refraction made at the ...
consequence their diameters ...
specu...
1 659 ...
of the
the th
light ...
eters are
Now the
are the ...
measuring them viz with $1\frac{11}{16}$ $2\frac{3}{8}$ $2\frac{11}{12}$ and $3\frac{3}{8}$ inches ... found in the third of these Observations ... found by computation
the theory of deriving these rings from the thickness of
which the speculum was made ... and therefore
agrees ...
of the ...
made b...
of all co...
other uniform ...
eters of the
the fits of t...
reflecting surface which caused those fits ... colours when equally inclined to the refracting or ... proportion of the intervals of
the rings made by the rays in the extremities and limits of the seven colours (red orange yellow green blue indigo violet) ... the diameters of
of the numbers 1 $\frac{8}{9}$ $\frac{5}{6}$ $\frac{3}{4}$ $\frac{2}{3}$ ... proportional ...
... those
... rings thus de...
diameters at equal distances fr...
convex plate of ...
in ... subdu...
this propor...
that the ... rings (like ... by experience ... plates of glass ...
the glass I procur...
on both sides to th...
... with the former plate ... plate of glass ground
parts of an inch and the diameters of the three first bright rings ... Its thickness was $\frac{5}{62}$
between the brightest parts of their orbit ...
glass were 3 $4\frac{1}{...}$ ...

light was still more increased these also vanished for the light which coming h hol n the window fell upon the speculum in several ad nd I lotted r if I rings ild be

bigger

OBS 12 When the colours of the prism were cast successively on the speculum, that ring which in the two last Observations was white was of the same bigness in all the colours but the rings without it were greater in the green than in the blue and still greater in the yellow and greatest in the red And on the contrary the rings within that white circle were less in the green than in the blue and still less in the yellow and least in the red For the angles of reflexion of those rays which made this ring being equal to their angles of incidence the fits of every reflected ray within the glass after reflexion are equal in length and number to the fits of the same ray within the glass before its incidence on the reflecting surface And, therefore since all the rays of all sorts at their entrance *into the glass were in a fit of transmission they were also in a fit of transmission* at their returning to the same surface after reflexion and by consequence were t to the white ring on th chart This is the reason

f

their colour in their progress from this white ring
increase or decrease by the greatest steps so that the rings of this colour without are greatest and within least And thus is the reason why in the last yterio tenor

These are the phenomena of thick convexo-concave plates which are everywhere of the same thickness There are yet other phenomena when these plates are a little thicker on one side than on the other and others when the plates are more or less concave than convex or plano-convex, or double-convex. For in all these cases the plates make rings of colours but after various manners all which so far as I have yet observed follow from the Propositions in the end of the third part of this book, and so conspire to confirm the truth of those Propositions But the phenomena are too various and the calculations whereby they follow from those Propositions too intricate to be here prosecuted. I content myself with having prosecuted this kind of phenomena so far as to discover their cause and by discovering it to ratify the Propositions in the third Part of this book.

OBS. 13 As light reflected by a lens quick-silvered on the backside makes the rings of colours above described, so it ought to make the like rings of colours in passing through a drop of water At the first reflexion of the rays within the drop some colours ought to be transmitted as in the case of a lens, and others to be reflected back to the eye For instance if the diameter of a small drop or r ed m king ray

yet their angles of reflexion were not in the same planes with their angles of incidence

OBS 11 The colours of the new rings were in a contrary order to those of the former and arose after this manner the white round spot of light in the middle of the rings continued white to the centre till the distance of the incident and reflected beams at the chart was about $/_8$ parts of an inch and then it began to grow *dark in the middle* And when that distance was about $1\frac{3}{16}$ of an inch the white spot was become a ring encompassing a dark round spot which in the middle inclined to violet and indigo And the luminous rings encompassing it were grown equal to those dark ones which in the four first Observations encompassed them that is to say the white spot was grown a white ring equal to the first of those dark rings and the first of those luminous rings was now grown equal to the second of those dark ones and the second of those luminous ones to the third of those dark ones and so on For the diameters of the luminous rings were now $1\frac{3}{16}$ $2\frac{1}{16}$ $2/_3$ $3\frac{3}{20}$ &c inches

When the distance between the incident and reflected beams of light became a little bigger there emerged out of the middle of the dark spot after the indigo a blue and then out of that blue a pale green and soon after a yellow and red And when the colour at the centre was brightest (being between yellow and red) the bright rings were grown equal to those rings which in the four first Observations next encompassed them that is to say the white spot in the middle of those rings was now become a white ring equal to the first of those bright rings and the first of those bright ones was now become equal to the second of those and so on For the diameters of the white ring and of the other luminous rings encompassing it were now $1\frac{11}{16}$ $2\frac{3}{8}$ $2\frac{11}{12}$ $3\frac{3}{8}$ &c or thereabouts

When the distance of the two beams of light at the chart was a little more increased *there emerged out of the middle in order after the red a purple a* blue a green a yellow and a red inclining much to purple and when the colour was brightest (being between yellow and red) the former indigo blue green yellow and red were become an iris or ring of colours equal to the first of those luminous rings which appeared in the four first Observations and the white ring which was now become the second of the luminous rings was grown equal to the second of those and the first of those which was now become the third [illegible] nd so on For their diameters [illegible] f the two beams of light and [illegible]

When these two beams became more distant there emerged out of the middle of the purplish red first a darker round spot *and then out of the middle of that* spot a brighter And now the former colours (purple blue green yellow and [illegible]

and the diameter of the white ring [illegible] about 3 inches

The colours of the rings in the middle began now to grow very dilute and if the distance between the two beams was increased half an inch or an inch more they vanished whilst the white ring with one or two of the rings next it on either side continued still visible But if the distance of the two beams of

BOOK THREE

Part I

nth flexions of the rays of light and

into a dark
light will be
larger than they *ought to be if the rays went on* straight lines
rarks of coloured

fraction of the air but without due examination of the matter
stances of the phenomena so far as I have observed them are as follows

Obs 1 I made in a piece of lead a small hole with a pin whose breadth was the 4 d part of an inch for 21 of those pins laid together took up the breadth of half an inch Through this hole I let into my darkened chamber a beam of the
such
oader
right

and at the distance of ten feet was the eighth part of an inch broad (that is 35 times broader)

Nor is it material whether the hair be encompassed with air or with any other pellucid substance For I wetted a polished plate of glass and laid the

hair G H I and Q R, S the places where the rays fall on a paper GQ IS the

tance from this middle ray round about it have 249 fits within the globul
and all the like rays at a certain farther distance
fits and all those at a cert ... f

... colours will make rings of other colours ... ough to be sensible ... And in like manner ... which passes
in a fair day the Sun shines through ... th ...
hail and that the ... Suppose ...

... And accordingly ... that of the second 10¼°
ger or less the rings shall be less or bigger as the globules of water are big
answers it For in June 1692 I saw by reflexion in a vessel of stagnating
water three halos crowns or rings of colours about the Sun like three little
rainbows concentric to his body The colours of the first or innermost crown
were blue next the Sun red without and white in the middle between the
blue and red Those of the second crown were purple and blue within and
pale red without and green in the middle And those ...
blue within and pale red ...
diately so that their colou...
outward blue white red ...
pale red The diameter of the second crown ... low and red pale blue
yellow and red on one side of the Sun to the middle of the same colour on the
other side was 9⅓ degrees or thereabouts The diameters of the first and third
I had not time to measure but that of the first seemed to be about five or
degrees and that of the third ...
abo...
I s...
abo... degree ... diamet...
about the Moon ...
which was of a bl...
witho...
outwa...
halo al...
... distant from the centre of the Moon ... time there appeared a
and its long diameter was perpendicular to the horizon verging below farthest
from the Moon It was elliptical
from the Moon I am told that the Moon has sometimes three or more concen
tric crowns of colours encompassing one another next about her body The
more equal the globules of water or ice are to one another the more crowns of
colours will appear and the colours will be the more lively The halo at the
distance of 22½ degrees from the Moon is of another sort By its being oval
and remoter from the Moon below than above I conclude that it was made
by refraction in some sort of hail or snow floating in the air in an horizontal
posture the refracting angle being about 58 or 60 degrees

— — planes of a obtuse IFBM

pass directly through the parallel planes of the glass and fall upon paper

Fig. 2

between I and M and all the light between the rays GO and HD be refracted by the oblique plane of the diamond-cut BD and fall upon the paper between K and L and the light which passes directly through the parallel planes of the glass and falls upon the paper between I and M will be bordered with three or more fringes at M

So by looking on the Sun through a feather or black ribband held close to the eye several rainbows will appear the shadows which the fibres or threads cast on the tunica retina being bordered with the like fringes of colours

Obs. 3 When the hair was twelve feet distant from this hole and its shadow fell obliquely upon a flat white scale of inches and parts of an inch placed half a foot beyond it and also when the shadow fell perpendicularly upon the same scale placed nine feet beyond it I measured the breadth of the shadow and fringes as accurately as I could and found them in parts of an inch as follows

At the distance of	Half a foot	Nine feet
The breadth of the shadow	$\frac{1}{54}$	$\frac{1}{9}$
The breadth between the middles of the brightest light of the innermost fringes on either side the shadow	$\frac{1}{38}$ or $\frac{1}{39}$	$\frac{7}{50}$
The breadth between the middles of the brightest light of the middlemost fringes on either side the shadow	$\frac{1}{23\frac{1}{2}}$	$\frac{4}{17}$
The breadth between the middles of the brightest light of the outmost fringes on either side the shadow	$\frac{1}{18}$ or $\frac{1}{18\frac{1}{2}}$	$\frac{3}{10}$
The distance between the middles of the brightest light of the first and second fringes	$\frac{1}{120}$	$\frac{1}{21}$
The distance between the middles of the brightest light of the second and third fringes	$\frac{1}{170}$	$\frac{1}{31}$
The breadth of the luminous part (green white yellow and red) of the first fringe	$\frac{1}{170}$	$\frac{1}{32}$
The breadth of the darker space between the first and second fringes	$\frac{1}{240}$	$\frac{1}{45}$
The breadth of the luminous part of the second fringe	$\frac{1}{290}$	$\frac{1}{55}$
The breadth of the darker space between the second and third fringes	$\frac{1}{340}$	$\frac{1}{63}$

breadth of the shadow of the hair cast on the paper and TI VS two rays passing to the points I and S without bending when the hair is taken away And it s manifest that all the light between these two rays TI and VS is bent in passing by the hair and turned aside from the shadow IS because if any part of this light were not bent it would fall on the paper within the shadow and there

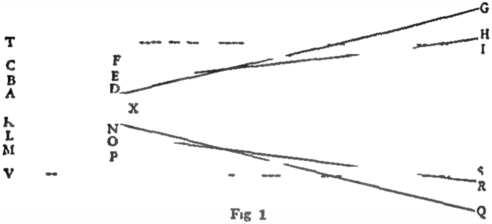

Fig 1

illuminate the paper contrary to experience And because when the paper is at a great distance from the hair the shadow is broad and therefore the rays TI and VS are at a great distance from one another it follows that the hair acts upon the rays of light at a good distance in their passing by it But the action is strongest on the rays which pass by at least distances and grows weaker and weaker accordingly as the rays pass by at distances greater and greater as is represented in the scheme For thence it comes to pass that the shadow of the hair is much broader in proportion to the distance of the paper from the hair when the paper is nearer the hair than when it is at a great distance from it

OBS 2 The shadows of all bodies (metals stones glass wood horn ice &c) in this light were bordered with three parallel fringes or bands of coloured light whereof that which was contiguous to the shadow was broadest and most luminous and that which was remotest from it was narrowest and so faint as not easily to be visible It was difficult to distinguish the colours unless when the light fell very obliquely upon a smooth paper or some other smooth white body so as to make them appear much broader than they would otherwise do And then the colours were plainly visible in this order the first or innermost fringe was violet and deep blue next the shadow and then light blue green and yellow in the middle and red without The second fringe was almost contiguous to the first and the third to the second and both were blue within and yellow and red without but their colours were very faint especially those of the third The colours therefore proceeded in this order from the shadow violet indigo pale blue green yellow red blue yellow red pale blue pale yellow and red The shadows made by scratches and bubbles in polished plates of glass were bordered with the like fringes of coloured light And if plates of looking glass sloped off near the edges with a diamond-cut be held in the same beam of light the light which passes through the parallel planes of the glass will be bordered with the like fringes of colours where those planes meet with the diamond-cut and by this means there will sometimes appear four or

... decreased gradually till it became insensible The whole length ...

I

... cause than the ...

was behind the knife and
pon its edge and that not
also when it was without
... h ndle This line
arrower than
... form
e
t

OBS 6 I placed another knife by this so that the ... I
... beam of light might fall upon both
n the distance
parted in the
middle and left a shadow between was so black
and dark that all the light which passed between the knives seemed to be bent
and turned aside to the one hand or to the other And as the knives still approached one another the shadow grew broader and the streams shorter at
th ... the contact of the
kn

o the inward
ends of the streams passes by the edges of the knives at the greatest distance
and this distance when the shadow begins to appear between the streams is
about the 800th part of an inch And the light which passes by the edges of the
knives ... more bent and goes to those

OBS 7 In the fifth Observation the fringes did not appear but by reason of
the breadth of the hole in the window became so broad as to run into one
...

made by the edge of one knife and three on the other side made by the edge of
the other knife They were distinctest when the knives were placed at the
greatest distance from the hole in the window and still became more distinct
by making the hole less insomuch that I could sometimes see a faint lineament
of a fourth ...

which was in the middle between them was grown very broad enlarging itself

These measures I took by letting the shadow of the hair at half a foot distance fall so obliquely on the scale as to appear twelve times broader than when it fell perpendicularly on it at the same distance and setting down in this Table the twelfth part of the measures I then took

OBS 4 When the shadow and fringes were cast obliquely upon a smooth white body and that body was removed farther and farther from the hair the first fringe began to appear and look brighter than the rest of the light at the distance of less than a quarter of an inch from the hair and the dark line or shadow between that and the second fringe began to appear at a less distance from the hair than that of the third part of an inch The second fringe began to appear at a distance from the hair of less than half an inch and the shadow between that and the third fringe at a distance less than an inch and the third fringe at a distance less than three inches At greater distances they became much more sensible but kept very nearly the same proportion of their breadths and intervals which they had at their first appearing For the distance between the middle of the first and middle of the second fringe was to the distance between the middle of the second and middle of the third fringe as three to two or ten to seven And the last of these two distances was equal to the breadth of the bright light or luminous part of the first fringe And this breadth was to the breadth of the bright light of the second fringe as seven to four and to the dark interval of the first and second fringe as three to two and to the like dark interval between the second and third as two to one For the breadths of the fringes seemed to be in the progression of the numbers 1 $\sqrt{\frac{1}{3}}$ $\sqrt{\frac{1}{5}}$ and their intervals to be in the same progression with them that is the fringes and their intervals together to be in the continual progression of the numbers 1 $\sqrt{\frac{1}{2}}$ $\sqrt{\frac{1}{3}}$ $\sqrt{\frac{1}{4}}$ $\sqrt{\frac{1}{5}}$ or thereabouts And these proportions held the same very nearly at all distances from the hair the dark intervals of the fringes being as broad in proportion to the breadth of the fringes at their first appearance as afterwards at great distances from the hair though not so dark and distinct

OBS 5 The Sun shining into my darkened chamber through a hole a quarter of an inch broad I placed at the distance of two or three feet from the hole a sheet of pasteboard which was blacked all over on both sides and in the middle of it had a hole about three-quarters of an inch square for the light to pass through And behind the hole I fastened to the pasteboard with pitch the blade of a sharp knife to intercept some part of the light which passed through the hole The planes of the pasteboard and blade of the knife were parallel to one another and perpendicular to the rays And when they were so placed that none of the Sun's light fell on the pasteboard but all of it passed through the hole to the knife and there part of it fell upon the blade of the knife and part of it passed by its edge I let this part of the light which passed by fall on a [illegible] of

he

he

n

a

one another and pretty nearly equal in length and [illegible] light Their light at that end next the Sun's direct light was pretty strong for the space of about a quarter of an inch or half an inch and in all its progress from

the middle of the light which passes between the knives where they are distant the 160th part of an inch, and the one half of that light passes by the edge of one knife at a distance no greater than the 320th part of an inch, and falling upon the paper makes the fringes of the shadow of that knife, and the other half passes by the edge of the other knife, at a distance not greater than the 320th part of an inch, and falling upon the paper makes the fringes of the shadow of the other knife. But if the paper be held at a distance from the knives greater than the third part of an inch, the dark lines above mentioned meet at a ... the fifth part of an inch from the end of the light which ... the

knives where the edges are distant above ...

For at another time, when the two knives were distant eight feet and five inches from the little hole in the window, made with a small pin as above, the light which fell upon the paper where the aforesaid dark lines met, passed between the knives, where the distance between their edges was as in the following Table, when the distance of the paper from the knives was also as follows.

Distances of the paper from the knives in inches	*Distances between the edges of the knives in millesimal parts of an inch*
1½	0·012
3⅓	0·020
8⅗	0·034
32	0·057
96	0·081
131	0·087

And hence I gather, that the light which makes the fringes upon the paper is not the same light at all distances of the paper from the knives, but when the paper is held near the knives, the fringes are made by light which passes by the edges of the knives at a less distance, and is more bent than when the paper is held at a greater distance from the knives.

OBS. 10. When the fringes of the shadows of the knives fell perpendicularly upon a paper at a great distance from the knives, they were in the form of hyperbolas, and their dimensions were as follows. Let CA, CB (Fig. 3) represent

...

from the point where the edges of the knives meet; *eis*, *fkt*, and *glv*, three hyperbolical lines representing the terminus of the shadow of one of the knives, the dark line between the first and second fringes of that shadow, and the dark line between the second and third fringes of the same shadow; *xip*, *ykq*, and *zlr*, three other hyperbolical lines representing the terminus of the shadow of the other knife, the dark line between the first and second fringes of that shadow, and the dark line between the second and third fringes of the same shadow

on both sides into the streams of light described in the fifth Observation the above-mentioned shadow began to appear in the middle of this line and divide it along the middle into two lines of light and increased until the whole light vanished This enlargement of the fringes was so great that the rays which go to the innermost fringe seemed to be bent above twenty times more when this fringe was ready to vanish than when one of the knives was taken away

And from this and the former Observation compared I gather that the light of the first fringe passed by the edge of the knife at a distance greater than the 800th part of an inch and the light of the second fringe passed by the edge of the knife at a greater distance than the light of the first fringe did and that of the third at a greater distance than that of the second and that of the streams of light described in the fifth and sixth Observations passed by the edges of the knives at less distances than that of any of the fringes

OBS 8 I caused the edges of two knives to be ground truly straight and pricking their points into a board so that their edges might look towards one another and meeting near their points contain a rectilinear angle I fastened their handles together with pitch to make this angle invariable The distance of the edges of the knives from one another at the distance of four inches from the angular point where the edges of the knives met was the eighth part of an inch and therefore the angle contained by the edges was about 1 degree 54 The knives thus fixed together I placed in a beam of the Sun's light let into my darkened chamber through a hole the 42d part of an inch wide at the distance of 10 or 15 feet from the hole and let the light which passed between their edges fall very obliquely upon a smooth white ruler at the distance of half an inch or an inch from the knives and there saw the fringes by the two edges of the knives run along the edges of the shadows of the knives in lines parallel to those edges without growing sensibly broader till they met in angles equal to the angle contained by the edges of the knives and where they met and joined they ended without crossing one another But if the ruler was held at a much greater distance from the knives the fringes where they were farther from the place of their meeting were a little narrower and became something broader and broader as they approached nearer and nearer to one another and after they met they crossed one another and then became much broader than before

Whence I gather that the distances at which the fringes pass by the knives are not increased nor altered by the approach of the knives but the angles in which the rays are there bent are much increased by that approach and that the knife which is nearest any ray determines which way the ray shall be bent and the other knife increases the bent

OBS 9 When the rays fell very obliquely upon the ruler at the distance of the third part of an inch from the knives the dark line between the first and second fringe of the shadow of one knife and the dark line between the first and second fringe of the shadow of the other knife met with one another at the distance of the fifth part of an inch from the end of the light which passed

... edges of the knives measured from the point of their concourse ...

In the full red Light they were totally red without any sensible blue or violet, and in the deep blue Light they were totally blue without any sensible red or
... Light they were totally green excepting a little
... compar
made in
... se made

... the red Light were largest, those made ... [illegible] ... in the green were of a middle bigness. For the Fringes with which the shadow of a man's hair were bordered, being measured across the shadow at the distance of six inches from the hair, the distance between the middle and most luminous part of the first or innermost Fringe on one side of the shadow and that of the like Fringe on the other side of the shadow was in the full red Light ... of an
... between the middle and
... the shadow was in the
... and these distances of the

Fringes had the same proportion at all distances from the hair without any sensible variation.

So then the rays which made these Fringes in the red Light passed by the hair at a greater distance than those did which made the like Fringes in the violet; and therefore the hair in causing these Fringes acted alike upon the red Light or least refrangible rays at a greater distance, and upon the violet or most refrangible rays at a less distance, and by those actions disposed the red Light into larger Fringes, and the violet into smaller, and the Lights of intermediate colours into Fringes of intermediate bignesses without changing the colour of any sort of Light.

When, therefore, the hair in the first and second of these Observations was held in the white beam of the Sun's Light, and cast a shadow which was bordered with three Fringes of coloured Light, those colours arose not from any new modifications impressed upon the rays of Light by the hair, but only from the various inflexions whereby the several sorts of rays were separated from one another, which before separation, by the mixture of all their colours, composed the white beam of the Sun's Light, but whenever separated compose Lights of the several colours which they are originally disposed to exhibit. In this 11th Observation, where the colours are separated before the Light passes by the hair, the least refrangible rays, which when separated from the rest make red, were inflected at a greater distance from the hair, so as to make three red Fringes at a greater distance from the middle of the shadow of the hair; and the most refrangible rays which when separated make violet, were inflected at a less distance from the hair, so as to make three violet Fringes at a less distance from the middle of the shadow of the hair. And other rays of intermediate degrees of refrangibility were inflected at intermediate distances from the hair, so as to make Fringes of intermediate colours at intermediate distances from the middle of the shadow of the hair. And in the second Observation, where all the colours are mixed in the white Light which passes by the hair, these colours are separated by the various inflexions of the rays, and the Fringes which they make appear all together, and the innermost Fringes being contiguous make one broad Fringe composed of all the colours in due order, the violet lying on the inside of the Fringe next the shadow, the red on the outside farthest from the shadow, and the blue, green, and yellow in the middle. And, in like manner, the middlemost Fringes of all the colours lying in order, and

And conceive that these three hy . . .
and cross them in the points z
terminated and distinguished luminous fringes by the lines ex and xzp until the meeting and crossing of the fringes and then tho e lines cross the fringes in the form of dark lines terminating the first luminous fringes

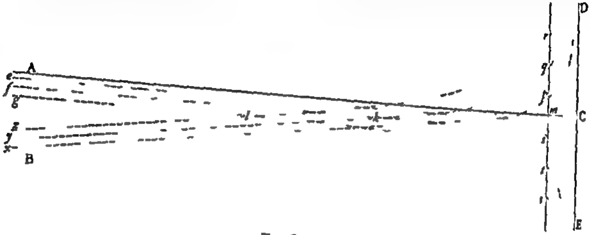

Fig 3

DE of these hyperbolas one asymptote is the line DE and their other asymptotes are parallel to the lines CA and CB Let rv represent a line drawn anywhere upon the paper parallel to the asymptote DE and let this line cross the right lines AC in m and BC . . . and the six dark . . .
es ps qt rv and
t mi and doing
ou may find as
now that these
. . . yperbolas differing little from the conical hyperbola And by measuring the lines Ci Ck Cl you may find other points of the e curves

For instance when the knives were distant from the hole in the window ten feet and the paper from the knives nine feet and the angle cont . . .
edges of
which w

. . . part of an inch or 0 00?5 inch) the sums np nq nr were 0 1828 0 3328 0 1978 inch I measured also the distances of the brightest parts of the fringes which run between pq and st qr and tv and next beyond r and v and found them 0 . 0 8 and 1 17 inches

OBS 11 The Sun shining into my darkened room through a small round hole made in a plate of lead with a slender pin as above I placed at the hole a prism to refract the light and form on the opposite wall the spectrum of colours described in the third experiment of the first book And then I found that the shadows of all bodies held in the coloured light between the prism and the wall were bordered with fringes of the colour of that light in which they were held

rapid rotation of the wheels and some [illegible] particles come together with an impetus as oil of vitriol distilled from its weight of nitre and then mixed with twice its weight of oil of anniseeds. So also a globe of glass about 8 or 10 inches in diameter being put into a frame where it may be swiftly turned round its axis will in turning shine where it rubs against the palm of one's hand applied to it. And if at the same time a piece of white paper or white cloth or the end of one's finger be held at the distance of about a quarter of an inch or half an inch from that part of the glass where it is most in motion, the electric vapour which is excited by the friction of the glass against the hand will (by dashing against the white paper cloth or finger) be put into such an agitation as to emit light and make the white paper cloth or finger appear lucid like a glow worm and in rushing out of the glass will sometimes push against the finger so as to be felt. And the same things have been found by rubbing a long and large cylinder of glass or amber with a paper held in one's hand and continuing the friction till the glass grew warm.

Qu. 9. Is not fire a body heated so hot as to emit light copiously? For what else is a red hot iron than fire? And what else is a burning coal than red hot wood?

Qu. 10. Is not flame a vapour fume or exhalation heated red hot, that is so hot as to shine? For bodies do not flame without emitting a copious fume and this fume burns in the flame. The ignis fatuus is a vapour shining without heat, and is there not the same difference between this vapour and flame as between rotten wood shining without heat and burning coals of fire. In distilling hot spirits, if the head of the still be taken off the vapour which ascends out of the still will take fire at the flame of a candle and turn into flame and the flame will run along the vapour from the candle to the still. Some bodies heated by motion or fermentation, if the heat grow intense, fume copiously, and if the heat be great enough the fumes will shine and become flame. [illegible]

being contiguous make another broad fringe composed of all the colours and the outmost fringes of all the colours lying in order and being contiguou make a third broad fringe composed of all the colours These are the three fringes of coloured light with which the shadows of all bodies are bordered in the second Observation

[illegible] in the foregoing Observations I designed to repeat most of them [illegible] ones for determining the [illegible] ge by bodies for making the fringes of colours with [illegible] em But I was then inter [illegible] and cannot now think of taking the e things into further consideration [illegible] of my design I shall conclude with

[illegible]

after separation to m [illegible]

[illegible] inflected to make those fringes

[illegible] nd sides of bodies [illegible] ke that of an eel? [illegible] arise from three

Qu 4 Do not [illegible] d are reflected or refracted begin [illegible] not re- flected refracted and inflected by one an [illegible]

[illegible] that is to say [illegible] it and light [illegible] ating motion

[illegible]

[illegible] than those of [illegible] ut

until it be stifled an [illegible]

Qu 7 Is not the strength and vigor of the [illegible] *phureous bodies observed above one reason* why sulphureous bodies take fire more readily and burn more vehemently than other bodies do?

Qu 8 Do not all fixed bodies when heated beyond a certain degree emit light and shine and is not this emission performed by the vibrating motions of their parts? And do not all bodies which abound with terrestrial part and especially with sulphureous ones emit light as often as those parts are suffi [illegible] whether that agitation be made by heat or by friction or [illegible] on or any other cause? As for [illegible]

a cat or neck of a [illegible]

and fish while they putrefy vapours arising [illegible] called *ignes fatui* stacks of moist hay or corn growing hot by fermentation glow worms and the eyes of some animals by vital motions the vulgar phos

rapid motion of the wheel, and some fluids when their parts come together with an impetus, as oil of vitriol distilled from its weight of nitre and then mixed with twice its weight of oil of aniseeds. So also a globe of glass about 8 or 10 inches in diameter, being put into a frame where it may be swiftly turned round its axis, will in turning shine where it rubs against the palm of one's hand applied to it: And if at the same time a piece of white paper or white cloth, or the end of one's finger be held at the distance of about a quarter of an inch or half an inch from that part of the glass where it is most in motion, the electric vapour which is excited by the friction of the glass against the hand will (by dashing against the white paper, cloth or finger) be put into such an agitation as to emit light and make the white paper, cloth or

in one's hand, and continuing the friction till the glass grew warm.

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smoke the flame is of several colours, as that of sulphur blue, that of copper opened with sublimate green, that of tallow yellow, that of camphor white. Smoke passing through flame cannot but grow red hot, and red hot smoke can have no other appearance than that of flame. When gunpowder takes fire, it goes away into flaming smoke. For the charcoal and sulphur easily take fire, and set fire to the nitre, and the spirit of the nitre being thereby rarified into vapour, rushes out with explosion much after the manner that the vapour of water rushes out of an Æolipile; the sulphur also being volatile is converted into vapour, and augments the explosion. And the acid vapour of the sulphur (namely that which distils under a bell into oil of sulphur) entering violently into the fixed body of the nitre, sets loose the spirit of the nitre, and excites a great fermentation, whereby the heat is further aug

mented and the fixed body of the nitre is also rarified into fume and the explosion is thereby made more vehement and ...
mixed ...

... ction of the vapour of the gun powder upon the salt of tartar whereby that ...
gunpowder arises therefore from the
being quickly and vehemently heated
vapour which vapour by the violence ... action becoming so hot as to shine appears in the form of flame

Qu 11 Do not great bodies conserve their heat the longest their parts heating one another and may not ...
beyond a certain degree emit light s...
of its light and the reflexions and re... of its rays within its pores to grow still hotter till it comes to a certain period of heat such as is that of the Sun? And are not the Sun and fixed stars great earths vehemently hot whose heat is conserved by the greatness of the bodies and the mutual action and reaction between them and the light which they emit and whose parts are kept from fuming away not only by their fixity but ...
of ...

... from them? For if ... warm in any pellucid vessel emptied of air that water in the vacuum will bubble and boil as vehemently as it would in the open air in a vessel set upon the fire till it conceives a much gre...
the ...
boil
a m... upon a red hot iron *in vacuo* emits a fume and flame but the same mixture in the open air by reason of the incumbent atmosphere does not so much as emit any fume which can be perceived by sight In like manner the great weight of the atmosphere which lies upon the globe of the Sun may hinder bodies there from rising up and going away from the Sun in the form of vapours and fumes unless by means of a far greater heat than that which on the surface of our Earth would very easily turn them into vapours and fumes And the same great weight may condense those vapours and exhalations as soon as they shall at any time begin to ascend from the Sun and make them presently fall back again into him and by that action increase his heat much after the manner that in our Earth the air increases the heat of a culinary fire And the same weight may hinder the globe of the Sun from being diminished unless by the emission of light and a very small quantity of vapours and exhalations

Qu 12 Do not the rays of light in falling upon the bottom of the eye excite vibrations in the *tunica retina*? Which vibrations being propagated along the solid fibres of the optic nerves into the brain cause the sense of seeing? For because dense bodies conserve their heat a long time and the densest bodies conserve their heat the longest the vibrations of their parts are of a lasting nature and therefore may be propagated along solid fibres of uniform dense matter to a great distance for conveying into the brain the impressions made upon all the organs of sense For that motion which can continue long in one and the same part of a body can be propagated a long way from one part to

another supposing the body homogeneal so that the motion may not be [illegible] hindered by any unevenness of the body [illegible] make vibrations of several bignesses [illegible] sensations of several colours much [illegible] of the air according to their several

[illegible] the least refrangible the largest for making a [illegible] the several intermediate sorts of rays, vibrations of several intermediate bignesses to make sensations of the several intermediate colours

Qu 14 May not the harmony and discord of colours arise from the proportions of the vibrations propagated through the fibres of the optic nerves into the brain, as the harmony and discord of sounds arise from the proportions of the vibrations of the air? For some colours if they be viewed together are agreeable to one another as those of [illegible] and indigo and others disagree

Qu 15 Are not the species of objects seen with both eyes united where the optic nerves meet before they come into the brain the fibres on the right side of both nerves uniting there and after union going thence into the brain in the nerve which is on the right side of the head and the fibres on the left side of both nerves uniting in the same place and after union going into the brain in the nerve which is on the left side of the head and these two nerves meeting in the brain in such a manner that their fibres make but one entire species or picture half of which on the right side of the sensorium comes from the right side of both eyes through the right side of both optic nerves to the place where the nerves meet and from thence on the right side of the head into the brain and the other half on the left side of the sensorium comes in like manner from the left side of both eyes For the optic nerves of such animals as look the same way with both eyes (as of men dogs sheep oxen &c) meet before they come into the brain but the optic nerves of such animals as do not look the same way with both eyes (as of fishes and of the chameleon) do not meet if I am rightly informed.

a quavering motion they appear again Do not these colours arise from such motions excited in the bottom of the eye by the pressure and motion of the finger as at other times are excited there by light for causing vision? And do not the motions once excited continue about a second of time before they cease? And when a man by a stroke upon his eye sees a flash of light are not the like motions excited in the retina by the stroke? And when a coal of fire moved

Qu 17 If a stone be thrown into stagnating water the waves excited thereby continue some time to arise in the place where the stone fell into the water and are propagated from thence in concentric circles upon the surface of the

mented and the fixed body of the nitre is also rarified into fume and the explosion is thereby made more vehement and quick. Fo
mixed

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gunpowder arises therefore from the
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vapour which vapour by the violence o action becoming so hot as to shine appears in the form of flame

Qu 11 Do not great bodies conserve thei he

and whose parts are kept from fuming away not only by their fixity but also by the vast weight and density of the atmospheres incumbent upon them and very strongly compressing them and condensing the vapours and exhalations which arise from them? For if water be made warm in any pellucid vessel emptied of air that water in the vacuum will bubble and boil as vehemently as it would in the open air in a vessel set upon the fire till it conceives a much greater heat For the weight of the incumbent atmosphere keeps down the vapours and hinders the water from boiling until it grow much hotter than is requisite to make it boil *in vacuo* Also a mixture of tin and lead being put upon a red hot iron *in vacuo* emits a fume and flame but the same mixture in the open air by reason of the incumbent atmosphere does not so much as emit any fume which can be perceived by sight In like manner the great weight of the atmosphere which lies upon the globe of the Sun may hinder bodies there from rising up and going from the Sun in the form of
than that which on the
vapours and fumes At weight may

culinary fire And the same
diminished unless by the em
and exhalations

Qu 12 Do not the rays of light in falling upon the bottom of the eye excite vibrations in the *tunica retina?* Which vibrations being propagated along the solid fibres of the optic nerves into the brain cause the sense of seeing? For
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their part

matter to a
upon all the or that motion which can continue long in one and the same part of a body can be propagated a long way from one part to

another, supposing the body homogeneal, so that the motion may not be reflected, refracted, interrupted or disordered by any unevenness of the body.

Qu. 13. Do not several sorts of rays make vibrations of several bignesses, which according to their bignesses excite sensations of several colours, much after the manner that the vibrations of the air, according to their several bignesses excite sensations of several sounds? And particularly do not the most refrangible rays excite the shortest vibrations for making a sensation of deep violet, the least refrangible the largest for making a sensation of deep red, and the several intermediate sorts of rays, vibrations of several intermediate bignesses to make sensations of the several intermediate colours?

Qu. 14. May not the harmony and discord of colours arise from the proportions of the vibrations propagated through the fibres of the optick nerves into the brain, as the harmony and discord of sounds arise from the proportions of the vibrations of the air? For some colours, if they be viewed together, are agreeable to one another, as those of gold and indigo, and others disagree.

Qu. 15. Are not the species of objects seen with both eyes united where the optick nerves meet before they come into the brain, the fibres on the right side of both nerves uniting there, and after union going thence into the brain in the nerve which is on the right side of the head, and the fibres on the left side of both nerves uniting in the same place, and after union going into the brain in the nerve which is on the left side of the head, and these two nerves meeting in the brain in such a manner that their fibres make but one entire species or picture, half of which on the right side of the sensorium comes from the right side of both eyes through the right side of both optick nerves to the place where the nerves meet, and from thence on the right side of the head into the brain, and the other half on the left side of the sensorium comes in like manner from the left side of both eyes. For the optick nerves of such animals as look the same way with both eyes (as of men, dogs, sheep, oxen, etc.) meet before they come into the brain, but the optick nerves of such animals as do not look the same way with both eyes (as of fishes, and of the chameleon) do not meet, if I am rightly informed.

Qu. 16. When a man in the dark presses either corner of his eye with his finger, and turns his eye away from his finger, he will see a circle of colours like those in the feather of a peacock's tail. If the eye and the finger remain quiet these colours vanish in a second minute of time, but if the finger be moved with a quavering motion they appear again. Do not these colours arise from such motions excited in the bottom of the eye by the pressure and motion of the finger, as at other times are excited there by light for causing vision? And do not the motions once excited continue about a second of time before they cease? And when a man by a stroke upon his eye sees a flash of light, are not the like motions excited in the retina by the stroke? And when a coal of fire moved nimbly in the circumference of a circle, makes the whole circumference appear like a circle of fire; is it not because the motions excited in the bottom of the eye by the rays of light are of a lasting nature, and continue till the coal of fire in going round returns to its former place? And considering the lastingness of the motions excited in the bottom of the eye by light, are they not of a vibrating nature?

Qu. 17. If a stone be thrown into stagnating water, the waves excited thereby continue some time to arise in the place where the stone fell into the water, and are propagated from thence in concentric circles upon the surface of the

water to great distances And the vibrations or tremors excited in the air by percussion continue a little time to move from the place of percussion in concentric spheres to great distances And in like manner when a ray of light falls upon the surface of any pellucid body and is there refracted or reflected may not waves of vibrations or tremors be thereby excited in the refracting or reflecting medium at the point of incidence and continue to arise there and to be propagated from thence as long as they continue to arise and be propagated, when they are excited in the bottom of the eye by the pressure or motion of the finger or by the light which comes from the coal of fire in the experiments above mentioned? And are not these vibrations propagated from the point of incidence to great distances? And do they not overtake the rays of light and by overtaking them successively do they not put them into the fits of easy reflexion and easy transmission described above? For if the rays endeavour to recede from the densest part of the vibration they may be alternately accelerated and retarded by the vibrations overtaking them

Qu 18 If in two large tall cylindrical vessels of glass inverted two little thermometers be suspended so as not to touch the vessels and the air be drawn out of one of these vessels and these vessels thus prepared be carried out of a cold place into a warm one the thermometer *in vacuo* will grow warm as much and almost as soon as the thermometer which is not *in vacuo* And when the vessels are carried back into the cold place the thermometer *in vacuo* will grow cold almost as soon as the other thermometer Is not the heat of the warm room conveyed through the vacuum by the vibrations of a much subtiler medium than air which after the air was drawn out remained in the vacuum? And is not this medium the same with that medium by which light is refracted and reflected and by whose vibrations light communicates heat to bodies and is put into fits of easy reflexion and easy transmission? And do not the vibrations of this medium in hot bodies contribute to the intenseness and duration of their heat? And do not hot bodies communicate their heat to contiguous cold ones by the vibrations of this medium propagated from them into the cold ones? And is not this medium exceedingly more rare and subtile than the air and exceedingly more elastic and active? And doth it not readily pervade all bodi

Qu

of this

denser parts of the medium? And is not the density thereof greater in free and open spaces void of air and other grosser bodies than within the pores of water glass crystal gems and other compact bodies? For when light passes through

s

and weakness thereof

Qu 20 Doth not this aethereal medium in passing out of water glass crystal and other compact and dense bodies into empty spaces grow denser and denser by degrees and by that means refract the rays of light not in a point but by bending them gradually in curved lines? And doth not the gradual condensation of this medium extend to some distance from the bodies and thereby cause the inflexions of the rays of light which pass by the edges of dense bodies at some distance from the bodies?

Qu. 21. Is not this medium much rarer within the dense bodies of the Sun, stars, planets and comets, than in the empty celestial spaces between them? And in passing from them to great distances, doth it not grow denser and denser perpetually, and thereby cause the gravity of those great bodies towards one another, and of their parts towards the bodies; every body endeavouring to go from the denser parts of the medium towards the rarer? For if this medium be rarer within the Sun's body than at its surface, and rarer there than at the hundredth part of an inch from its body, and rarer there than at the fiftieth part of an inch from its body, and rarer there than at the orb of Saturn; I see no reason why the increase of density should stop anywhere, and not rather be continued through all distances from the Sun to Saturn, and beyond. And though this increase of density may at great distances be exceeding slow, yet if the elastic force of this medium be exceeding great, it may suffice to impel bodies from the denser parts of the medium towards the rarer, with all that power which we call gravity. And that the elastic force of this medium is exceeding great, may be gathered from the swiftness of its vibrations. Sounds move about 1140 English feet in a second minute of time, and in seven or eight minutes of time they move about one hundred English miles. Light moves from the Sun to us in about seven or eight minutes of time, which distance is about 70 000 000 English miles, supposing the horizontal parallax of the Sun to be about 12″. And the vibrations or pulses of this medium, that they may cause the alternate fits of easy transmission and easy reflexion, must be swifter than light, and by consequence above 700 000 times swifter than sounds. And therefore the elastic force of this medium, in proportion to its density, must be above 700 000 × 700 000 (that is, above 490 000 000 000) times greater than the elastic force of the air is in proportion to its density. For the velocities of the pulses of elastic mediums are in a subduplicate ratio of the elasticities and the rarities of the mediums taken together.

As attraction is stronger in small magnets than in great ones in proportion to their bulk, and gravity is greater in the surfaces of small planets than in those of great ones in proportion to their bulk, and small bodies are agitated much more by electric attraction than great ones; so the smallness of the rays of light may contribute very much to the power of the agent by which they are refracted. And so if any one should suppose that aether (like our air) may contain particles which endeavour to recede from one another (for I do not know what this aether is) and that its particles are exceedingly smaller than those of air, or even than those of light: the exceeding smallness of its particles may contribute to the greatness of the force by which those particles may [illegible]

by endeavouring to expand itself.

Qu. 22. May not planets and comets, and all gross bodies, perform their motions more freely, and with less resistance in this aethereal medium than in any fluid, which fills all space adequately without leaving any pores, and by consequence is much denser than quick-silver or gold? And may not its resistance be so small, as to be inconsiderable? For instance; if this aether (for so I will call it) should be supposed 700 000 times more elastic than our air, and above 700 000 times more rare; its resistance would be above 600 000 000 times

less than that of water And so small a resistance would scarce make any sensible alteration in the motions of the planets in ten thousand years If any one would ask how a medium can be so rare let him tell me how the air in the upper parts of the atmosphere can be above a hundred thousand thousand times rarer than gold Let him also tell me how an electric body can by friction emit an exhalation so rare and subtile and yet so potent as by its emission to cause no sensible diminution of the weight of the electric body, and to be expanded through a sphere whose diameter is above two feet and yet to be able to agitate and carry up leaf copper or leaf gold at the distance of above a foot from the electric body? And how the effluvia of a magnet can be so rare and subtile as to pass through a plate of glass without any resistance or diminution of their force and yet so potent as to turn a magnetic needle beyond the glass?

Qu 23 Is not vision performed chiefly by the vibrations of this medium excited in the bottom of the eye by
the solid pellucid and uniform capil
of sensation? And is not hearing per by the vibrations either of this or some other medium excited in the auditory nerves by the tremors of the air and propagated through the solid pellucid and uniform capillaments of those nerves into the place of sensation? And so of the other senses

Qu 24 Is not animal motion performed by the vibrations of this medium excited in the brain by the power of the will and propagated from thence through the solid pellucid and uniform
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the nerves are each of them soli
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I ufficiently uniform I suppose them to be pellucid when viewed singly tho the reflexions in their cylindrical surfaces may make the whole nerve (composed of many capillaments) appear opaque and white For opacity arises from reflecting surfaces such as may disturb and interrupt the motions of this medium

Qu 25 Are there not other original properties of the rays of light besides those already described? An instance of another
the refraction of Island crystal described firs
afterwards more exactly by Huygens in his bo
a pellucid fissile stone clear as water or crystal of the rock and without colour enduring a red heat without losing its transparency and in a very strong heat calcining without fusion Steeped a day or two in water it loses its natural polish Being rubbed on cloth it attracts pieces of straws and other light things like amber or glass and with *aqua fortis* it makes an ebullition It an oblique parallelepiped
The obtuse angles of the
degrees and 52 minutes the acute ones 78 degrees and 8 minutes Two of the solid angles opposite to one another as C and E are compassed each of them with three of these obtuse angles and each of the other six with one obtuse and two acute ones [Fig 4] It cleaves easily in planes parallel to any of its sides and not in any other planes It cleaves with a glossy polite surface not perfectly plane but with some little

scratched, and by reason of its softness it takes a polish very difficultly. It polishes better upon polished Looking-glass than upon metal, and perhaps better upon pitch, leather or parchment. Afterwards it must be rubbed with a little oil or white of an egg, to fill up its scratches; whereby it will become very transparent and polished. But for several experiments, it is not necessary to polish it. If a piece of this crystalline stone be laid upon a book, every letter of the book seen through it will appear double, by means of a double refraction. And if any beam of light falls either perpendicularly, or in any oblique angle upon any surface of this crystal, it becomes divided into two beams by means of the same double refraction. Which beams are of the same colour with the incident beam of light, and seem equal to one another in the quantity of their light, or very nearly equal. One of these refractions is performed by the usual rule of Opticks, the sine of incidence out of air into this crystal being to the sine of refraction, as five to three. The other refraction, which may be called the unusual refraction, is performed by the following rule.

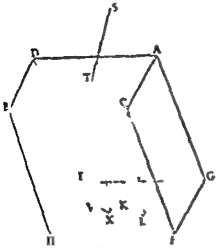

Fig. 4.

Let ADBC represent the refracting surface of the crystal, C the biggest solid angle at that surface, GEHF the opposite surface, and CK a perpendicular on that surface. This perpendicular makes with the edge of the crystal CF an angle of 19 degrees 3'. Join KF, and in it take KL, so that the angle KCL be 6 degrees 40', and the angle LCF 12 degrees 23'. And if ST represent any beam of light incident at T in any angle upon the refracting surface ADBC, let TV be the refracted beam determined by the given portion of the sines 5 to 3, according to the usual rule of Opticks. Draw VX parallel and equal to KL. Draw it the same way from V in which L lieth from K; and joining TX, this line TX shall be the other refracted beam carried from T to X, by the unusual refraction.

If therefore the incident beam ST be perpendicular to the refracting surface, the two beams TV and TX, into which it shall become divided, shall be parallel to the lines CK and CL; one of those beams going through the crystal perpendicularly, as it ought to do by the usual laws of Opticks, and the other TX by an unusual refraction diverging from the perpendicular, and making with it an angle VTX of about 6⅔ degrees, as is found by experience. And hence, the plane VTX, and such like planes which are parallel to the plane CFK, may be called the planes of perpendicular refraction. And the coast towards which the lines KL and VX are drawn, may be called the coast of un-

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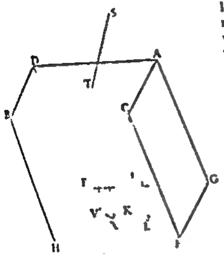

Fig 4

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[illegible]

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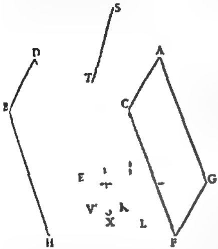

Fig. 4

or in any oblique angle upon any surface of this crystal it becomes divided into two beams by means of the same double refraction Which beams are of the same colour with the incident beam of light and seem equal to one another in the quantity of their light or very nearly equal One of these refractions is performed by the usual rule of Optics the sine of incidence out of air into this crystal being to the sine of refraction as five to three The other refraction which may be called the unusual refraction is performed by the following rule

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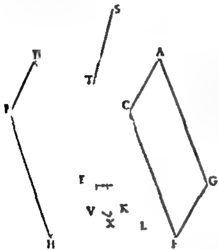

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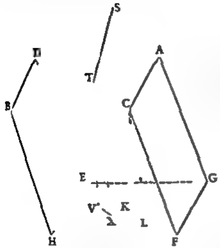

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In like manner crystal of the rock has a double refraction but the difference of the two refractions is not so great and manifest as in island crystal

When the beam ST incident on island crystal is divided into two beams TV and TX and these two beams arrive at the farther surface of the glass the beam TV which was refracted at the first surface after the usual manner shall be again refracted entirely after the usual manner at the second surface and the beam TX which was refracted after the unusual manner in the first surface shall be again refracted entirely after the unusual manner in the second surface so that both these beams shall emerge out of the second surface in lines parallel to the first incident beam ST

And if two pieces of island crystal be placed one after another in such manner that all the surfaces of the latter be parallel to all the corresponding surfaces of the former the rays which are refracted after the usual manner in the first surface of the first crystal shall be refracted after the usual manner in all the following surfaces and the rays which are refracted after the unusual manner in the first surface shall be refracted after the unusual manner in all the following surfaces And the same thing happens though the surfaces of the crystals be any ways inclined to one another provided that their planes of perpendicular refraction be parallel to one another

And therefore there is an original difference in the rays of light by means of which some rays are in this experiment constantly refracted after the usual manner and others constantly after the unusual manner for if the difference be not original but arises from new modifications impressed on the rays at their first refraction it would be altered by new modifications in the three following refractions whereas it suffers no alteration but is constant and has the same effect upon the rays in all the refractions The unusual refraction is therefore performed by an original property of the rays And it remains to be enquired whether the rays have not more original properties than are yet discovered

Qu 26 Have not the rays of light several sides endued with several original properties? For if the planes of perpendicular refraction of the second crystal be at right angles with the planes of perpendicular refraction of the first crystal the rays which are refracted after the usual manner in passing through the first crystal will be all of them refracted after the unusual manner in passing through the second crystal and the rays which are refracted after the unusual manner in passing through the first crystal will be all of them refracted after the usual manner in passing through the second crystal And therefore there

refraction For one and the same ray is here refracted [illegible] usual and sometimes after the unusual manner according to the position which its sides have to the crystals If the sides of the ray are posited the same way to both crystals it is refracted after the same manner in them both but if that side of the ray which looks towards the coast of the unusual refraction of the first crystal be 90 degrees from that side of the same ray which looks toward the coast of the unusual refraction of the second crystal (which may be effected by varying the position of the second crystal to the first and by consequence to the rays of light) the ray shall be refracted after several man

ners in the several crystals There is nothing more required to determine whether the rays of light which fall upon the second crystal shall be refracted after the usual or after the unusual manner but to turn about this crystal so that the coast of this crystal s unusual refraction may be on this or on that side of the ray And therefore every ray may be considered as having four sides or quarters two of which opposite to one another incline the ray to be refracted after the unusual manner as often as either of them are turned towards the coast of unusual refraction and the other two whenever either of them are turned towards the coast of unusual refraction do not incline it to l

tion of the rays in their passage through those surfaces and the rays were refracted by the same laws in all the four surfaces it appears that those dispositions were in the rays originally and suffered no alteration by the first refraction and that by means of those dispositions the rays were refracted at their incidence on the first surface of the first crystal some of them after the usual and some of them after the unusual manner accordingly as their sides of unusual refraction were then turned towards the coast of the unusual refraction of that crystal or sideways from it

Every ray of light has therefore two opposite sides originally endued with a property on which the unusual refraction depends and the other two opposite sides not endued with that property And it remains to be enquired whether there are not more properties of light by which the sides of the rays differ and are distinguished from one another

In explaining the difference of the sides of the rays above mentioned I have supposed that the rays fall perpendicularly on the first crystal But if they fall obliquely on it the success is the same Those rays which are refracted after the usual manner in the first crystal will be refracted after the unusual manner in the second crystal supposing the planes of perpendicular refraction to be at right angles with one another as above and on the contrary

If the planes of the perpendicular refraction of the two crystals be neither parallel nor perpendicular to one another but contain an acute angle the two beams of l g t wh h

tion, and some of them their other sides turned towards the coast of the un u l

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those phenomena depend not upon new modifications as has been supposed but upon the original and unchangeable properties of the rays

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If light consisted only in pression propagated without actual motion it

would not be able to agitate and heat the bodies which refract and reflect it If it consisted in motion propagated to all distances in an instant it would require an infinite force every moment

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wh it pressure of wa y tends every way with equal force and is propagated as readily and with as much force sideways as downwards and through crooked passages as through straight ones The waves on the surface of stagnating water passing by the sides of a broad obstacle which stops part of them bend afterwards and dilate themselves gradually into the quiet water behind the obstacle The waves pulses or vibrations of the air wherein sounds consist bend manifestly though not so m

crooked passages nor to bend shadow For the fixed stars by the interposition of any of the planets cease to be seen And so do the parts of the sun by the interposition of the Moon Mercury or Venus The rays which pass very near to the edges of any body are bent a little by the action of the body as we shewed above but this bending is not towards but from the shadow and is performed only in the passage of the ray by the body and at a very small distance from it So soon as the ray is past the body it goes right on

To explain the unusual refraction of island crystal by pression or motion propagated has not hitherto been attempted (to my knowledge) except by Huygens who for that end supposed two several vibrating mediums within that crystal But when he tried the refractions in two successive pieces of that crystal and found them such as is mentioned above he a loss fo expl

body

experi t have different properties in their different sides He suspected that the pulses of æther in passing through the first crystal might receive certain new modifications which might determine them to be propagated in this or that medium within the second crystal according to the position of that crystal But what modifications those might be he could not say nor think of anything satisfactory in that point And if he had known that the unusual refraction depends not on new modifications but on the original and unchangeable dispositions of the rays he would have found it as difficult to explain how those dispositio

beginning To me at least this seems inexplicable if light be nothing else than pression or motion propagated through æther

And it is as difficult to explain by these hypotheses ho nately in fits of easy suppose that there a

th vibrat.ons of one of them constitute light and the vibrations of the other are swifter and as often as they overtake the vibrations of the first put them r those fit. But how two æthers can be diffused through all pace one or without re-

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mot ons of the planets and comet in all manner of courses tu ugu heav ens For thence it is manifest that the heavens are void of all sensible resi tance and by consequence of all sensible matter

For the resistin power of fluid mediums arises partly from the attrition of the parts of th medium and partly from the *vis inertiæ* of the matter That part of the resistance of a pherical body which arises from the attrition of the parts of the medium is very nearly as the diameter or at the most as the *factum of the diameter* and the velocity of the pherical body together And that part of the resistance which arises from the *vis inertiæ* of the matter is as the square of that *factum* And by this difference the two sorts of resistance may be distinguished from one another in any medium and these being distinguished, it will be found that almost all the resistance of bodies of a competent magnitude moving in air water quick-silver and such like fluid with a competent velocity arises from the *vis inertiæ* of the parts of the fluid

Now that part of the resisting power of any medium which arises from the

h part of h med m ma be diminished

is proportional to the density of the matter and cannot be diminished by dividin the matter into smaller part nor by any other means than by decreasing the density of the medium And for these reasons the density of fluid mediums is very nearly proportional to their resistance Liquors which differ n t m ch in density as water spirit of wine spirit of turpentine hot oil differ

found by experiments made with pendulums. The open air in which we breathe is eight or nine hundred times lighter than water and by consequence eight or nine hundred times rarer and accordingly its resistance is less than that of water in the same proportion or thereabouts as I have also found by experiments made with pendulum And in thinner air the resistance is still less and at length by rarefying the air becomes insensible For small feathers falling in the open air meet with great resistance but in a tall glass well emptied of air they fall as fast as lead or gold as I have seen tried several times Whence the resistance seems till to decrease in proportion to the density of the fluid For I do not find by any experiment that bodies moving in quick-silver water or air meet with any other sensible resistance than what arises from the density and tenacity of those sensible fluids as they would do if the pores of those and all other paces were filled with a dense and subtile fluid Now if the resistance in a vessel well emptied of air was but a hundred times less than in the open air it would be about a million of times less than in quick-silver But it seems to be much less in such a vessel and still much less in the heavens

would not be able to agitate and heat the bodies which refract and reflect it If it consisted in motion propagated to all distances in an instant it would require an infinite force every moment in every shining particle to generate that motion And if it consisted in pression or motion propagated either in an instant or in time it would bend into the shadow For pression or motion cannot be propagated in a fluid in right lines beyond an obstacle which stops part of the motion but will bend and spread every way into the quiescent medium which lies beyond the obstacle Gravity tends downwards but the pressure of water arising from gravity tends every way with equal force and is propagated as readily and with as much force sideways as downwards and through crooked passages as through straight ones The waves on the surface of stagnating water passing by the sides of a broad obstacle which stops part of them bend afterwards and dilate themselves gradually into the quiet water behind

ough crooked pipes as thro
crooked passages nor to bend
rposition of any of the planets
cease to be seen And so do the parts of the sun by the interposition of the Moon Mercury or Venus The rays which pass very near to the edges of any body are bent a little by the action of the body as we shewed above but this bending is not towards but from the shadow and is performed only in the passage of the ray by the body and at a very small distance from it So soon as the ray is past the body it goes right on

To explain the unusual refraction of island crystal by pression or motion propagated has not hitherto been attempted (to my knowledge) except by Huygens who for that end supposed two several vibrating mediums within that crystal But when he tried the refractions in two successive pieces of that
lf at
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body through an uniform medium must be on all sides alike ... experiments it appears that the rays of light have different properties in their different sides He suspected that the pulses of æther in passing through the first crystal might receive certain new modifications which might determine them to be propagated in this or that medium within the second crystal according to the position of that crystal But what modifications those might be he could not say nor think of anything satisfactory in that point And if he had known that the unusual refraction depends not on new modifications but on the original and unchangeable dispositions of the rays he would have found it as difficult to explain how those dispositions which he supposed to be impressed on the rays by the first crystal could be in them before their incidence
shining bodies can have
at least this seems in
tion propagated through
æther

And it is as difficult to explain by these hypotheses how rays can be alternately in fits of easy reflexion and easy transmission unless perhaps one might suppose that there are in all space two æthereal vibrating mediums and that

ral and not only to unfold the mechanism of the world but chiefly to resolve these and such like questions What is th re in places almost empty of matter

To what end are comets and whence is it that planets move all one and the same way in orbs concentric while comets move all manner of ways in orbs very eccentric and what hinders the fixed stars from falling upon one another? How came the bodies of animals to be contrived with so much art and for what ends were their several parts? Was the eye contrived without skill in Optics, and the ear without knowledge of sounds? How do the motions of the body follow from the will and whence is the instinct in animals? Is not the sensory of animals that place to which the sensitive substance is present and into which the sensible species of things are carried through the nerves and brain, that there they may be perceived by their immediate presence to that substance And these things being rightly dispatched does it not appear from phenomena that there is a Being incorporeal living intelligent omnipresent who in infinite space (as it were in his sensory) sees the things themselves intimately and throughly perceives them and comprehends them wholly by their immediate presence to himself? Of which things the images only carried

ties unchanged in passing through several mediums which is another condition of the rays of light. Pellucid substances act upon the rays of light at a distance in refracting reflecting and inflecting them and the rays mutually agitate the parts of those substances at a distance for heating them and this action and reaction at a distance very much resembles an attractive force between bodies If refraction be performed by attraction of the rays the sines of incidence must be to the sines of refraction in a given proportion as we shewed in our principles of philosophy And this rule is true by experience The rays of light in going out of glass into a vacuum are bent towards the glass and if they fall too obliquely on the vacuum they are bent backwards into the glass and totally reflected and this reflexion cannot be ascribed to the resistance of an absolute vacuum but must be caused by the power of the glass attracting the rays at their going out of it into the vacuum and bringing them back. For if the farther surface of the glass be moistened with water or clear oil or liquid and clear honey the rays which would otherwise be reflected will go into the water oil or honey and, therefore are not reflected before they arrive at the farther surface of the glass and begin to go out of it If they go out of it into the water oil or honey they go on because the attraction of the glass is almost balanced and rendered ineffectual by the contrary attraction of the liquor But if they go out of it into a vacuum which has no attraction to balance that of the glass the attraction of the glass either bends and refracts them or brings them back and reflects them.

at the height of three or four hundred miles from the Earth or above For Mr Boyle has shewed that air may be rarified above ten thousand times in vessel of glass and the heavens are much emptier of air than any vacuum we can make below For since the air is compressed by the weight of the incumbent atmosphere and the density of air is proportional to the force compressing it it follows by computation that at the height of about seven and a half English miles from the Earth the air is four times rarer than at the surface of the Earth and at the height of 15 miles it is sixteen times rarer than that at the surface of the Earth and at the height of 22½ 30 or 38 miles it is respectively 64 256 or 1 024 times rarer or thereabouts and at the height of 76 152 228 miles it is about 1 000 000 1 000 000 000 000 or 1 000 000 000 000 000 000 times rarer and so on

Heat promotes fluidity very much by diminishing the tenacity of bodies It makes many bodies fluid which are not fluid in cold and increases the fluidity of tenacious liquids as of oil balsam and honey and thereby decreases their resistance But it decreases not the resistance of water considerably as it would do if any considerable part of the resistance of water arose from the attrition or tenacity of its parts And therefore the resistance of water arises principally and almost entirely from the *vis inertiæ* of its matter and by consequence if the heavens were as dense as water they would not have much less resistance than water if as dense as quick silver they would not have much less resistance than quick silver if absolutely dense or full of matter without any vacuum let the matter be never so subtile and fluid they would have a greater resistance than quick silver A solid globe in such a medium would lose above half its motion in moving three times the length of its diameter and a globe not solid (such as are the planets) would be retarded sooner And therefore to make way for the regular and lasting motions of the planets and comets it s necessary to empty the heavens of all matter except perhaps some very thin vapours steams or effluvia arising from the atmospheres of the Earth planets and comets and from such an exceedingly rare æthereal medium as we described above A dense fluid can be of no use for explaining the phenomena of Nature the motions of the planets and comets being better explained without it It serves only to disturb and retard the motions of those great bodies and make the frame of Nature languish and in the pores of bodies it serves only to stop the vibrating motions of their parts wherein their heat and activity ... of Nature and make ... therefore it ough ... light consists in pression or motion propagated through such a medium are rejected with it

And for rejecting such a medium we have the authority of those the oldest and most celebrated philosophers of Greece and Phœnicia who made a vac ... the first principles of their philoso- ... her cause than dense matter Later ... uch a cause out of natural philoso- ... things mechanically and referring other cause to metaphy ... main bu ... of natural philosophy ... to argue from phenomena without feigning hypotheses and to deduce causes from effect till we come to the very first cause which certainly is not mechan

Qu 30 Are not gross bodies and light convertible into one another and may

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...ke upon their part as we shewed above I know no body less apt to shine than water and yet water by frequent distillations change into fixed earth as Mr Boyle has tried and then this earth being enabled to endure a sufficient heat shines by heat like other bodies

The changing of bodies into light and light into bodies is very conformable to the course of Nature which seems delighted with transmutations Water which is a very fluid tasteless salt she changes by heat into vapour which is a sort of air and by cold into ice which is a hard pellucid brittle fusible stone and this stone returns into water by heat and vapour returns into water by cold E...

...

...times in the form of a hard brittle metal sometimes in the form of a corrosive pellucid salt called sublimate sometimes in the form of a tasteless pellucid volatile white earth called *Mercurius d...*

in that of a re...

it turns into v...

these changes it turns again into its first form of mercury Eggs grow from insensible magnitudes and change into animals tadpoles into frogs and worms into flies All birds beasts and fishes insects trees and other vegetables with their several parts grow out of water and watery tinctures and salts and by putrefaction return again into watery substances And water standing a few days in the open air yields a tincture which (like that of malt) by standing longer yields a sediment and a spirit but before putrefaction is fit nourishment for animals and vegetables And among such various and strange transmutations, why may not Nature change bodies into light and light into bodies?

Qu 31 Have not the small particles of bodies certain powers virtues or forces by which they act at a distance not only upon the rays of light for reflecting refracting and inflecting them but also upon one another for producing a great part of the phenomena of Nature? For it s well known that bodies act one upon another by the attractions of gravity magnetism and electricity and these instances shew the tenor and course of Nature and make it not improbable but that there may be more attractive powers than these For Nature is very consonant and conformable to herself How these attractions may be performed I do not here consider What I call attraction may be performed by impulse or by some other means unknown to me I use that word here to signify only in general any force by which bodies tend towards one another whatsoever be the cause For we must learn from ...

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small distances as hitherto...

tion may reach to such ma...

And this is still more evident by laying together two prisms of glass or two object glasses of very long telescopes the one plane the other a little convex and so compressing them that they do not fully touch nor are too far asunder For the light which falls upon the farther surface of the first glass where the interval between the glasses is not above the ten hundred thousandth part of an inch will go through that surface and through the air or vacuum between the glasses and enter into the second glass as was explained in the first fourth and eighth Observations of the first part of the second book But if the second glass be taken away the light which goes out of the second surface of the first glass into the air or vacuum will not go on forwards but turns [illegible]

[illegible] producing all the variety of colours and degrees of refrangibility than that the rays of light be bodies of different sizes the least of which may take violet the weakest and darkest of the colours and be more easily diverted by refracting surfaces from the right course and the rest as they are bigger and bigger may make the stronger and more lucid colours (blue green yellow and red) and be more and more difficultly diverted Nothing more is requisite for putting the rays of light into fits of easy reflexion and easy transmission than that they be small bodies which by their attractive powers or some other force stir up vibrations in what they act upon which vibrations being swifter than the rays overtake them successively and agitate them so as by turns to increase and decrease their velocities and thereby put them into those fits And lastly the unusual refraction of island crystal looks very much as if it were performed by some kind of attractive virtue lodged in certain sides both of the rays and of the particles of the crystal For were it not for some kind of disposition or virtue lodged in some sides of the particles of the crystal and not in their other sides and which inclines and bends the rays towards the coast of unusual refraction the rays which fall perpendicularly on the crystal would not be refracted towards that coast rather than towards any other coast both at their incidence and at their emergence so as to emerge perpendicularly by a contrary situation of the coast of unusual refraction at the second surface the crystal acting upon the rays after they have passed through it and are emerging into the air or if you please into a vacuum And since the crystal by this disposition or virtue does not act upon the rays unless when one of their sides of unusual refraction looks towards that coast this argues a virtue or disposition in those sides of the rays which answers to and sympathizes with that virtue or disposition of the crystal as the poles of two magnets answer to one another And as magnetism may be intended and remitted and is found only in the magnet and in iron so this virtue of refracting the perpendicular rays is greater in island crystal less in crystal of the rock and is not yet found in other bodies I do not say that this virtue is magnetical it seems to be of another kind I only say that whatever it be it's difficult to conceive how the rays of light unless they be bodies can have a permanent virtue in two of their sides which is not in their other sides and this without any regard to their position to the space or medium through which they pass

What I mean in this Question by a vacuum and by the attractions of the rays of light towards glass or crystal may be understood by what was said in the 18th 19th and 20th Questions.

fire and flame So when a drachm of the above-mentioned compound spirit of n̄ was poured upon half a drachm of oil of caraway seeds *in vacuo* the mixture immediately made a flash like gunpowder and burst the exhausted receiver which was a glass six inches wide and eight inches deep And even the

m one motions by a very potent principle which acts upon them only when they approach one another and causes them to meet and clash with great vi

into pieces and

solution of any metal, precipitates the metal and makes it fall down to the bottom of the liquor in the form of mud does not this argue that the acid particles are attracted more strongly by the salt of tartar than by the metal and by the stronger attraction go from the metal to the salt of tartar? And so when a solution of iron in *aqua fortis* dissolves the *lapis calaminaris* and lets go the iron or a solution of copper dissolves iron immersed in it and lets go the copper or a solution of silver dissolves copper and lets go the silver or a solution of mercury in *aqua fortis* being poured upon iron copper tin or lead dissolves the metal and lets go the mercury—does not this argue that the acid particles of the *aqua fortis* are attracted more strongly by the *lapis calaminaris* than by iron and more strongly b

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... of vitriol in the form of spirit of vitriol and this spirit (being poured upon iron, copper or salt of tartar) unites with the body and lets go the water

For when salt of tartar runs *per* ...
between the particles of the salt of ...
float in the air in the form of vapou ... does not common salt or salt petre or vitriol run *per deliquium* but for want of such an attraction? Or why does not salt of tartar draw more water out of the air than in a certain proportion to its quantity but for want of an attractive force after it is satiated with water? And whence is it but from this attractive power that water which alone distils with a gentle luke warm heat will not distil from salt of tartar without a great heat? And is it not from the like attractive power between the particles of oil of vitriol and the particles of water that oil of vitriol draws to it a good quantity of water out of the ai ...
distillation lets go the water ve ...
poured successively into the sar ... very not in the mixing does not this heat argue a great motion in the parts of the liquors? And does not this motion argue that the parts of the two liquors in mixing coalesce with violence and by consequence rush towards one another with an accelerated motion? And when *aqua fortis* or spirit of vitriol poured upon filings of iron dissolves the filings with a great heat and ebullition is not this heat and ebullition effected by a violent motion of the parts and does not that motion argue that the acid parts of the liquor rush towards the parts of the metal with violence and run forcibly into its pores till they get between its outmost particles and the main mass of the metal and surrounding those particles loosen them from the main mass and set them at liberty to float off into the water? And when the acid particles which alone would distil with an easy heat will not separate from the particles of the metal without a very violent heat does not this confirm the attraction between them?

When spirit of vitriol poured upon common salt or saltpetre makes an ebullition with the salt and unites with it and in distillation the spirit of the common salt or saltpetre comes over much easier than it would do before and the acid part of the spirit of vitriol stays behind does not this argue that the fixed alkali of the salt attracts the acid spirit of the vitriol more strongly than its own spirit and not being able to hold them both lets go its own? And when oil of vitriol is drawn off from its weight of nitre and from both the ing ...
compound spirit of nitre ...
one part of oil of cloves or ...
or animal substances or o ... with a little balsam of sulphur and the liquors grow so very hot in mixing as presently to send up a burning flame—does not this very ... at the two liquors mix with violence a ... ds one another with an accelerated m ... ? And is it not for the same reason tha ... spirit of wine poured on the same compound spirit flashes and that the *pulvis fulminans* composed of sulphur nitre and salt of tartar goes off with a more sudden and violent explosion than gunpowder the acid spirits of the sulphur and nitre rushing towards one another and towards the salt of tartar with so great a violence as by the shock to turn the whole at once into vapour and flame? Where the dissolution is slow it makes a slow ebullition and a gentle heat and where it is quicker it makes a greater ebullition with more heat and where it is done at once the ebullition is contracted into a sudden blast or violent explosion with a heat equal to that of

... compound spirit of
... in vacuo the mix
... the exhausted re-

hala ions hurricanes and spouts we may learn that sulphureous steams abound in the bowels of the Earth and ferment with mineral and sometimes take fire with a sudden coruscation and explosion and if pent up in subterraneous cav

canes and sometimes causes the land to slide or the sea to boil and carries up the water thereof in drops which by their weight fall down again in spouts Also some sulphureous steams at all times when the Earth is dry ascending into the air ferment there with nitrous acids and sometimes taking fire cause lightning and thunder and fiery meteors For the air abounds with acid vapours fit to promote fermentations as appears by the rusting of iron and copper in it the kindling of fire by blowing and the beating of the heart by means of respiration Now the above-mentioned motions are so great and violent as to shew that in fermentations the particles of bodies which almost rest are put into new motions by a very potent principle which acts upon them only when they approach one another and causes them to meet and clash with great violence and grow hot with the motion and dash one another into pieces and vanish into air and vapour and flame

When salt of tartar *per deliquium* being poured into the solution of any metal precipitates the metal and makes it fall down to the bottom of the liquor in th ...

in *aqua fort* ...
tion
of a

fort s being poured upon iron copper tin or lead dissolve the metal and lets go the mercury—does not this argue that the acid particles of the *aqua fortis* are attracted more strongly by the *lapis calaminaris* than by iron and more strongly by iron than by copper and more strongly by copper than by silver and more strongly by iron copper tin and lead than by mercury? And is it not for the same reason that iron requires more *aqua fortis* to dissolve it than copper and copper more than the other metals and that of all metals iron is dissolved most easily and is most apt to rust and next after iron copper?

When oil of vitriol is mixed with a little water or ... run *per deliquium* and in distillation the water ascends difficultly and brings over with it some part of the oil of vitriol in the form of spirit of vitriol and this spirit (being poured upon iron copper or salt of tartar) unites with the body and lets go the water

—doth not this shew that the acid spirit is attracted by the water and more attracted by the fixed body than by the water and therefore lets go the water to close with the fixed body? And is it not fo

lets go the water? And is it not ... from a mutual attraction that the spirits of soot and sea salt unite and compose the particles of sal ammoniac

particles of sulphur compose cinnabar and that the particles of spirit of wine and spirit of urine well rectified unite and letting go the water which dissolved them compose a consistent body and that in subliming cinnabar from

the

spirit of salt lets go the mercury and unites with the antimonial metal which attracts it mo

with it till the he

the

is subtile enough to penetrate gold as well as silver but wants the attractive force to give it entrance and that *aqua regia* is subtile enough to penetrate silver as well as gold but wants the attractive force to give it entrance? For *aqua regia* is nothing else than *aqua fortis* mixed with some spirit of salt or with sal ammoniac and even common salt dissolved in *aqua fortis* enables the menstruum to dissolve gold though the salt be a gross body When therefore spirit of salt precipitates silver out of *aqua fortis* is it not done by attracting and mixing with the *aqua fortis* and not attracting or perhaps repelling silver? And when water precipitates antimony out of the sublimate of antimony and sal ammoniac or out of butter of antimony is it not done by its dissolving mixing with and weakening the sal armoniac or spirit of

mony? And is it no

and oil of quick sil

not from the same principle that heat congregates homogeneal bodies and separates heterogeneal ones?

When arsenic with soap gives a regulus and with mercury sublimate a volatile fusible salt like butter of antimony doth not this shew that arsenic which is a substance totally volatile is compounded of fixed and volatile parts strongly cohering by a mutual attraction so that the volatile will not ascend without carrying up the fixed? And so when an equal weight of spirit of wine and oil of vitriol are digested together and in distillation yield two fragrant and volatile

spirits which will not mix with one another and a fixed black earth remains behind—doth not this shew that oil of vitriol is composed of volatile and fixed

The three first were found not much unequal to one another the fourth in so small a quantity as scarce to be worth considering The acid salt dissolved in water is the same with oil of sulphur *per campanam* and abounding much in

these minerals and that the bitumen carries up the other ingredients of the sulphur which without it would not sublime? And the same question may be put concerning all or almost all the gross bodies in Nature For all the parts of animals and vegetables are composed of substances volat[illegible] fixed

do not these things shew that salts are dry earth and watery acid united by attraction and that the earth will not become a salt without so much acid as makes it dissolvable in w[illegible]

As gravity makes the sea flo[illegible]
[illegible] of the Earth so the attracti[illegible]
denser and compacter particles of
otherwise the acid would not do t[illegible]

common water for making salts dissolvable in the water nor would salt of tar tar readily draw off the acid from dissolved metals nor metals the acid from mercury Now as in the great globe of the Earth and sea the densest bodies by their gravity sink down in water and always endeavour to go towards the centre of the globe so in particles of salt the densest matter may always en deavour to approach the centre of the particle so that a particle of salt may be compared to a chaos being dense hard dry, and earthy in the centre and rare soft moist and watery in the circumference And hence it seems to be that salts are of a lasting nature being scarce destroyed unless by drawing away their watery parts by violence or by letting them soak into the pores of the central earth by a gentle heat in putrefaction until the earth be dissolved by the water and separated into smaller particles which by reason of their small ness make the rotten compound appear of a black colour Hence also it may be that the parts of animals and vegetables preserve their several forms and as- similate their nourishment the soft and moist nourishment easily [illegible]

[illegible] us in confusion putrefaction and death

If a very small quantity of any salt or vitriol be dissolved in a great quantity of water the particles of the salt or vitriol will not sink to the bottom though they be heavier in species than the water but will evenly diffuse themselves into all the water so as to make it as saline at the top as at the bottom And does not this imply that the parts of the salt or vitriol recede from one another and endeavour to expand themselves and get as far asunder as the quantity of water in which they float will allow? And does not this endeavour imply that they have a repulsive force by which they fly from one another or at least that they attract the water more strongly than they do one another? For as all things ascend in water which are less attracted than water by the gravitating power of the Earth so all the particles of salt which float in water and are less attracted than water by any one particle of salt must recede from that particle and give way to the more attracted water

When any saline liquor is evaporated to a cuticle and let cool the salt con cretes in regular figures which argues that the particles of the salt before they concreted floated in the liquor at equal distances in rank and file and by con sequence that they acted upon one another by some power which at equal dis- tances is equal at unequal distances unequal For by such a power they will range themselves uniformly and without it they will float irregularly and come together as irregularly And since the particles of Island crystal act all the same way upon the rays of light for causing the unusual refraction may it not be sup- posed that in the formation of this crystal the particles not only ranged them selves in rank and file for concreting in regular figures but also by some kind of polar virtue turned their homogeneal sides the same way

The parts of all homogeneal hard bodies which fully touch one another stick together very strongly And for explaining how this may be some have in vented hooked atoms which is begging the question and others tell us that bodies are glued together by rest (that is by an occult quality or rather by nothing) and others that they stick together by conspiring motions (that is by relative rest amongst themselves) I had rather infer from their cohesion

that their particles attract one another by some force which in immediate con tact is exceeding strong at small distances performs the chemical operations above mentioned and reaches not far from the particles with any sensible ef fec

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ury and evaporating the phlegm spirit of wine and p v ...
phlegming and mixing them and spirit of urine and pirit of salt by subliming them together to make sal ammoniac Even the rays of light eem to be hard bod es fo other wise they would not retain different properties in their differ

its besides a large experience without an experimental exception o v t t m pound bodies are o very hard as we find some of them to be and yet are very porous and con i t of parts which are only laid together the imple particles hich are void of pores and were never yet divided mu t be much harder For such hard particles being heaped up together can carce touch one another in more than a few point and therefore must be separable by much less force than i requi ite to break a olid particle v hose parts touch in all the space be tween them without any pores or interstices to weaken their cohesion And

another is very difficult to conceive

The same thin I infer also from the cohering of two polished marbles in vacuo and from the standin of quick-silver in the barometer at the height of 50 60 or 0 inches or above whenever it is well purged of air and carefully poured in so that its parts be everywhere contiguous both to one another and to the glass The atmosphere by its eight presses the quick-silver into the glass to the he ght of 29 or 30 inches And some other agent raise it higher not by pres ng it into the glass but by making its parts stick to the glass and to one an ther For upon any l -continuation of parts made either by bubbles or by

common water for making salts dissolvable in the water nor would salt of tartar readily draw off the acid from dissolved metals nor metals the acid from mercury Now as in the great globe of the Earth and sea the densest bodies by their gravity sink down in water and always endeavour to go towards the centre of the globe so in particles of salt the densest matter may always endeavour to approach the centre of the particle so that a particle of salt may be compared to a chaos being den e hard dry and earthy in the centre and rare soft moist and watery in the circumference And hence it seems to be that salts are of a lasting nature being scarce destroyed unless by drawing away their watery parts by violence or by letting them soak into the pores of the central earth by a gentle heat in putrefaction until the earth be dissolved by the water and separated into smaller particles which by reason of their smallness make the rotten compound appear of a l l k colo r
that the parts of animals and ve
similate their nourishment the s
texture by a gentle heat and mo it becomes like the dense hard dry and durable earth n the c

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quantity of any salt or vitriol be di solved in a great quantity of water the particles of the salt or vitriol will not sink to the bottom though they be heavier in species than the water but will evenly diffu e themselves into all the water so as to make it as saline at the top as at the bottom And does not th mpl h h

d ow And does not this endeavour imply that they have a repulsive force by which they fly from one another or at least that they attract the water more strongly than they do one another? For as all things ascend in water which are less attracted than water by the gravitating power of the Earth so all the particles of salt which float in water and are less attracted than water by any one particle of salt must recede from that particle and give way to the more

When any
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concreted flo ed in the liquor at equal distances in rank and file and by consequence that they acted upon one another by some power which at equal distances is equal at unequal distances unequal For by such a power they will range themselves uniformly and without it they will float irregularly and come together as irregularly And since the particles of island crystal act all the same way upon the rays of light for causing the unusual refraction may it not be supposed that in the formation of this crystal the particles not only ranged themselves in rank and file for concreting in regular figures but also by some kind of polar virtue turned their homogeneal sides the same way

The parts of all homogeneal hard bodies which fully touch one another stick together very strongly And for explaining how this may be some have invented hooked atoms which is begging the question and others tell us that bodies are glued together by rest (that is by an occult quality or rather by nothing) and others that they tick together by con piring motions (that is by relative rest amongst themselves) I had rather infer from their cohesion

the excesses of coloured plates of water between two glasses are set down the thickness of the plate where it appears very black is three-eighths of the ten hundred thousandth part of an inch And where the oil of oranges between the glasses is of this thickness the attraction collected by the foregoing rule seems to be so strong as within a circle of an inch in diameter to suffice to hold up a weight equal to that of a cylinder of water of an inch in diameter and two or three furlongs in length And where it is of a less thickness the attraction may be proportionally greater and continue to increase until the thickness do not exceed that of a single particle of the oil. There are therefore agents in nature able to make the particles of bodies stick together by very strong attractions. And it is the business of experimental philosophy to find them out

Now the smallest particles of matter may cohere by the strongest attractions and compose bigger particles of weaker virtue and many of these may cohere and compose bigger particles whose virtue is still weaker and so on for divers successions until the progression end in the biggest particles on which the operations in chemistry and the colours of natural bodies depend and which by cohering compose bodies of a sensible magnitude If the body is compact and bends or yields inward to pression—without any sliding of its parts it is hard and elastic returning to its figure with a force rising from the mutual attraction of its parts If the parts slide upon one another the body is malleable or soft If they slip easily and are of a fit size to be agitated by heat and the heat is big enough to keep them in agitation the body is fluid and if it be apt to stick to things it is humid and the drops of every fluid affect a round figure by the mutual attraction of their parts as the globe of the Earth and sea affects a round figure by the mutual attraction of its parts by gravity

Since metals dissolved in acids attract but a small quantity of the acid, their attractive force can reach but to a small distance from them And as in algebra where affirmative quantities vanish and cease there negative ones begin so in mechanics where attraction ceases there a repulsive virtue ought to succeed. And that there is such a virtue seems to follow from the reflexions and inflexions of the rays of light For the rays are repelled by bodies in both these cases without the immediate contact of the reflecting or inflecting body It seems also to follow from the emission of light the ray so soon as it is shaken off from a shining body by the vibrat

beyond the reach of attracti

ry For that force which i

cient to emit it. It seem

The partic es wh

so soon as they

from it and also

there so as some ... to take up above a million of times more space than they did before in the form of a dense body Which vast contraction and expansion seems unintelligible by feigning the particles of air to be springy and ramous or rolled up like hoops or by any other means than a repulsive power The particles of fluids which do not cohere too strongly and are of such a smallness

a very little d

between the planes ... ments succeed after the same manner *in vacuo* as in the open air (as hath been tried before the Royal Society) and therefore are not influenced by the weight or pressure of the atmosphere

And if a large pipe of glass be filled with sifted ashes well pres ... the glass and one end of the p ...

... above the stag ... sts up to this h ...

... downwards as upwards ... the action of the particles is very strong ... the ashes being not ... not so strong as th ... of 60 or 70 inches ... pended to the height of abo ...

By the ... of animal ... juices fro ...

If two p ... polished plates of glass three or four inches broad and twenty or twenty five long be laid one of them parallel to the horizon the other upon the first so as at one of their ends to touch one another and contain an angle of about 10 or 15 minutes and the same be first m ... with a cle ... or two of ... the upper ... as to touch it at one end as above and to touch the drop at the other end making with the lower gl s an angle of about 10 or 15 minutes the drop ll begin to move towards the con co ... e with an accelerated motion till ... the two glasses attract the drop ... in that way towards which the attractions incline And if when the drop is in motion you lift up that end of the glasses where they meet and towards which the drop moves the drop will ascend between the gl ... therefore is attracted And as y ... will ascend slower and slower ... by its weight as much as upwar ... means you may know the force by which the drop is attracted at all distances from the concourse of the glasses

Now by some experiments of this kind (made by Mr Hauksbee) it has been found that the attraction is almost reciprocally in a duplicate proportion of the distance of the middle of the drop from the concourse of the gl ...

what they recover from their elasticity If it be said that they can lose no motion but what they communicate to other bodies the consequence is that in

ten pitch were each of them as large as those which some suppose to revolve

tinue lon er in motion but unless the matter were void of all tenacity and attrition of parts and communication of motion (which is not to be supposed) the motion would constantly decay Seeing therefore the variety of motion which we find in the world is always decreasing there is a necessity of conservin and recruiting it by active principles such as are the cause of gravity by which planets and comets keep their motions in their orbs

remain in their orbs

ormed them and harder than any never to wear or

changed Water and earth composed of old worn particles and fragments of particles would not be of the same nature and texture now with water and earth composed of entire particles in the beginning And therefore that Nature may be la t n

variou

ticles

but n

It se ms t me further that these particles have not only a vis inertiæ accompanied with such passive laws of motion as naturally result from that force

as renders them most susceptible to those agitations which keep liquors in a Fluor are most easily separated and rarefied into vapour and in the language of the chemists they are volatile rarefying with an easy heat and condensing with cold But those which are grosser and so less susceptible of agitation or cohere by a stronger attraction are not separated without a stronger heat or perhaps not without fermentation And these last are the bodies which chemists call fixed and being rarefied by fermentation become true permanent air those particles receding from one another with the greatest force and being most difficultly brought together which upon contact cohere most strongly And because the particles of permanent air are grosser and arise from denser substances than those of vapours thence it is that true air is more ponderous than vapour and that a moist atmosphere is lighter than a dry one quantity for quantity From the same repelling power it seems to be that flies walk upon the water without wetting their feet and that the object glasses of long telescopes lie upon one another without touching and that dry powders are difficultly made to touch one another so as to stick together unless by melting them or wetting them with water which by exhaling may bring them together and that two polished marbles which by immediate contact stick together are difficultly brought so close together as to stick

And thus Nature will be very conformable to herself and very simple performing all the great motions of the heavenly bodies by the attraction of gravity which intercedes those bodies and almost all the small ones of their particles by some other attractive and repelling powers which intercede the particles The *vis inertiæ* is a passive principle by which bodies persist in their motion or rest receive motion in proportion to the force impressing it and resist as much as they are resisted By this principle alone there never could have been any motion in the world Some other principle was necessary for putting bodies into motion and now they are in motion some other principle is necessary for conserving the motion For from the various composition of two motions tis very certain that there is not always the same quantity of motion in the world For if two globes joined by a slender rod revolve about their common centre of gravity with a uniform motion while that centre moves on uniformly in a right line drawn in the plane of their circular motion the sum of the motions of the t right line described by their comn sum of their motions when they By this instance it appears that motion may be got or lost But by reason of the tenacity of fluids and attrition of their parts and the weakness of elasticity in solids motion is much more apt to be lost than got and is always upon the decay For bodies which are either absolutely hard or so soft as to be void of elasticity will not rebound from one another Impenetrability makes them only stop If two equal bodies meet directly *in vacuo* they will by the laws of motion stop where they meet and lose all their motion and remain in rest unless they be elastic and receive new motion from their spring If they have so much elasticity as suffices to

H ... no more the soul of them than the soul of man is the ... oul of the species of th ...

the species of things in its sen..orium but only for conveying them thither and God has no need of such organs He being everywhere present to the things themselves ...

... conclusions but such as are taken from experiments or other certain truths For hypotheses are not to be regarded in experimental philosophy And although the arguing from experiments and observations by induction be no demonstration of general conclu..ions yet it is the best way of arguin which the nature of things admits of and may be looked upon as ... much the stronger by how much the induction is more general And if no exception occur from phenomena the conclusion may be pronounced generally But if at any time afterwards any exception ...hall occur from experiments it may then begin to be p...

analysis we may ...
the forces produc...
from particular c...
gen... ral This is th...
the causes discov...
the phenomena pr...

In the two first ...
cover and prove t...
gibility reflexibili...
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which their reflexions and ...
may be ...
arguin...
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than ...
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philosophy in all its part...
fected, the bound...
can know by natu...
over us and what ...
as well as that to ...
And no doubt if t...
moral philosophy ...

but also that they are moved by cert ... that of gravity ... These princ ... to result from the pecific ... but as general laws of nature by which the things themselves are formed their truth appearing to us by phenomena tho ... causes be not yet discovered For the ... onl ...

... ... mutest effects Such as would be th ... of gravity and of magnetic and ele ... tions if we ... known t ... cult ... fo ... on ...

all corpo ... step in p ... principles were not yet discovered And ... therefore I scruple not to propose the principles of motion above mentioned they being of very general extent and leave their causes to be found out

Now by the help of these principles all material things seem to have been composed of the hard and solid particles above mentioned vario ... ated in the first creation by the ... Him who cre ... cal to seek fo ... out of a chao ... being once formed it may continue by those laws for many ages For while comets move in very eccentric orbs in all manner of positions blind fate could never make all the planets move one and the same way in orbs concentric some inconsiderable irregularities excepted which may have risen from the mut ... upon one ...

... ... alike and on either side of their bodies two legs behind and either two arms or two legs or two wings before upon their shoulders and between their shoulders a neck running down into a backbone and a head upon it and in the head two ears two eyes a nose a mouth and a tongue alike situated Also the first contrivance of those very artificial parts of animals the eyes ears brain muscles heart lungs midriff glands larynx hands wing swimming bladders natural spectacles and other organs of sense and motion and the instinct of brutes an ... effe t of noth ...

... yet ... body of God ... f as the ... and th ...

TREATISE ON LIGHT

and instead of teaching the transmigration of souls and to worship the Sun and Moon and dead heroes they would have taught us to worship our true Author and Benefactor as their ancestors did under the government of Noah and his sons before they corrupted themselves

BIOGRAPHICAL NOTE

Christiaan Huygens 1629–1695

In fam… to which Christiaan Huygens … Ap 1 14 1629 at The Hague was … of the most eminent in both the political … development of the Dutch Renaissance. The father of these … Constantijn …

… England where he was knighted in … he became the friend of D… whose poetry he began translating into Dutch. As one of the leaders of the Amsterdam school, he was the intimate friend of V…, the Dutch national poet, and was himself a … foremost classical poet.

Constantijn, who was … distinguished … musician and a mathematician, … himself the preliminary instruction … Christiaan, the second son, was … as a boy in languages, drawing and … At thirteen he began the study of mechanics, which together with mathematics … his chief interest. But before de… entire attention to these subjects, he … to Leyden to study law with Vinnius, … his famous commentary … to him. In 1646 Huygens … to Breda, where his father directed … and two years later he took his degree in law. In both places he continued his pursuit of mathematics, particularly with Van Schooten, who included some of Huygens' … in his edition of Descartes' *Geometry*.

At seventeen Huygens communicated his first mathematical discovery to Mersenne, … introduced him to the learned world as … and soon after he … correspondence with the leading scientists of Europe. Descartes, … been shown … mathematical paper of Huygens, declared his … that he would excel in this science … I see hardly anyone who knows any… Although Descartes frequented Sir Constantijn's house, it does not appear that he ever met his son. They … exchanged letters. Des… Huygens' son … blood …

… Huygens was travelling in Denmark … with the Count of Nassau, he regretted that time and weather did not permit his crossing over to Sweden to visit Descartes, who was then living there at the invitation of Queen Christina.

…

later he sent to Van Schooten his work on probability, which while recognizing the pr…

… as … his elder brother in astronomy. They found a new method of grinding and polishing lenses which overcame the de…

…m.

Huygens' reputation now became international. As early as 1655 the University of Angers had distinguished him with an honorary degree of doctor of laws. In 1663, on the occasion of a visit to England, he was elected a fellow of the Royal Society. Two years later, on the establishment of the French Royal Academy of Sciences, Colbert invited him to be its first foreign resident, and for the next fifteen years Huygens made his home in France. He received a handsome pension from Louis XIV and lived at Paris in the Biblio-

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tific research His treatises on Dioptrics and the concussion of elastic bodies were hailed not only for their discoveries but also for the style in which they were presented and Newton claimed that among modern writers he had most closely approximated the style of the ancients His greatest work the *Horologium oscillatorium* (1673) dealt with the problems raised by the pendulum clock and contained original discoveries sufficient for several important treatises

Twice during his residence in Paris Huygens returned to Holland in the hope that his native air would restore his health and in 1681 perhaps because of the revocation of the Edict of Nantes he severed his connections and left France Upon his return to Holland Huygens took up again the study of optics physics and astronomy He had always been interested in useful inventions and in addition to the pendulum clock had already improved the air pump and the barometer provided the first idea of the micrometer and introduced the use of a spiral band for a watch spring In Holland he turned again to the construction of telescopes Using lenses of long focal distance mounted on poles he produced what were called aerial telescopes He also succeeded in constructing an almost perfectly achromatic eye-piece still known by his name His researches in optics finally led him to publish in 1690 his *Treatise on Light* which had been written in French in 1678 while at Paris In response to the need for some means of repre-

work found among his posthumous papers called *Cosmotheoros* and translated into English under the title *The celestial worlds discovered or conjectures concerning the inhabitants plants and productions of the worlds in the planets*

Worn out by his great and varied activity and the burden of an enormous correspondence Huygens died at The Hague June 8 1695 at the age of sixty six

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Worn out by his great and varied activity and the burden of an enormous correspondence Huygens died at The Hague June 8 1695 at the age of sixty six

PREFACE

I wrote this treatise during my sojourn in France twelve years ago and I [illegible] the learned persons who then composed [illegible]

member having been present when I [illegible] them who applied themselves particularly to the study of mathematics [illegible] of whom I cannot cite more than the celebrated gentlemen Cassini, Römer, and de la Hire. And, although I have since corrected and changed some parts, the copies which I had made of it at that time may serve for proof that I have yet added nothing to it save some conjectures touching the formation of Iceland crystal, and a novel observation on the refraction of rock crystal. I have desired to relate these particulars to make known how long I have meditated the things which now I publish, and not for the purpose of detracting from the merit of those who, without having seen anything that I have written, may be found to have treated of like matters: as has in fact occurred to two eminent geometricians, Messieurs Newton and Leibnitz, with respect to the problem of the figure of glasses for collecting rays when one of the surfaces is given.

One may ask why I have so long delayed to bring this work to the light. The reason is that I wrote it rather carelessly in the Language in which it [illegible] with the intention of translating it into Latin, so doing in order to [illegible] to the things [illegible] which I proposed to myself to give [illegible] the treatise on dioptrics, in which I explain the effects of telescopes and other things which belong more to that science. But the pleasure [illegible] being past, I have put off from time to time the execution of this design, and I know not when I shall ever come to an end of it, being often turned [illegible] by business or by some new study. Considering which I have finally judged [illegible] to publish this writing [illegible] by waiting longer [illegible]

There will be seen in it demonstrations of those kinds which do not produce as great a certitude as those of Geometry, and which even differ much therefrom [illegible]

[illegible]

PREFACE

I wrote this treatise during my sojourn in France twelve years ago, and I communicated it in the year 1678 to the learned persons who then composed the Royal Academy of Science, to the membership of which the King had done me the honour of calling me. Several of that body who are still alive will remember

de la Hire. And, although I have since corrected and changed some parts, the copies which I had made of it at that time may serve for proof that I have yet added nothing to it save some conjectures touching the formation of Iceland crystal, and a novel observation on the refraction of rock crystal. I have desired to relate these particulars to make known how long I have meditated the things which now I publish, and not for the purpose of detracting from the merit of those who, without having seen anything that I have written, may be found to have treated of like matters: as has in fact occurred to two eminent geometricians, Messieurs Newton and Leibnitz, with respect to the problem of the figure of glasses for collecting rays when one of the surfaces is given.

One may ask why I have so long delayed to bring this work to the light. The reason is that I wrote it rather carelessly in the Language in which it appears, with the intention of translating it into Latin, so doing in order to obtain greater attention to the thing. After which I proposed to myself to give it out along with another tre

telescopes and those things

of novelty being past, I h

design, and I know not when I shall ever come to an end of it, being often turned aside either by business or by some new study. Considering which I have finally judged that it was better worth while to publish this writing, such as it is, than to let it run the risk, by waiting longer, of remaining lost.

There will be seen in it demonstrations of those kinds which do not produce as great a certitude as those of geometry, and which even differ much therefrom, since whereas the geometers prove their propositions by fixed and incontestable principles, here the principles are verified by the conclusions to be drawn from them; the nature of these things not allowing of this being done otherwise. It is always possible to attain thereby to a degree of probability which very often is scarcely less than complete proof. To wit, when things which have been demonstrated by the principles that have been assumed correspond perfectly to the phenomena which experiment has brought under observation; especially when there are a great number of them, and further, principally, when one can imagine and foresee new phenomena which ought to follow from the hypotheses which one employs, and when one finds that therein the fact corresponds to our prevision. But if all these proofs of probability are met with in that which I propose to discuss, as it seems to me they are, this ought to be a very strong confirmation of the success of my inquiry; and it

CHAPTER ONE

On Rays Propagated in Straight Lines

... in which geometry is applied to matter the ... drawn from expe-

tain than the preceding ...

The majority of those who have written touching the various parts of Optics have contented themselves with presuming these truths But some more in- ... estimate the origin and the causes considering ... anced

do not ... for better and ... desire to propound what I have meditated on the subject so as to contribute as much as I can to the ... planation of this department of natural science ... hich not without reason is reputed to be one of its most difficult parts. I recognize m... to be much indebted to those who were the first to begin ... dissipate the strange obscurity in which these things were enveloped and to ... hope that they might be explained by intelligible reasoning But on the other hand I am ... also that even here these have often been willing ... as assured and demonstrative reasonings which were far from conclu ... For I do not find that any one has yet given a probable explanation of ... phenomena of light namely why it is not propa ...

I h... in this book to ... some clearer and more probable ... philosophy of the present day ...

transparent ... the different densities of the atmosphere ... causes of the strange refraction of a certain ... brought from Iceland. And finally I shall treat of the ... transparent and reflecting bodies by which rays are collected ... turned and in various ways. From this it will be seen with ... our new theory we find not only the ellipses hyperbo- ... curves which M Descartes has ingeniously invented for this pur ... but also those which the surface of a glass lens ought to possess when its ... surface ... as spherical or plane or of any other figure that may be ...

It is ... to doubt that light consists in the motion of some sort of matter For whether one considers its production one sees that here upon the

must be ill if the facts are not pretty much as I represent them I would believe then that those who love to know the causes of things and who are able to admire the marvels of light will find some satisfaction in these various speculations regarding it and in the new explanation of its famous property which is the main foundation of the construction of our eyes and of those great inventions which extend so vastly the use of them I hope also that there will be some who by following these beginnings will penetrate much further into this question than I have been able to do since the subject must be far from being exhausted This appears from the
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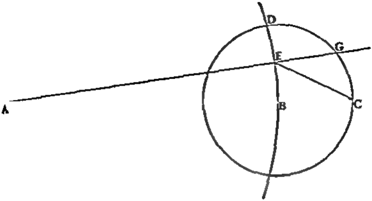
The Hague *January* 8 1690



earth ABC a straight line which I suppose to meet the orbit of the moon which is represented by the circle CD at C

Now f ... ch ... sr ... I I

I

have arrived at the point C but will only arrive there an hour after It will then be one hour after reckoning from the moment when the earth was at B that the moon, arriving at C will be obscured but this obscuration or interruption of the light will not reach the earth till after another hour Let us suppose that the earth in these two hours will have arrived at E. The earth then being at E, will see the eclipsed moon at C which it left an hour before and at the same time will see the sun at A For it being immovable as I suppose with Copernicus and the light moving always in straight lines it must always appear where it is But one has always observed we are told that the eclipsed moon appears at the point of the ecliptic opposite to the sun and yet here it would appear in arrear of that point by an amount equal to the angle GEC the supplement of AEC This however is contrary to experience since the angle GEC would be very sensible and about 33 degrees Now according to our computation, which is given in the treatise on the causes of the phenomena of Saturn the distance BA between the earth and the sun is about twelve thousand diameters of the earth and hence four hundred times greater than BC the distance of the moon which is 30 diameters Then the angle ECB will be nearl fo ... h ...
ly the
angle I
greater ... minutes.

But it must be noted that the speed of light in this argument has been assumed such that it takes a time of one hour to make the passage from here to the moon. If one supposes that for this it requires only one minute of time then it is manifest that the angle CEG will only be 33 minutes and if it requires onl ten seconds of t m ... he ...
will not be easy
consequently wi
light is instantan ous

I is true that we are here supposing a strange velocity that would be a hun-

earth it is chiefly engendered by fire and flame which cont ...

... un one conceives the causes of all n ... l effects in term of ... do ...

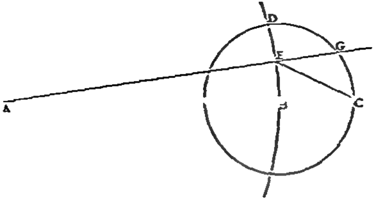
of ... pr sion of some movement of a kind of matter which acts on the nerves at the back of our eyes there is here yet one reason more for believing that light consists in a movement of the matter which exists between us and the luminous body

Further when one considers the extreme speed with which light spreads on every side and how when it comes from different regions even from those directly opposite the rays traverse one another without hindrance one may well understand that when we see a luminous object it cannot be by any transport of matter coming ... arrow traverses the a ... properties of light es ... cond of them It is then in some other way that light spreads and that which can lead u to co ...

... has been produced by a movement which is passed on successively from one part of the air to another and that the spreading of this movement taking place equally rapidly on all sides ought to form spherical surfaces ever enlarging and which strike our ears Now there is no doubt at all that light also comes from the luminous body to our eyes by some movement impressed on the matter which is betw ... since as we h ... pa es from ... which we are ... —it will follow that this movement impressed on the intervening matter is successive and consequently it spread as sound does by spherical surfaces and waves for I call them waves from their resemblance to tho e which are seen to be formed in water when a stone is thrown into it and which present a successive spreading as circles though the arise from another cause and are only in a flat surface

To see then whether the spreading of light takes time let us consider first whether there are any facts of experience which can convince us to the contrary As to tho e which can be made here on the earth by striking lights at great distances although they prove that light takes no sensible time to pass over these distances one may say with good rea on that they are too small and that the only conclusion to be drawn from them is that the passage of light is extremely rapid M Descarte who was of opinion that it is instantaneous founded his views not ... from the eclipses of the ... convincing I will set ... make the conclusion more comprehensible

Let A be the place of the sun BD a part of the orbit or annual path of the



are ABC a straight line which I suppose to meet the orbit of the moon, which is represented by the circle CD at C.

Now if light requires time, for example one hour, to traverse the space which is between the earth and the moon, it will follow that the earth having arrived at B, the shadow which it casts, or the interruption of the light, will not yet have arrived at the point C, but will only arrive there an hour after. It will then be one hour after, reckoning from the moment when the earth was at B, that the moon, arriving at C, will be obscured; but this obscuration or interruption of the light will not reach the earth till after another hour. Let us suppose that the earth in these two hours will have arrived at E. The earth then, being at E, will see the eclipsed moon at C, which it left an hour before, and at the same time will see the sun at A. For it being immovable, as I suppose with Copernicus, and the light moving always in straight lines, it must always appear where it is. But one has always observed, we are told, that the eclipsed moon appears at the point of the ecliptic opposite to the sun; and yet here it would appear in arrear of that point by an amount equal to the angle GEC, the supplement of AEC. This, however, is contrary to experience, since the angle GEC would be very sensible, and about 33 degrees. Now according to our computation, which is given in the treatise on the causes of the phenomena of nature, the distance BA between the earth and the sun is about twelve hundred diameters of the earth, and hence four hundred times greater than BC the distance of the moon, which is 30 diameters. Then the angle ECB will be nearly four hundred times greater than BAE, which is five minutes, namely the path which the earth travels in two hours along its orbit; and thus the angle BCE will be nearly 33 degrees; and likewise the angle CEG, which is greater by five minutes.

But it must be noted that the speed of light in this argument has been assumed such that it takes a time of one hour to make the passage from here to the moon. If one supposes that for this it requires only one minute of time, then it is manifest that the angle CEG will only be 33 minutes; and if it requires only ten seconds of time, the angle will be less than six minutes. And then it will not be easy to perceive anything of it in observations of the eclipse; nor consequently will it be permissible to deduce from it that the movement of light is instantaneous.

It is true that we are here supposing a celerity of light that would be a hun-

earth it is chiefly engendered by fire and fl... bodie...

e

c

...s of all natural ... motions This in my opinion we must necessarily do or else renounce all hopes of ever compre...

And a ... of sight ... ter ... m...

... one considers the extreme speed with which light spreads on every side and how when it comes from different regions even from those directly opposite the rays traverse one another without hindrance one may well understand that when we see a luminous object it cannot ... port of matter coming to ... arrow traverses the a... properties of light es... it is then in some ot... that light spreads and that ...

... produced by a movement ... on successively from one part of the air to another and that the spreading of this movement taking place equally rapidly on all sides ought to form spherical surfaces ever enlarging and which strike our ears Now there is no doubt at all that light also comes from the luminous body to our eyes by some movement impressed on the matter which is between the two since as we have already seen it cannot be by the transport of a body which passes from one to the other If in addition light takes time for its passage—which we are now going to examine—it will follow that this movement impressed on the intervening matter is successive and consequently it ... as sound does by spheric... their resem... thrown in ... spreading as circles though the e... at surface

To ... time let us consider first wheth... any facts of experience which can convince us to the contrary As to those which can be made here on the earth by striking lights at great distances although they prove that light takes no sensible time to pass over these distances one may say with good reason that they are too small and that the only conclusion to be drawn from them is that the passage of light is extremely rapid M Descartes who was of on... f...

Let A be the place of the sun BD a part of the orbit or annual path of the

— ... when the earth has come to E from D while approach
... be observed at E
... d at D

Now in quantities of observations ... ring ten con
secutive years these differences have been found to be very considerable such as ten minutes and more and from them it has been concluded that in order to traverse the whole diameter of the annual orbit KL, which is double the distance from here to the sun light requires about 22 minutes of time

The movement of Jupiter in his orbit while the earth passed from B to C
... calculation and this makes it evident that
... illuminations or the anticipation
... ecurring in the movement of the
E... planet or to its eccentricity

If one considers the vast size of the diameter KL, which according to me is some 24 thousand diameters of the earth one will acknowledge the extreme velocity of Light For supposing that KL is no more than 22 thousand of these diameters it appears that being traversed in 22 minutes this makes the speed
... diameters in one second or in
... hundred times a hundred
... ontains 2 865 leagues reck
oned at 25 to the degree and each league is 2,282 toises according to the exact measurement which Mr Picard made by order of the King in 1669 But sound as I have said above only travels 180 toises in the same time of one second hence the velocity of light is more than six hundred thousand times greater
... from her ... tan

... I have said that it spreads by spherical waves like the movement of sound

But if the one resembles the other in this respect they differ in many other things to wit in the first production of the movement which causes them in the matter in which the movement spreads and in the manner in which it is propagated. As to that which occurs in the production of sound one knows that it is occasioned by the agitation undergone by an entire body or by a considerable part of one which shakes all the contiguous air But the movement of the light must originate as from each point of the luminous object else we should not be able to perceive all the different parts of that object as will be more evident in that which follows And I do not believe that this movement can be better explained than by supposing that all those of the luminous bodies which are liquid such as flames and apparently the sun and the stars are composed of particles which float in a much more subtle medium which

...

...king similarly against the ethereal matter The agitation moreover of the particles which engender the light ought to be much more prompt and more rapid than is that of the bodies which cause sound since we do not see that the tremors of a body which is giving out a sound are capable of giving rise to

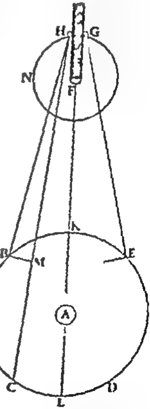
speed but of a successive m ... others I have then ... posing that the er ...

... ays seemed to me th ... has been to ... telligibly and ... him has said ... or even incon ... dealing with lig ... s properties

But th ... seeming ... ous proof of M ... which I ... for it ...

... least six ... stated

... he makes use of the eclipses suffered by the little planets which revolve around Jupiter and which often enter his shadow and see what is his reasoning Let A be the sun BCDE the annual orbit of the earth F Jupiter GN the orbit of the nearest of his satellites for it is this one which is more apt for this investigation than any of the other three because of the quickness of its revolution Let G be this satellite entering into the shadow of Jupiter H the same satellite emerging from the shadow



Let it be then supposed the earth being at B some time before the last quadrature that one has seen the said satellite emerge from the shadow it must needs be if the earth remains at the same place that after 42½ hours one would again see a similar emergence because that is the time in which it makes the round of its orbit and when it would come again into opposition to the sun And if the earth for instance were to remain always at B during 30 revolutions of this satellite one would see it again emerge from the shadow after 30 times 42½ hours But the earth having been carried along during this time to C increasing thus its distance from Jupiter it follows that if light requires time for its passage the illumination of the little planet will be perceived later at C than it would have been at B and that there must be added to this time of 30 times 42½ hours that which the light has required to traverse the space MC the difference of the spaces CH BH Similarly at

been stirred And even that one which was used to strike remains motionless with them Whence one sees that the movement passes with an extreme velocity

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pa Moreover there are experiments which demonstrate that all the bodies h h we real

ge and quite thick piece of the same substance which had a flat surface lightly
mained round marks of sm
weak or trong This makes
meet and sprin back and for this time must be required

Now in applying this kind of movement to that which produces light there is nothing to hinder us from estimating the particles of the ether to be of a substance as nearly approaching to perfect hardness and possessing a pringi ness prompt as we choose It is not necessary to examine here the causes of this hardne or of h
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s infinite eries of different sizes of cor uses having different degrees of velocity of which Nature makes use to produce so many marvellous effects

But though we hall
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light even as the movement of the hand in the air is not capable of pro l sound

Now if one ex ~ ~
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One sees here not only that our air which
is the matter by

demonstrated even more clearly by the celebrated experiment of Torricelli in which the tube of glass from which the quicksilver has withdrawn itself remaining void of air transmits light just the same as when air is in it For this proves that a matter different from air exists in this tube and that this matter must have penetrated the glass or the quicksilver either one or the other though they are both impenetrable to th I
when in the same experiment one m
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thro

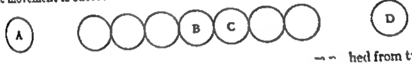
A nt modes in which I have said the movements of sound and of light are communicated one may sufficiently comprehend how this occurs in the case of sound if one considers that the air is of such a nature that it can be compressed and reduced to a much smaller space than that which it ordinarily occupies And in proportion as it is compres ed the more does it exert an effort to regain its volume for this property along with its penetrability which remains notwithstanding its compression seems to prove that it is made up of small bodies which float about and which are agitated very rapidly in the ethereal matter compo ed of much smaller parts So that the cau e of the preading of sound is the effort which the e little bodies make in colli ions with one another to regain freedom when they are a little more squeezed together in the circuit of the e waves than elsewhere

But the extreme velocity of light and other properties which it has cannot admit of such a propagation of motion and I am about to how here th ay in which I conceive it must occur Fo
made of some very hard
arranges them in a straight line so that they touch one another one finds on striking with a similar sphere again t the first of these spheres that the motion passes as in an instant to the la t of them which separates itself from the row without one s being able to perceive that the others have

them. And one must imagine the same about every point of the surface and of the part within the flame

But as the percussions at the centres of these waves possess no regular uc ... that the waves themselves follow one another ... to be uch ... al intervals ... ame centre ... nother with

the movement is successive This may be pro ...

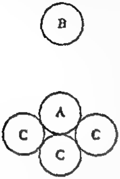


... hed from two ... will see each ... yet the whole ... long its whole length twice over And if these contrary movements ... meet one another at the middle phere B or at ome other such as C that phere will yield and act as a spring at both sides and so will serve at the same instant to transmit these two movements

But what may at first appear full strange and even incredible is that the undulations produced by such small movements and corpu cles should pread to such immense di tances as for example from the sun or from the stars to us For the force of these waves must grow feeble in proportion as they move a ay from their origin so that the action of each one in particular will without doubt become incapable of making itself felt to our ight But one will cease to be astonished by considering ho at a great di tance from the luminous body an ... body ... e only ... is this ... points of a fixed tar b g it may be as the un make practically only one ... e wave which may well have force enough to produce an impression on our eyes More- ... wave in ... orpu cles rendering their action more sensible

There is the further consideration in the emanation of these waves that each particle of matter in which a wave spreads ought not to communicate its mot on only to the next particle which is in the straight line drawn from the luminous point but that it al o imparts some of it necessarily to all the others wh ch touch it and which oppose them elves to its movement So it arises that around each particle there is made a wave of which that particle

And it must be known that although the particles of the ether are not ranged thus in straight lines as in our row of spheres but conf ... one of them touches several others ... ting their movement and from sprea ...



be remarked that ... of motion erving for this propagation and verifiable by experiment It is that when a sphere such as A here touches everal other imilar spheres CCC if it is struck by another sphere B in such a way as to exert an impulse against all the spheres CCC which touch it it transmits to them the whole of its movement and remains after that motionless like the phere B And without supposing that the ethereal particles are of spherical form (for I see indeed no need to suppose them so) one may well understand that this property of communicating an impulse does not fail to contribute to the aforesaid propagation of movement

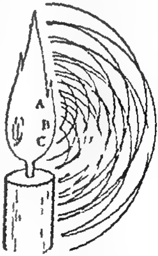
Equality of size seems to be more necessary because otherwise there ought to be some reflexion of movement backwards when it passes from a smaller particle to a larger one according to the *Laws of Percussion* which I published some years ago

However one will see hereafter that we have to suppose ... not so much as a necessity fo the ...

... are equal for a purpose ...

... of light at least in th ... region of atmo ... sun and t ...

I have ... n in what manner one may conceive light to spread successively by spherical waves and how it is possible that this spreading is accomplished with as great a velocity as that which experiments and celestial observations demand Whence it may be further remarked that although the particles are supposed to be in continual movement (for there are many reasons for this) the successive propagation of the waves cannot be hindered by this because the propagation consists nowise in the transport of those particles but merely in a small agitation which they cannot help communicating to those surrounding notwithstanding any movement which may act on them causing them to be changing positions amongst themselves



But we must consider still more particularly the origin of these waves and the manner in which they spread And first it follows from what has been said on the production of light that each little region of a luminous body such as the sun a candle or a burning coal generates its own waves of which that region is the centre Thus in the flame of a candle having distinguished the points A B C concentric circles described about each of these points represent the waves which come from

the straight lines AC AE as has just been shown the parts of the partial waves which spread outside the space ACE being too feeble to produce light there

Now however small we make the opening BG there is always the same reason causing the light there to pass between straight lines since this opening is always large enough to contain a great number of particles of the ethereal matter which are of an inconceivable smallness so that it appears that each little portion of the wave necessarily advances following the straight line which comes from the luminous point Thus then we may take the rays of light as if they were straight lines

It appears moreover by what has been remarked touching the feebleness of the particular waves that it is not needful that all the particles of the ether should be equal amongst themselves though equality is more apt for the propagation of the movement For it is true that inequality will cause a particle by pushing against another larger one to strive to recoil with a part of its movement but it will thereby merely generate backwards towards the luminous point some partial waves incapable of causing light and not a wave compounded of many as CE was

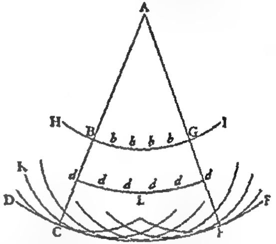
Another property of waves of light and one of the most marvellous is that when some of them come from different or even from opposing sides they produce their effect across one another without any hindrance Whence also it comes about that a number of spectators may view different objects at the same time through the same opening and that two persons can at the same time see one another's eyes. Now according to the explanation which has been given of the action of light how the waves do not destroy nor interrupt one another when they cross one another these effects which I have just mentioned are easily conceived But in my judgement they are not at all easy to explain according to the views of M Descartes who makes light to consist in a continuous pressure merely tending to movement For this pressure not being able to act from two opposite sides at the same time against bodies which have no inclination to approach one another it is impossible so to understand what I have been saying about two persons mutually seeing one another's eyes or how two torches can illuminate one another

CHAPTER TWO

On Reflexion

HAVING explained the effects of waves of light which spread in a homogeneous matter we will examine next that which happens to them on encountering other bodies We will first make evident how the reflexion of light is explained by these same waves and why it preserves equality of angles.

is the centre Thus if DCF is a wave emanating from the luminous point A which is its centre the particle B one of those comprised within the sphere DCF will have made its particular or partial wave KCL which will touch the wave DCF at C at the same moment that the principal wave emanating from the point A has arrived at DCF and it is clear that it will be only the region C of the wave KCL which will touch the wave DCF to wit that which is in the straight line drawn through AB Similarly the other particles of the sphere DCF such as bb dd etc will each make its own wave But each of these waves can be infinitely feeble only as compared



with the wave DCF to the composition of which all the others contribute by the part of their surface which is most distant from the centre A

One sees in addition that the wave DCF is determined by the distance attained in a certain space of time by the movement which started from the point A there being no movement beyond this wave though there will be in the space which it encloses namely in parts of the particular waves those parts which do not touch the sphere DCF ... fraught with ... that all the ... ing to its reflexion and its refracti... ... gained in principle by this means This is a matter which has been quite unknown to those who hitherto have begun to consider the waves of light amongst whom are Mr Hooke in his *Micrographia* and Father Pardies who in a treatise of which he let me see a portion and which he was unable to complete as he died sho... prove by the ... dation whic... on ...

... properties of light ...

wave ought to ... same str... ... the portion BC of the wave ... luminous point A as its centre will spread into the arc CE bounded by the straight lines ABC AGE For although the particular waves produced by the particles comprised within the space CAE spread also outside this space they yet do not concur at the same instant to compose a wave which terminates the movement as they do precisely at the circumference CE which is their common tangent

And hence one sees the reason why light at least if its rays are not reflected or broken spreads only by straight lines ... ex... For ... CE ...

... will always be terminated by

the straight lines AC AE as has just been shown the parts of the partial waves which spread outside the space ACE being too feeble to produce light there

Now however small we make the opening BG there is always the same reason causing the light there to pass between straight lines since this opening is always large enough to contain a great number of particles of the ethereal matter which are of an inconceivable smallness so that it appears that each little portion of the wave necessarily advances following the straight line which comes from the luminous point Thus then we may take the rays of light as if they were straight lines

It appears moreover by what has been remarked touching the feebleness of the particular waves th [illegible] should be equal amon [illegible] on of the movem [illegible] but it will thereby merely generate backward towards the luminous point some partial waves incapable of causing light and not a wave compounded of many as CE was.

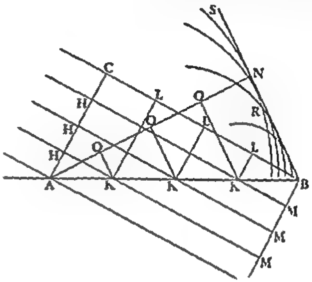
Another property [illegible] when one of them [illegible] produce [illegible] effect [illegible] comes about [illegible] same time through [illegible] opening and thus two persons can at the same time see one another's eyes. Now according to the explanation which has been given [illegible] how the waves do not destroy nor interrupt one another when they cross one another these effects which I have just mentioned are easily conceived. But in my judgement they are not at all easy to explain according to the views of Mr Descartes, who makes light to consist in a continuous pressure [illegible] For this pressure not being [illegible] [illegible]

CHAPTER TWO

On Reflexion

[illegible]

not be exempt) and let a line AC inclined to AB represent a portion of a wave of light the centre of which is so distant that this portion AC may be considered as a straight line for I consider all this as in one plane imagining to myself that the plane in which this figure is cuts the sphere of the wave through its centre and intersects the plane AB at right angles This explanation will suffice once for all



The piece C of the wave AC will in a certain space of time advance as far as the plane AB at B following the straight line CB which may be supposed to come from the luminous centre and which in consequence is perpendicular to AC Now in this same space of time the portion A of the same wave which has been h nd leas

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considers further the other pieces H of the wave AC it appears that they will not only have reached the surface AB by straight lines HK parallel to CB but that in addition they will have generated in the transparent air from the centres K K K particular spherical waves repre ented here by circumferences the semi-diameters of that the

ues of which A is the

ce as is easy to see

ne BN (compri ed between B and the point N where the perpendicular from the point A falls) which is as it were formed by all these circumferences and which terminates the movement which is made by the reflexion of the wave AC and it is also the place where the movement occurs in much greater quantity than anywhere else Wherefore according to that which has been explained BN is the propagation of the wave AC at the moment when the piece C of it has arrived at B For there is no other line which like BN is a common tangent to all the afore aid circles except BG below the plane AB which line BG would be the propagation of the wave if the movement could have pread in a medium homogeneous with that which is ab ve the plane And if one wi hes to see how th

BN

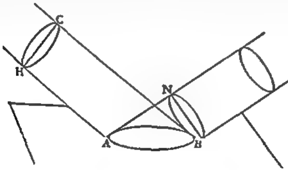
wave AC has become broken up into all the OKL parts successively and that it has become straight again at NB

h h d f fl xion i made equal to the angle

But in considering the preceding demonstration one might aver that it is indeed true that BN is the common tangent of the circular waves in the plane of this figure but that these waves being in truth spherical have still an infinitude of similar tangents namely all the straight lines which are drawn

I say then that the wave AC being regarded only as a line produces no light For a visible ray of light however narrow it may be has always some width

demonstration that each small piece of this wave HC having arrived at the plane AB and there generating each one its particular wave these will all have when C arrives at B a common plane which will touch them namely a circle BN similar to CH and this will be intersected at its middle and at right



angles by the same plane which likewise intersects the circle CH and the ellipse AB

One sees also that the said spheres of the partial waves cannot have any common tangent plane other than the circle BN so that it will be this plane where h

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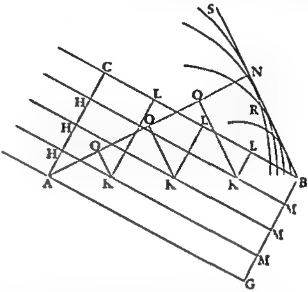
plane AB or at least not wholly Whence it is to be remarked that though the

not be exempt) and let a line AC inclined to AB represent a portion of a wave of light the centre of which is so distant that this portion AC may be considered as a straight line for I consider all this as in one plane imagining to myself that the plane in which this figure is cuts the sphere of the wave through its centre and intersects the plane AB at right angles This explanation will suffice once for all



The piece C of the wave AC will in a certain space of time advance as far as the plane AB at B following the straight line CB which may be supposed to come from the luminous centre and which in consequence is perpendicular to AC Now in this same space of time the portion A of the same wave which has been hindered from communicating its movement beyond the plane AB or at least partly so ought to have continued its movement in the matter which is above this plane and this along a distance equal to CB making its own partial spherical wave according to what has been said above Which wave is here represented by the circumference SNR the centre of which is A and its semi diameter AN equal to CB

If one considers further the other pieces H of the wave AC it appears that they will not only have reached the surface AB by straight lines HK parallel to CB but that in addition they will have generated in the transparent air from the centres K K K particular spherical waves represented here by circumferences the semi-diameters of which are equal to KM that is to say to the continuations of HK as far as the line BG parallel to AC But all these circumferences have as a common tangent the straight line BN namely the same which is drawn from B as a tangent to the first of the circles of which A is the centre and AN the semi-diameter equal to BC as is easy to see

It is then the line BN (comprised between B and the point N where the perpendicular from the point A falls) which is as it were formed by all these circumferences and which terminates the movement which is made by the reflexion of the wave AC and it is also the place where the movement occurs in much greater quantity than anywhere else Wherefore according to that ... mo- hich the ove-ment could have spread in a medium homogeneous with that which is above the plane And if one wishes to see how the wave AC has come successively to BN one has only to draw in the same figure the straight lines KO parallel to BN and the straight lines KL parallel to AC Thus one will see that the straight

wave AC has become broken up into all the OKL parts successively and that it has become straight again at NB

Now it is apparent here that the angle of reflexion is made equal to the angle of incidence For the triangles ACB BNA being rectangular and having the de AB common and the ide CB equal to NA it follows that the angles these sides will be equal and therefore al o the angles CBA NAB
— he incident ray ■
the reflected ray

ion one might aver that it is
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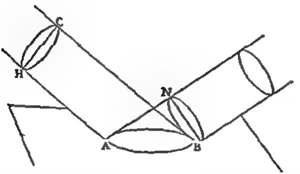
uth pherical have still an in
raight lines which are drawn
from the point B in the surface generated by the straight line BN about the axis BA It remains therefore to demonstrate that there is no difficulty herein and by the same argument one will see why the incident ray and the reflected ray are always in one and the same plane perpendicular to the reflecting plane I say then that the wave AC being regarded only as a line produces no light r — hl ro f l ht howe er narrow it may be has always some width



angles by the same plane which likewise intersects the circle CH and the ellipse AB

One sees also that the aid pheres of the partial waves cannot have any common tangent plane other than the circle BN ■ that it will be this plane where there will be more reflected movement than anywhere else and which w

plane AB or at least not wholly Whence it is to be remarked that though the

movement of the ethereal matter might communicate itself partly to the [illegible] of the reflecting body this could in nothing [illegible]
the [illegible]

[illegible] matter This comes [illegible] y of bodies which act as springs of which we have spoken above namely that whether compressed little or much they recoil in equal times Equally so in every reflexion of the light against whatever body it may be the angles of reflexion and incidence ought to be equal notwithstanding that the body might be of such a nature that it takes away a portion of the movement made by the in[illegible] in [illegible]

[illegible] as has been supp[illegible]

[illegible] one another which particles [illegible] all those of the ethereal matter as will appear by what we shall say in treating of the transparency and opacity of bodies For the su[illegible] sisting thus of particles put togethe[illegible]

[illegible] and the thing is expl[illegible] the particles of quicksilver for [illegible] millions of them in [illegible] of grains of sand wh[illegible] this surface then bec[illegible] although it always re[illegible] evident that the cent[illegible] have spoken are almost in one uniform p[illegible] gent can fit to them as perfectly as is requ[illegible] this alone is requisite in our method of [illegible] construction to cause equality of the said angles without the remainder of the movement reflected from all parts being able to produce any contrary effect

CHAPTER THREE

On Refraction

In the same way as the effects of reflexion have been explained by waves of light reflected at the surface of polished bodies we will explain transparency and the phenomena of refraction by waves which spread within and across diaphanous bodies both solids such as glass and liquids such as water oils etc But in order that it may not seem strange to suppose this passage of waves in the interior of these bodies I will first show that one may conceive it possible in more than one mode

cannot penetrate transparent bodies at

... since they are of a nature to act as a [illegible] they being composed of detached particles as in water and other transparent liquids. But it may seem more difficult as regards glass and other transparent and hard bodies, because their solidity does not seem to permit them to receive movement except in their whole mass at the same time. This, however, is not necessary because this solidity is not such as it appears to us, it being probable rather that these bodies are composed of particles merely placed close to one another and held together by some pressure from without of some other matter, and by the irregularity of their shapes. For primarily their rarity is shown by the facility with which there passes through them the matter of the vortices of the magnet, and that which causes gravity. Further, one cannot say that these bodies are of a texture similar to that of a sponge or of light bread, because the heat of the fire makes them flow and thereby changes the situation of the particles amongst themselves. [illegible]

[illegible]

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But it may be objected here that if water is a body of so great rarity
its particles occupy so small a portion
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ho essayed to employ
its liquidity while subjected to this pressure

This is no small difficulty It may however be resolved by saying that the very violent and rapid motion of the subtle matter which renders water liquid by agitating the particles of which it is composed maintains this liquidity in spite of the pressure which hitherto any one has been minded to apply to it

The rarity of
conceives that t
fills the interstic
one may believe that the progression of the waves ought to be a little slower in the interior of bodies by reason of the small detours which the same particles cause In which different velocity of light I shall show the cause of refraction to consist

Before doing so I will indicate the third and last mode in which transparency may be conceived which is by supposing
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particles and so it will be these secondary particles which will receive the movement from those of the ether

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...ower in the interior of such bodies than it is out ... hich the waves of

easy penetrability by the ethereal matter one might also prove that the same penetrability obtains for metals and for every other sort of body For this sphere bein for example of silver it i certain that it contains ome of the ethereal matter which serves for light since this was there as well as in the air when the openin of the phere was closed Yet being closed and placed upon a horizontal plane it resist the movement which one wishes to give to it ... o that on mu t

ought also to be transparent which however is not the case

Whence then one will say does their opacity come Is it because the particles which compose them are oft that is to say these particles being composed of others that are maller are they capable of changing their figure on receiving the pressure of the ethereal particles the motion of which they thereby damp and so hinder the continuance of the waves of light? That cannot be for if the particles of the metal are oft how is it that polished silver and mercury reflect light so tron ly What I find to be most probable herein is to say that metallic bodies which are almost the only really opaque ones have mixed amongst their hard particles ome soft ones so that ome serve to cause reflexion and the others to hinder transparency while on the other hand transparent bodies contain only hard particles which have the faculty of recoil and serve together with those of the ethereal matter for the propagation of the waves of light as has been said

The chief property of refraction is that a ray of light such as AB being in the air and falling obliquely upon the polished urface of a transparent body such as FG is broken at the point of incidence B in such a way that with the strai...

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trary one finds that the sphere re
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b
w that by this process
th y may be inferred also as relating to opaque bodies

The second mode then of explaining transparency and one which appears more probably true is by saying that the waves of light are carried on in the ethereal matter which continuously occupies the interstices or pores of transparent bodies For since it passes through them continuously
follows that they are always full of

that force is required to
horizontal velocity on bodies in proportion as they contain coherent matter and if the proportion of this force follows the law of weights as is confirmed by experiment then the quantity of the constituent matter of bodies also follows the proportion of their weights Now we see that water weighs only one fourteenth part as much as an equal portion of quicksilver therefore the matter of the water does not
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follow of the magnet and of that which cause of gravity pass very freely thro

But it may be

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ing even its entire liquidity while subjected to this pressure

This is no small difficulty It may however be resolved by saying that the very violent and rapid motion of the subtle matter which renders water liquid by agitating the particles of which it is composed maintains this liquidity in spite of the pressure which hitherto any one has been minded to apply to it

The rarity of transparent bodies being then such as we have said one easily conceives that the waves might be carried on in the ethereal matter which fills the interstices of the particles And moreover one may believe that the progression of these waves ought to be a little slower in the interior of bodies by reason of the which differ
ent velocity of li

Before doing third and last mode in which transparency may be conceived which is by supposing that the movement of the waves of light is transmitted indifferently both in the particles of the ethereal matter which occupy the interstices of bodies and in the particles which compose them so that the movement passes from one to the other And it will be seen hereafter that this hypothesis serves excellently to explain the double refraction of certain transparent bodies

Should it be objected that if the particles of the ether are smaller than those of transparent bodies (since they pass through their intervals) it would follow that they can communicate to them but little of their movement it may be replied that the particles of these bodies are in turn composed of still smaller

partial waves represented here by circumferences the semi-diameters of which

stances had been of the same penetrability

Now all these circumferences have for a common tangent the straight line BN namely the same line which is drawn as a tangent from the point B to the circumference SNR which we considered first For it is easy to see that all the

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which

terminates the movement that the wave AC has communicated within the transparent body and where this movement occurs in much greater amount than anywhere else And for that reason this line in accordance with what has been said more than once is the propagation of the wave AC at the moment when its piece C has reached B For there is no other line below the plane AB which is like BN a common tangent to all these partial waves And if one would know how the wave AC has come progressively to BN it is necessary only to draw

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is towards AF therefore also the sine of the angle DAE will be to the sine of the angle NAF the same as the said velocities of light

To see consequently what the refraction will be when the waves of light pass into a substance in which the movement travels

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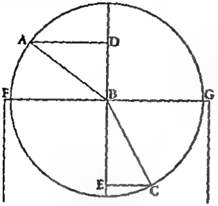
wave

from the points of intersection upon the straight line DE which are called the sines of the angles ABD CBE have a certain ratio between themselves which ratio is always the same for all inclinations of the incident ray at least for a given transparent body This ratio is in glass very nearly as 3 to 2 and in water very nearly as 4 to 3 and [redacted] likewise different in other diaphanous bodies

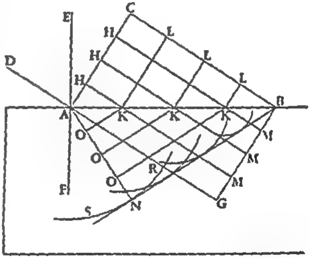


Another property similar to this is that the refractions are reciprocal between the rays entering into a transparent body and those which are leaving it That is to say that if the ray AB in entering the transparent body [redacted] refracted into BC then likewise CB being taken as a ray in the interior of this body will be refr... ing out into BA

let

... when I say plane that

... p rfect even ness but such as has been understood in treating of reflexion and for the same reason Let the line AC represent a portion of a wave of light the centre of which is supposed so distant that this portion may be considered as a straight line The piece C then of the wave AC will in a certain space of time have advanced as far as the plane AB following the straight line CB which may be imagined as coming from the luminous centre and which consequently will cut AC at right angles Now in the same time the piece A would have come to G along the straight line AG equal and parallel to CB and all the portion of wave AC would be at GB if the matter of the transparent body transmitted the movement of the wave as quickly as the matter of the ether But let us suppose that it transmits this movement less quickly by one third for instance Movement will then be spread from the point A in the matter of the transparent body through a distance equal to two-thirds of CB making its own particular spherical wave according to what has been said before This wave is then represented by the circumference SNR the centre of which [redacted] A and its semi-diameter equal to two-thirds of CB Then if one considers in order the other pieces H of the wave AC it appears that in the same time that the piece C reaches B they will not onl ... t the surface AB ... tion they



circles having radii equal to ⅔ of the lengths LB to which they correspond For all these circles will be enclosed in one another and will all pass beyond the point B

Now it is to be remarked that from the moment when the angle DAQ is
— d — DA to pass into the other
on which occurs at the
to realize by experi
ment with a triangular prism and for this our theory can afford this reason When the angle DAQ is still large enough to enable the ray DA to pass it is evident that the light from the portion AC of the wave is collected in a minimum space when it reaches BN It appears also that the wave BN becomes o

AC is entirely reduced to the same point B Similarly when the piece H has

re-enforced the partial waves which produce the interior reflexion against the
— explained
nce DAQ causes the
(for this angle being
49 degrees 11 minutes in the glass the angle BAN is still 11 degrees 21 minutes and the same angle being reduced by one degree only the angle BAN is reduced to zero and so the wave BN reduced to a point) thence it comes about that the interior reflexion from being obscure becomes suddenly bright so soon as the angle of incidence is such that it no longer gives passage to the refraction

Now as concerns ordinary external reflexion that is to say which occurs when the angle of incidence DAQ is still large enough to enable the refracted ray to penetrate beyond the surface AB this reflexion should occur against the particles of the substance which touches the transparent body on its outside And it apparently occurs against the particles of the air or others mingled with the ethereal particles and larger than they So on the other hand the external reflexion of these bodies occurs against the particles which compose them and which are also larger than those of the ethereal matter since the latter flows in their interstices It is true that there remains here some difficulty in those experiments in which this interior reflexion occurs without the particles of air being able to contribute to it as in vessels or tubes from which the air has been extracted

Experience moreover teaches us that these two reflexions are of nearly equal force and that in different transparent bodies they are so much the stronger as the refraction of these bodies is the greater Thus one sees manifestly that the reflexion of glass is stronger than that of water and that of diamond — —

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AC will have advanced as far as BN so that BC the nor
to AN the perpendicular on BN is
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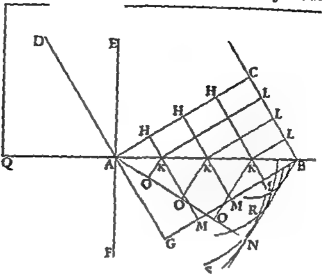
namely that if NA



fall upon 1

is equal to
⅔ of AB or is g cannot form one side of the triangle ANB since it becomes equal to or greater than AB so that the portion of wave BN cannot be found anywhere neither consequently can AN which ought to be perpendicular to it And thus the incident ray DA does not then pe the surface AB

Wh
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tc as it is very nearly in
w s angle DAQ must exceed 41 degrees 24 minutes And this accords perfectly with experiment

But it might here be asked since the meeting of the wave AC
surface AB ought to prod
side why does no
what has been sai ates an infinitude of partial waves in the matter which is at the other side of AB these waves never have a common tangent line (either straight or curved) at the same moment and so there no line terminating the propagation of the wave AC beyond the plane AB nor any place where the movement is gathered together in sufficiently great quantity to produce light And one will easily see the truth of this namely that CB being larger than ⅔ of AB the waves excited beyond the plane AB will have no common tangent if about the centres K one then draws

Here BL and KM are the sines of angles BKL KBM that is to say of the angles PBA QBC and therefore they are to one another as the velocity of light in the medium A is to the velocity in the medium C Then the time along LB is equal to the time along KM and since the time along BC is equal to the time alor [illegible]
But the [illegible]
AKN is [illegible]
alon AKC will exceed by as much more the time along ABC Hence it appears that the time along ABC is the shortest possible which was to be proven

CHAPTER FOUR

On the Refraction of the Air

WE have shown how the movement which constitutes light spreads by spherical waves in any homogeneous matter And it is evident that when the matter is not homogeneous but of such a constitution that the movement is communicated in it more rapidly toward one side than toward another these waves cannot be spherical but that they must acquire their figure according to the different distances over which the successive movement passes in equal times

It is thus that we shall in the first place explain the refractions which occur in the air which extends from here to the clouds and beyond The effects of which refractions are very remarkable for by them we often see objects which the rotundity of the earth ought otherwise to hide such as islands and the tops of mountains when one is at sea Because also of them the sun and the moon appear as risen before in fact they have and appear to set later so that at times the moon has been seen eclipsed while the sun appeared still above the horizon And so also [illegible]
the stars always a[illegible]
these same refract[illegible] is one experiment which renders this refraction very evident which is that of fixing a telescope on some spot so that it views an object such as a steeple or a house at a distance of half a league or more If then you look thro[illegible]
the day leavin[illegible] it always fixed [illegible]
spots of the object will not alwa[illegible]
telescope but that generally in the morning and in the evening when there are more vapours near the earth these objects seem to rise higher so that the half or more of them will no longer be visible and so that they seem lower toward mid-day when these vapours are dissipated

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wise at the plane
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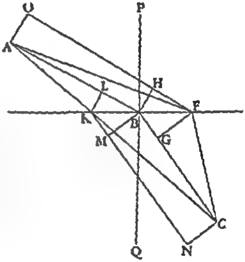
air But he assumed
I sines which we have just proved by these dif
f nt degrees of velocity alone or rather what is equivalent he assumed not only that the velocities were different but that the light took the least time possible for its passage and thence deduced the
His demonstration
of letters of M Desc ere another which is simpler and easier

Let KF be the plane
t
t
t by B to C having been refracted at B according to the law demonstrated a little before that is to say that having drawn PBQ which cuts the plane at right angles let the sine of the angle ABP have to the sine of the angle CBQ the same ratio as the velocity of light in the medium where A is to the velocity of light in the medium where C is It is to be shown that the time of passage of light along AB and BC taken together is the shortest that can be Let us assume that it may have come by other lines and in the first place along AΓ FC so that the point of refraction Γ may be farther from B than the point A and let AO be a line perpendicular to AB and ΓO parallel to AB BH perpendicular to ΓO and FG to BC



Since then the angle HBF is equal to PBA and the angle BΓC equal to QBC it follows that the sine of the angle HBΓ will also have the same ratio to the sine of BΓG as the veloc its velocity in
we take BΓ as
to one anoth r
t and there
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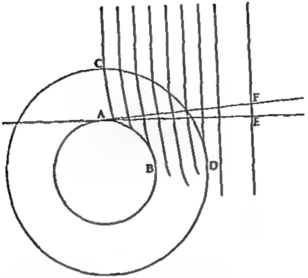
the interior
OH theref
the time al n the time along OΓC will be long long ABC But AΓ is longe n the time along AFC will by ju

Now let us assume KC the point of refraction K an the point B is and let CN be the perpendicular upon BC KN parallel to BC BM perpendicular upon KN and KL upon BA



spectator at A, its region C will be the farthest advanced and the straight line AF which intersects this wave at right angles and which determines the ap-

be perceived in the line AF by refraction. But this angle EAF is scarcely ever

to be seen, although the spot from which it is viewed is always the same But the reason for this effect will be still more evident from what we are going to remark touching the curvature of rays It appears from the things explained above that the progression or propagation of a small part of a wave of light is properly what one calls a ray Now these rays instead of being straight as they are in homogeneous media ought to be curved in an atmosphere of unequal penetrability For they necessarily follow from the object to the eye the line

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curve does not hinder the point A from being seen Now according as the air near the earth exceeds in density that which is higher the curvature of the ray AEB becomes greater so that at certain times it passes above the summit E, which allows the point A to be perceived by the eye at B and at other times it is intercepted by the same tower E which hides A from this same eye

up higher the density of air diminishes in proportion Now whether the particles of water and those of air take part by means of the particles of ethereal matter in the movement which constitutes light but have a less prompt recoil than these or whether the encounter and hindrance which the e particles of air and water offer to the propagation of movement of the ethereal progress retard the progression it follows that both kinds of particles flying amidst the ethereal particles must render the air from a great height down to the earth gradually less easy for the spreading of the waves of light

Whence the configuration of the waves ought to become nearly such as this

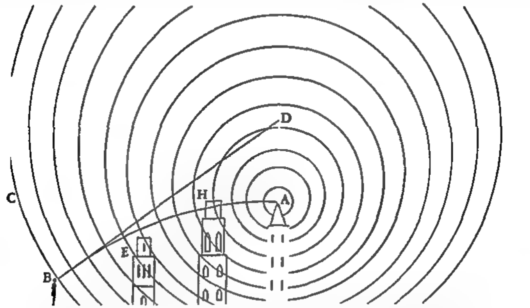
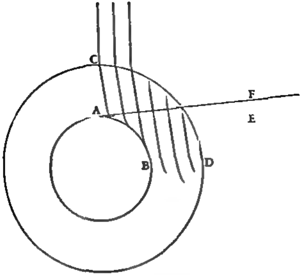


figure represents namely if A is a light or the visible point of a steeple the waves which start from it ought to spread more widely upwards and less widely downwards but in other directions more or less as they approximate to these two extremes This being so it necessarily follows that every line intersecting one of these waves at right angles will pass above the point A always excepting the one line which is perpendicular to the horizon

Let BC be the wave which brings the light to the spectator who is at B and let BD be the straight line which inter ects this wave at right angles Now because the ray or straight line by which we judge the spot where the object appears to us is nothing else than the perpendicular to the wave that reaches our eye as will be understood by what was said above it is manifest that the point A will be perceived as being in the line BD and therefore higher than in fact it is

Similarly if the earth be AB and the top of the atmosphere CD which probably is not a well defined spherical surface (since we know that the air



spectator at A its region C will be the farthest advanced and the straight line AF which intersects this wave at right angles and which determines the apparent place of the sun will pass above the real sun which will be seen along the line AE. And so it may occur that when it ought not to be visible in the ab ence of vapours because the line AE encounters the rotundity of the earth it will be perceived in the line AF by refraction But this angle EAF is carcely ever more than half a degree because the attenuation of the vapours alters the waves of light but little Furthermore these refractions are not altogether con stant in all weathers particularly at mall elevations of 2 or 3 degrees which results from the different quantity of aqueous vapours rising above the earth

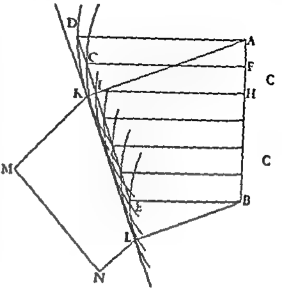
And this same thing is the cause why at certain times a distant object will be hidden behind another less distant one and yet may at another time be able to be see...

... homogeneous media ought to be curved in an atmosphere of unequal penetrability For they necessarily follow from the object to the eye the line which intersect...

figure [p...

which det...

seeing the object For although the point of the steeple A appears raised to D it would yet not appear to the eye B if the tower H was between the two because it crosses the curve AEB But the tower E which is beneath this curve does not hinder the point A from being seen Now according as the air near the earth exceeds in density that which is higher the curvature of the ray AEB becomes greater so that at certain times it passes above the summit E, which allows the point A to be perceived by the eye at B and at other times it is intercepted by the same tower E which hides *A from this same eye*



But to demonstrate this curvature of the rays conformably to all our preceding theory let us imagine that AB is a small portion of a wave of light coming from the side C which we may consider as a straight line Let us also suppose that it is perpendicular to the horizon the portion B being nearer to the earth than the portion A and that because the vapours are less hindering at A than at B the particular wave which comes from the point A spreads through a certain space AD while the particular wave which starts from the point B spreads through a shorter space BE AD and BE b

lines FG

line AB an BE (which is straight or may be considered as such) let the different penetrabilities at the different heights in the air between A and B be represented by all these lines so that the particular wave originating from the point F will spread across the space FG and that from the point H across the space HI while that from the point A spreads across the space AD

Now if about the centres A B one describes the circles DK EL which represent the spreading of the waves which originate f om b

and if one draws the

to see that this s

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AI KL which will terminate the movement of the particular waves originating from the points of the wave AB and this movement will be stronger between the points KL th

instant

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has bee of the ordinary refraction Now it appears that AK and BL dip down toward the side where the air is less easy to penetrate for AK being longer than BL and parallel to it it follows that the lines AB and KL being prolonged would meet at the side L But the angle K is a right angle hence KAB is necessarily acute and consequently less than DAB If one investigates in the same way the progression of the part

KL one

ner that

And this the ray will continue along the curved line which intersects all the waves at right angles as has been said

CHAPTER FIVE

On the Strange Refractions of Iceland Crystal

1 There is brought from Iceland which is an island in the North Sea in the latitude of [illegible] degrees a kind of crystal or transparent stone very remarkable for its figure and other qualities but above all for its strange refractions The causes of this have seemed to me to be worthy of being carefully investigated the more so because amongst transparent bodies this one alone does not follow the ordinary rules with respect to rays of light I have even been under some necessity to make this research because the refractions of this crystal seemed to overturn our preceding explanation of regular refraction which explanation on the contrary they strongly confirm as will be seen after they have been brought under the same principle In Iceland are found great lumps

[illegible]

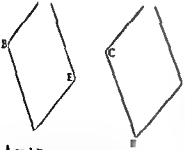
to Mr Eras-
[illegible] ent on of Iceland crystal and of its

[illegible]

and those which I have made for I have applied myself with great exactitude [illegible] before under

[illegible] hich it has of [illegible] f talc than of crystal For an iron pike effects an entrance into it as easily as into any other talc

[illegible] ave [illegible] ed each of the six faces being a parallelogram



[illegible] all the six faces are equal and similar rhombuses The figure here added represents a piece of this crystal The obtuse angles of all the parallelograms as C D here are angles of 101 degrees 52 minutes and consequently the acute angles such as A and B are of 78 degrees 8 minutes

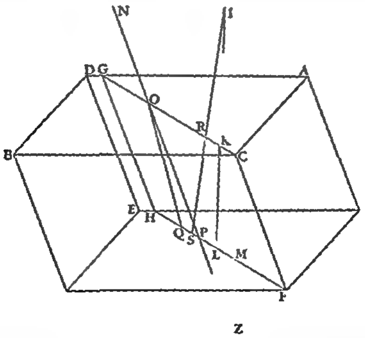
5 Of the solid angles there are two opposite to one another such as C and E, which are each composed of three equal obtuse plane angles The other six are composed of two acute angles and one obtuse All that I have just said has

been likewise remarked by Mr Bartholinus in the aforesaid treati ■ if we differ it is only slightly about the values of the angles He recounts moreover some other properties of this crystal to wit that when rubbed against cloth it attracts straws and other light things as do amber diamond glass and Spanish wax Let a piece be covered with water for ■ day or more the surface loses its natural polish When aquafortis is poured on it it produces ebullition especially as I have found if the crystal has been pulverized I have also found by experiment that it may be heated to redness in the fire without being in anywise altered or rendered less transparent but ■ very violent fire calcines it nevertheless Its transparency is scarcely less than that of water or of rock crystal and devoid of colour But rays of light pass through it in another fashion and produce those marvellous refractions the causes of which I am now going to try to explain reserving for the end of this treatise the statement of my conjectures touching the formation and extraordinary configuration of this crystal

6 In all other transparent bodies that we know there ■ but one sole and simple refraction but in this substance there are two different ones The effect ■ that objects seen through it especially such as are placed right against it appear double and that a ray of sunlight falling on one of its surfaces p rts itself into two rays and traverses the crystal th

7 It is again a mon

falls perpendicul t on without suffering refraction and tha ique ray is always refracted But in this crystal the perpendicular ray suffers refraction and there are oblique rays which pass through it quite straight



8 But in order to explain these phenomena more particularly let there be in the first place a piece ABΓΓ of the same crystal and let the obtuse angle ACB one of the three which constitute the equilateral solid angle C be di

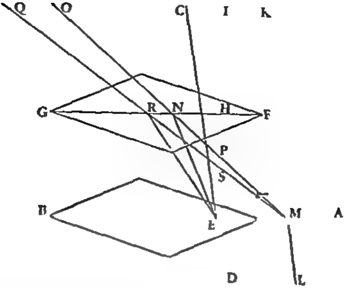
vided into two equal parts by the traight line CG and let it be conceived that the crystal is intersected by a plane which passes through this line and through the ... CF which plane will necessarily be perpendicular to the surface AB and its section in the crys al will form a parallelogram GCFH We will call this section the principal section of the crystal.

9 Now if one covers the surface AB leaving there only a small aperture at ... K situated in the traight line CG and if one exposes it to the sun ... ray IK will divide itself ... traight by KL and ... which is in the plane ... degrees 40 minutes ... ig from the other side ... Z And ... in this ex traordinary refraction the point M is seen by ... ed ray MKI which I consider as going to the eye at I it necessarily follows that the point L by virtue of the same refraction will be seen by the refracted ray LRI so that LP will be parallel to MK if the distance from the eye KI is supposed very great The point L appears then as being in the straight line IRS but the same point appears also by ordinary refraction to be in the straight line IK hence it is necessarily judged to be double And similarly if L be a small hole in a sheet of paper or other substance which is laid against the crystal it will appear when turned towards daylight as if there were two holes which will seem the wider apart from one another the greater the thickness of the crystal

10 Again, if one turns the crystal in such wise that an incident ray NO of sunlight, which I suppose to be in the plane continued from GCFH makes with GC an angle of 73 degrees and 20 minutes and is consequently nearly ... CF ... with FH an angle of 70 degrees 57 min ... e end it will divide ... tinue along OP in a ... er side of the crystal without any refraction but the other will be refracted and will go along OQ And it must be noted that it is special to the plane through GCF and to those which are parallel to it that all incident rays which are in one of these planes continue to be in it ... they have entered the crystal and have become dou ble for it is quite otherwise for rays in all other planes which intersect the crystal, as we shall see afterwards.

11 I recognized at first by these experiments and by some others that of the two refractions which the ray suffers in this crystal, there is one which follows the ordinary rules and it is this to which the rays KL and OQ belong. This is why I have distinguished this ordinary refraction from the other and having measured it by exact observation, I found that its proportion, considered as to the sines of the angles which the incident and refracted rays make with the perpendicular was very precisely that of 5 to 3 as was found also by ... Bartholinus, and consequently much greater than that of rock crystal, or of glass which is nearly 3 to 2.

12 The mode of making these observations exactly is as follows. Upon a leaf of paper fixed on a thoroughly flat table there is traced a black line AB and two others, CED and KML, which cut it at right angles and are more or less distant from one another according as it is desired to examine a ray that is



more or less oblique Then place the crystal upon the intersection E so that the line AB concurs with that '
face or with some line parallel t
line AB it will appear single c
through the crystal and the portions which appear outside it meet together in a straight line but the line CD will appear double and one can distingui h the image which is due to regular refraction by the circumstance that when one views it with both eyes it seems raised up more than the other or again by the circumstance that when the crystal is turned around on the paper this image remains stationary whereas the other image shifts and moves entirely around Afterwards let the eye be placed at I (remaining always in the plane perpendicular through AB) so that it views the image which is formed by regular refraction of the line CD making a straight line with the remainder of that line which is outside the crystal And then marking on the surface of the crystal the point H where the intersection L appears this point will be directly above E Then draw back the eye towards O keeping always in the plane perpendicular through AB so that the image of the line CD which is formed by ordinary refraction may appear in a straight line with the line KI viewed without refraction and then mark on the crystal the point N where the point
13
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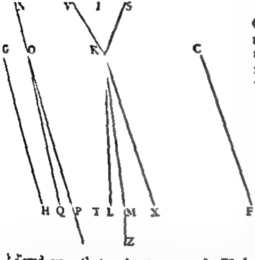
of the refraction will be that of I N to NP becau e the e lines are to one an other as the sines of the angles NPH NI P which are equal to tho e which the incident ray ON and its refraction NE mal e with the perpendicular to the surface This proportion as I have said is sufficiently precisely as 5 to 3 and is always the same for all inclinations of the incident ray

14 The same mode of ob ervation has also served me for examining the extraordinary or irregular refraction of this cry tal For the point H having been found and marked as aforesaid directly above the point E I observed the appearance of the line CD which is made by the extraordinary refraction

having placed the eye at Q so that this appearance made a straight line with the line KL viewed without refraction I ascertained the triangles REH ... the angles RSH REs which the incident and the



ratio of FR to RS was not constant varied with the varying obliquity of the incident ray

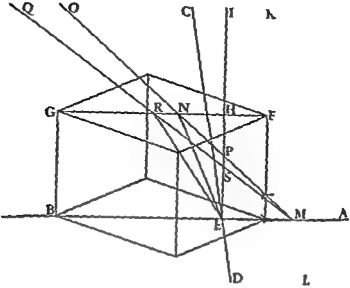
16 I found also that when QRE made a straight line that is when the incident ray entered the crystal without being refracted (as I ascertained by the circumstance that then the point E viewed by the extraordinary refraction appeared in the line CD as seen without refraction) I found I say then that the angle QRG was 3 degrees 20 minutes as has been already remarked and so it is not the ray parallel to the edge of the crystal which crosses it in a straight line without being refracted as Mr Bartholinus believed, since that inclination is only 70 degrees 5 minutes as was stated above. And this is to be noted in order that no one may search in vain for the cause of the singular property of this ray in its parallelism to the edges mentioned.

17 Finally continuing my observations to discover the nature of this refraction, I learned that it obeyed the following remarkable rule Let the parallelogram GCFH made by the principal section of the crystal as previously de-

perpendicular ray Ih falls and this occurs also for refractions in other sections of this crystal. But before speaking of those which have also other particular properties we will investigate the causes of the phenomena which I have already reported.

It was after having explained the refraction of ordinary transparent bodies by means of the spherical emanations of light as above that I resumed my examination of the nature of this crystal wherein I had previously been unable to discover anything

18 As there were two different refractions I conceived that there were also two different emanations of waves of light and that one could occur in the ethereal matter extending through the body of the crystal Which matter being present in much larger quantity than is that of the particles which compose it was alone capable of causing transparency according to what has been explained heretofore I attributed to this emanation of waves the regular re-



more or less oblique Then place the crystal upon the intersection E so that the line AB concurs with that which bisects the obtuse angle of the lower sur face or with some line parallel to it Then by placing the eye directl o line AB it will appear single only nd on l through the —
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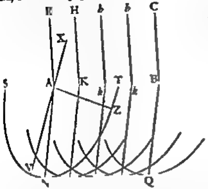
the circ u on the paper this image r ueleas the other image shifts and moves entirely around Afterwards let the eye be placed at I (remaining always in the plane perpendicular through AB) o that it views the image which is formed by regular refraction of the line CD making a straight line with the remainder of that line which is outside the crystal And then marking on the urface of the crystal the point H where the intersection E appears this point will be directly above E Then draw back the eye towards O keeping always in the plane perpendicular through AB so that the image of the line CD which is formed by ordinary refraction m he line KL

HE w being traced sepa tel

(us sines of the angles NPH NEP are equal to those which the incident ray ON and its refraction NE make with the perpendicular to the surface This proportion as I have said is sufficiently precisely as 5 to 3 and is always the same for all inclinations of the incident ray

14 The same mode of ob ervation has also served me for examining the extraordinary or irregular refraction of this cry tal For the point H having been found and marked as aforesaid directly above the point E I observed the appearance of the line CD which is made by the extraordinary refraction

falls perpendicularly on the flat surface of a transparent body in which they should spread in this manner I took AB for the exposed region of the surface And, since a ray perpendicular to a plane and coming from a very distant source of light is nothing else according to the precedent theory than the incidence of a portion of the wave parallel to that plane I supposed the straight line RC parallel and equal to AB to be a portion of a wave of light



the hemispherical partial waves which in a body of ordinary refraction would spread from each of these last point as we have above explained in treating of refraction, these must here be hemi-spheroids The axes (or rather the major diameters) of these I supposed to be oblique to the plane AB as is AV the semi axis or semi major diameter of the spheroid SVT which represents the partial wave coming from the point A after the wave RC has reached AB I say axis or major diameter because the same ellipse SVT may be considered as the section of a spheroid of which the axis is AZ perpendicular to AV But for the present without yet deciding one or other we will consider these spheroids only in those sections of them which make ellipses in the plane of this figure Now taking a certain space of time during which the wave SVT

occurs in much greater amount than anywhere else being made up of arcs of an infinity of ellipses the centres of which are along the line AB

24 Now it appeared that this common tangent NQ was parallel to AB and of the same length but that it was not directly opposite to it since it was comprised between the lines AN BQ which are diameters of ellipses having

tion on entering a transparent body seeing that the wave RC having come

face DC and passes through the edge CF another perpendicular to the face BF passing through the edge CA and the third perpendicular to the face AF

fraction which is observed in this stone by supposing these waves to be ordinarily of spherical form and having a slower progression with ... than they have outside it whence ...

19 ...

I w ... would do anc ... spread indifferently both in the ethereal matter diffused throughout the crystal and in the particles of which it is composed according to the last mode in which I have explained transparency It seemed to me that the disposition or regular arrangement of these particles co... tribute to form spheroidal waves (not ... that the ... one d ...

... at the end of this t ... conjectures and some experiments which confirm them

20 The double emission of waves of light which I had imagined became more probable to me after I had observed a certain phenomenon in the ordinary [rock] crystal which occurs in hexagonal form and which because of this regularity seems also to be composed of particles of definite figure and ranged in order This was that this crystal as well as that from Iceland has a double refraction though less evident For having had cut from it some well polished prisms of different sections I remarked in all in viewing through them the flame of a candle or the lead of window panes that everything appeared double though with images not very distant from one another Whence I understood the reason why this substance though so transparent is useless for telescopes when they have ever so little length

21 Now this double refraction according to ... hereinbefore established seem... ...ght both of them spherical ... one series a little slower on ... thus the phenomenon ... quite naturally explained by postulating substance ... I ... a ... same body And since it m... have been objected that in composing these two kinds of crystal of equal particles of a certain figure regularly piled the interstices which these particles leave and which contain the ethereal matter would scarcely suffice to transmit the waves of light which I have localized there I removed this difficulty by regarding these particles as being of a very rare texture or rather as composed of other much smaller particles between which the ethereal matter passes quite freely This moreover necessarily follows from that which has been already demonstrated touching the small quantity of matter of which the bodies are built up

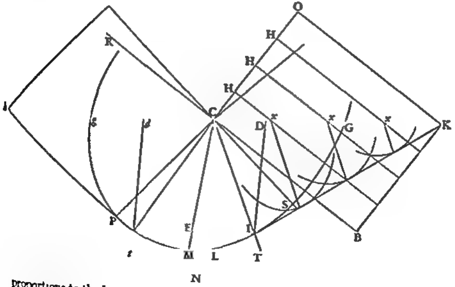
22 Supposing then these spheroidal waves besides the spherical ones I began to examine whether they could serve to explain the phenomena of the irregular refraction and how by these same phenomena I could determine the figure and position of the spheroids as to which I obtained at last the desired success by proceeding as follows

23 I considered first the effect of waves so formed as respects the ray which

about its smaller diameter I found also the value of CG the semi-diameter parallel to the tangent ML to be 98 779

25 Now passin to the investigation of the refractions which obliquely inci dent rays must undergo according to our hypothesis of pheroidal waves I saw that these refractions depended on the ratio between the velocity of move-

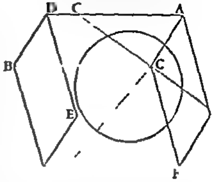
sphere the semi-diameter of which i equal to the line N which will be deter mined hereafter the following is the way of finding the refraction of the inci dent rays. Let there be such a ray RC falling upon the surface CK. Make CO perpendicular to RC and across the angle KCO adjust OK equal to N and perpendicular to CO then draw KI which touches the ellipse GSP and from the point of contact I join IC which will be the required refraction of the ray PC The demonstration of this i it will be seen entirely similar to that of which we made use in explaining ordinary refraction For the refraction of the ray PC is nothing else than the progression of the portion C of the wave CO



proportions to the l ... to CO

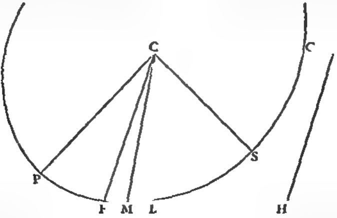
...ction. ... has been demonstrated in ordinary

passing through the edge BC I knew that the refractions of the incident rays belonging to these three planes were all similar But there could be no position of the spheroid which would have the same relation to the three sections except that in which the axis was also the axis of the solid angle C Consequently I saw that the axis of this angle that is to say the straight line which traversed the crystal from the point C with equal inclination to the edges CF CA CB was the line which determined the position of the axis of all the spheroidal waves which one imagined to originate from some point taken within or on the surface of the crystal since all these spheroids ought to be alike and have their axes parallel to one another



26 Considering after this the plane of one of these three sections namely that through GCF the angle of which is 109 degrees 3 minutes since the angle F was shown above to be 70 degrees 57 minutes and imagining a spheroidal wave about the centre C I knew because I have just explained it that its axis must be in the same plane the half of which axis I have marked CS in the next figure and seeking by calculation (which will be given with others at the end of this discourse) the value of the angle CGS I found it 45 degrees 20 minutes

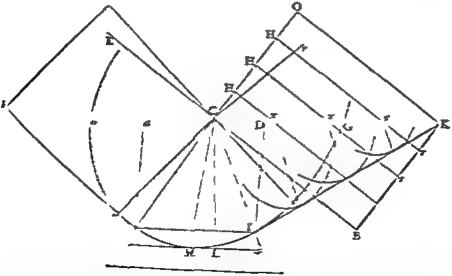
27 To know from this the form of this spheroid that is to say the proportion of the semi diameters CS CP of its elliptical section which are perpendicular to one another I considered that the point M where the ellipse is touched by the straight line FH parallel to CG ought to be so situated that CM makes with the perpendicular CL an angle of 6 degrees 40 minutes since this being so this ellipse satisfies what has been said about the refraction of the ray perpendicular to the surface CG which is inclined to the perpendicular CL



by the same angle This then being thus disposed and taking CM at 100 000 parts I found by the calculation which will be given at the end the semi major diameter CP to be 105 032 and the semi axis CS to be 93 410 the ratio of which numbers is very nearly 9 to 8 so that the spheroid was of the kind which resembles a compressed sphere being generated by the revolution of an ellipse

about its smaller diameter. I found also the value of CG the semi-diameter perpendicular to the tangent ML to be 98779.

28. Now passing to the investigation of the refractions which obliquely incident rays must undergo, according to our hypothesis of spheroidal waves, I saw that these refractions depended on the ratio between the velocity of movement of the light outside the crystal in the ether, and that within the crystal. For supposing, for example, this proportion to be such that while the light in the crystal forms the spheroid GSP, as I have just said, it forms outside a sphere the semi-diameter of which is equal to the line N which will be determined hereafter, the following is the way of finding the refraction of the incident rays. Let there be such a ray RC falling upon the surface CK. Make CO perpendicular to RC, and across the angle KCO adjust OK, equal to N and perpendicular to CO; then draw KI which touches the ellipse GSP, and from the point of contact I join IC, which will be the required refraction of the ray RC. The demonstration of this is, it will be seen, entirely similar to that of which we made use in explaining ordinary refraction. For the refraction of the ray RC is nothing else than the progression of the portion C of the wave CO, continued in the crystal. Now the portions H of this wave, during the time that O comes to K, will have arrived at the surface CK along the straight lines Hx, and will moreover have produced in the crystal around the centres x some hemi-spheroidal partial waves similar to the hemi-spheroidal GSP, and similarly disposed, and of which the major and minor diameters will bear the same



proportion to the lines xv (the continuations of the lines Hx up to KB parallel to CO) that the diameters of the spheroid GSP bear to the line CB, or N. And it is quite easy to see that the common tangent of all these spheroids, which are here ellipses, will be the straight line IK, which consequently will be the propagation of the wave CO; and the point I will be that of the point C, conformably with that which has been demonstrated in ordinary refraction.

Now as to finding the point of contact I it is known that one must find CD a third proportional to the lines CK CG and draw DI parallel to CM previously determined which is the conjugate diameter to CG for then by drawing KI it touches the ellipse at I

29 Now as we have found CI the refraction of the ray RC similarly one will find Ci the refraction of the ray rC which comes from th ... by making Co perpendicular to C ... t on ...

t ...

te ... prolongs the ... at the tangent ML at T and t the distances ... Mt will also be equal And so by our hypothesis we explain perfectly the phenomenon mentioned above to wit that when th ... rays equally inclined but coming from ... the ... ray ... divergences in the direction ... surface of the crystal

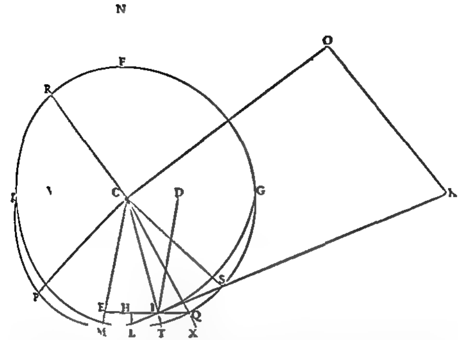
30 To find the length of the line N in proportion to CP CS CG it must be determined by observations of the irregular refraction which occurs ... section of the crystal and I find th ... le ... of ... di ... thus pr ... may be called the *Proportion of ... tion* similarly as in glass that of 3 to 2 as will be manifest when I shall have explained a short process in the preceding way to find the irregular refractions

31 Supposing then in the next figure as previously the surface of the crystal gG the ellipse GPg and the line N and CM the refraction of the perpendicular ray FC from which it diverges by 6 degrees 40 minutes Now let there be some other ray RC the refraction of which must be found

About the centre C with semi diameter CG let the circumference gRG be described cutting the ray RC at R and let RV be the perpendicular on CG Then as the line N is to CG let CV be to CD and let DI be drawn parallel to CM cutting the ellipse gMG at I then joining CI this will be the required refraction of the ray RC Which is demonstrated thus

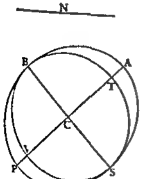
Let CO be perpendicular to CR an ... ted ... I ... e ... CI is the refraction of the ray RC ... ow since the angle RCO is a right angle it is easy to see that the right-angled triangles RCV KCO are similar As then CK is to KO so also is RC to CV But KO is equal to N and RC to CG then as CK is to N so will CG be to CV But as N is to CG so by ... CG so is CG to CD And because ... to CG it follows that KI touches ... which remained to be shown

32 One ... ees then that as there is in the refraction of ordinary media a



certain constant proportion between the sines of the angles which the incident ray and the refracted ray make with the perpendicular so here there is such a proportion between CN and CD or IE that is to say between the sine of the angle which the incident ray makes with the perpendicular and the horizontal intercept in the ellipse between the refraction of this ray and the diameter CM For the ratio of CN to CD is as has been said the same as that of N to the semi-diameter CG

I will add here before passing away that in comparing together the regular and irregular refraction of this crystal there is this remarkable fact



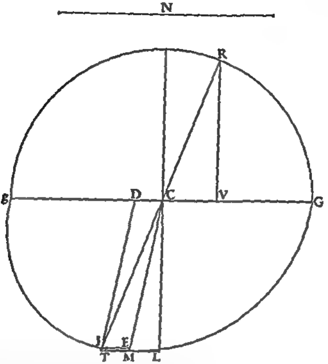
of time of the light which serves for the regular refraction

For we have stated before this that the line N being the radius of a spherical wave of light in air while in the crystal it spread through the spheroid ABPS the ratio of N to CS will be 156 96 to 93 410 But it has also been stated that the proportion of the regular refraction was 5 to 3 that is to say that N being the radius of a spherical wave of light in air its extension in the crystal would in the same space of time form a sphere the radius of which would be to N as 3 to 5 Now 156 962 is to 93 410 as 5 to 3 less $\frac{1}{41}$ So that it is sufficiently nearly and may be exactly the

sphere BVST which the light describes for the regular refraction in the crystal while it describes the spheroid BPSV for the irregular refraction and while it describes the sphere of radius N in air outside the crystal

Although then there are accord n

one of these propagations
than the other but that they have an equal velocity in



the other direction namely in that parallel to the same axis BS which is also the axis of the obtuse angle of the crystal

34 The proportion of the refraction being what we have just seen I will now show that there necessarily follo

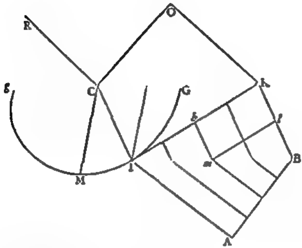
and that the ray RC
the same surface gG the angle RCG of 73 degrees 20 minutes inclining to the same side as the crystal (of
abo
one
ray is proved as
follo ation

CG or CR being as precedently 98 779 CM being 100 000 and the angle RCV 73 degrees 20 minutes CV will be 28 330 But because CI is the refraction of the ray RC the proportion of CV to CD is 156 962 to 98 779 namely that of N to CG then CD is 17 828

Now the rectangle gDC is to the square of DI as the square of CG is to the square of CM hence DI or CF will be 98 353 But as CI is to EI so will CM be to MT which will then be 18 127 And being added to MI which is 11 609 (namely the sine of the angle LCM which is 6 degrees 40 minutes taking CM

100 000 as radius) we get LT 27 936 and this is to LC 99,324 as CV to VR th is to say as 29 938 the tangent of the complement of the angle RCV which is 73 degrees 20 minutes is to the radius of the tables Whence it appears that PCIT is a strai ht line which was to be proved

35. Further it will be seen that the ray CI in emerging through the oppo-ite ... ow ... ob- ... nat



Let the same things be supposed as before that is to say let CO perpendicular to CR represent a portion of a wave the continuation of which in the crystal is IK, so that the piece C will be continued on along the traight line CI while O comes to K. Now if one takes a second period of time equal to the first the piece K of the wave IK will in this second period have advanced along the straight line KB equal and parallel to CI because every piece of the wave CO on arrivin at the surface CK ought to go on in the crystal the same as the piece C and in this ame time there will be formed in the air from the point I ...

passes the remainder lB there will tart from the point m a partial wave the semi-diameter of which mn will have the same ratio to lB as IA to KB Whence ...

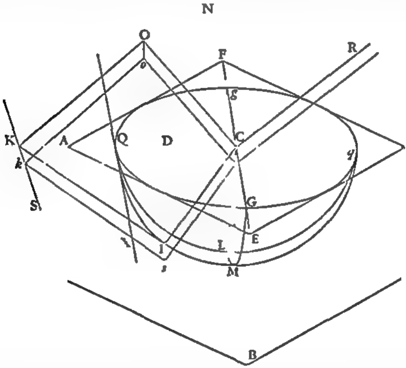
all the points of the wa e IK against the surface of the ether IB It is then precisely the tangent BA which will be the continuation of the wave IK, out side the crystal when the piece K has reached B And in consequence IA

which is perpendicular to BA will be the refraction of the ray CI on emerging from the crystal Now it is clear that IA is parallel to the incident ray RC since IB is equal to CK and IA equal to KO and the angles A and O are right angles

It is seen then that according to our hypothesis the reciprocal relation of refraction holds good in this crystal as well as in ordinary transparent bodies as is thus in fact found by observation

36 I pass now to the con ideration of other ections of the crystal and of the refractions there produced on which as will be seen some other very remarkable phenomena depend

Let ABH be a parallelepiped of crystal and let the top surface AEHF be a perfect rhombus the obtu e angles of which are equally divided by the straight line EF and the acute angles by the straight line AH perpendicular to FE



The section which we have hitherto considered is that which passes through the lines EF EB and which at the same time cuts the plane AFHF at right angles Refractions in this section have this in common with the refractions in ordinary media that the plane which is drawn through the incident ray and which also intersects the surface of the crystal at right angles is that in which the refracted ray al o is found But the refractions which appertain to every other ection of this crystal have this strange property that the refracted ray always quits the plane of the incident ray *perpendicular to the surface* and turns away towards the side of the slope of the cry tal For which fact we shall show the rea on in the first place for the section through AH and we shall show at the same time how one can determine the refraction according to our hypothesis Let there be then in the plane which pa ses through AH and which is perpendicular to the plane AFHE the incident ray RC it is required to find its refraction in the crystal

37 About the centre C which I suppose to be in the intersection of AH and FE, let there be imagined a hemi-spheroid QGgqM such as the light would
h eye l nd l t it section by the plane AEHF form

— — r —

that of 98 779 to 105 032

h n d ... th t m n

pe — — h h h l
CN
wil

it is certain according to what has been explained above Article 27 that a plane which would touch the spheroid at the point M where I suppose the straight line CM to meet the surface would be parallel to the plane QGq If
h

will necessarily be in the ellipse QMq because this plane through KS as well as the plane which touches the spheroid at the point M are parallel to QN the tangent of the spheroid for this consequence will be demonstrated at the end of this treatise Let this point of contact be at I then making KC QC DC proportionals draw DI parallel to CM also join CI I say that CI will be the required refraction of the ray RC This will be manifest if in considering CO which is perpendicular to the ray RC as a portion of the wave of light we can demonstrate that the continuation of its piece C will be found in the crystal at I when O has arrived at K

38 Now as in the chapter on reflexion in demonstrating that the incident and reflected rays are —
surface we considered
here consid h h

... crystal at KK all the points of the wave COoc will have arrived at the rectangle Kc along lines parallel to OK and from the points of their incidences there will originate beyond that in the crystal partial hemi-spheroids similar to th h ...

spheroids all those which have their centres along the line CK touch this plane in the line KI (for this is to be shown in the same way as we have demonstrated the refraction of the oblique ray ...

and all those which have their cent

KI in the line Ii all these being sim

the parallelogram Ki is that which

lelogram will be precisely the continuation of the wave COoc in the crystal when Oo has arrived at Kk because it forms the termination of the movement and because of the quantity of movement ...

where else and thus it appears that the pie

tinuation at I that is to say that the ray ...

From this it is to be noted that the proportion of the refraction for this section of the crystal is that of the line N to the semi diameter CQ by which one will easily find the refractions of all incident rays in the same ...

was there the same as

very nearly as 8 to 5

... to CQ the major semi-diameter of the spheroid that is to say as 156 962 to 105 032 very nearly as 3 to 2 but just a little less Which still agrees perfectly with what one finds by observation

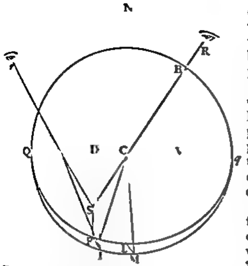
39 For the rest this diversity of proportion of refraction produces a very singular effect in this crystal which is that when it is placed upon a sheet of paper on which there are letters or anything else marked if one views it from above with the two eyes situated in the plane of the section through FI one sees the letters raised up by this irregular refraction more than when one puts one s eyes in the plane of section through AH and the difference of these elevations appears by comparison with the other ordinary refraction of the crystal ...

... position of the eyes namely when they are in the plane through AH these two stages are four times more distant from one another than when the eyes are in the plane through EF

We will show that this effect follows from the refractions and it will enable us at the same time to ascertain the apparent place of a point of an object placed immediately under the crystal according to the different situation of the eyes

40 Let us see first by how much the irregular refraction of the plane through AH ought to lift the bottom of the crystal Let the plane of this figure represent separately the section through Qq and CL in which section there is also the ray RC and let the semi-elliptic plane through Qq and CM be inclined to the former as previously by an angle of [illegible] degrees 10 minutes and in this plane CI is then the refraction of the ray RC

If now one considers the point I as at the bottom of the crystal and that it is viewed by the rays ICR Icr refracted equally at the points Cc which should ...



drawn the perpendicular IP which will lie at the bottom of the crystal the length SP will be the apparent elevation of the point I above the bottom

Let there be described on Qq a semicircle cutting the ray CR at B from which BV is drawn perpendicular to Qq and let the proportion of the refraction for this section be as before that of the line N to the semi-diameter CQ

Then as N is to CQ so is VC to CD as appears by the method of finding the refraction which we have shown above Article 31 but as VC is to CD so is VB to DS Then as N is to CQ so is VB to DS Let ML be perpendicular to CL And because I suppose the eye Rr to be distant about a foot or so from the crystal, and consequently the angle RSr very small VB may be considered as equal to the semi-d[illegible] m CQ d DP

so is CQ to DS

and CQ 10s 03

the com leme

supposed as ra

99 324 to 0,28 and so the elevation of the point I by the refraction of this section is known.

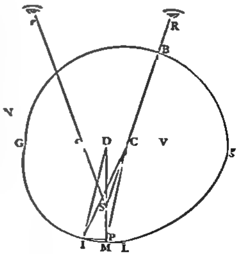
41. Now let there be represe ed b h

be o e the preceding one

des 27 and 28 which is

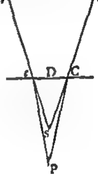
Let the point I taken in this emp e

be imagined again at the bottom of the crystal and let it be viewed by the refracted rays ICR Ir which go to the two eyes CR and or being equally inclined to the surface of the crystal Gg This being so if one draws ID parallel to CM which I suppose to be the refraction of the perpendicular ray incident at the point C the distances DC Dc will be equal as is easy to see by that which has been demonstrated in Article 28 Now it is certain that the point I should appear at S where the straight lines PC rc meet when prolonged and that this point will fall in the line



DP perpendicular to Gg If one draws IP perpendicular to this DP it will be the distance PS which will mark the apparent elevation of the point I Let there be described on Gg a semicircle cutting CR at B from which let BV be drawn perpendicular to Gg and let N to GC be the proportion of the refraction in this section as in Article 28 Since then CI is the refraction of the radius BC and DI is parallel to CM VC must be to CD as N to GC according to what has been demonstrated in Article 31 But as VC is to CD so is BV to DS Let ML be *drawn perpendicular to CL* And because I consider again the eyes to be distant above the crystal BV is deemed equal to the semi diameter CG and hence DS will be a third proportional to the lines N and CG also DP will be deemed equal to CL Now CG consisting of 98 778 parts of which CM contains 100 000 N is taken as 156 962 Then DS will be 62 163 But CL is also determined and contains 99 324 parts as has been said in Articles 34 and 40 Then the ratio of PD to DS will be as 99 324 to 62 163 And thus one knows the elevation of the point at the bottom I by the refraction of this section and it appears that this elevation is greater than that by *the refraction of the preceding section* since the ratio of PD to DS was there as 99 324 to 70 283

But by the regular refraction of the crystal of which we have above said *that the proportion is* 5 to 3 the elevation of the point I or P from the bottom will be $\frac{2}{5}$ of the height DP as appears by this figure where the point P being viewed by the rays PCR *Pcr* refracted equally at the surface *Cc* this point must needs appear to be at S in the perpendicular PD where the lines RC *rc* meet when prolonged and one knows that the line PC is to CS as 5 to 3 since they are to one another as the sine of the angle CSP or DSC is to the sine of the angle SPC And because the ratio of PD to DS is deemed the same as that of PC to CS the two eyes R*r* being suppo ed very far above the crystal the elevation PS will thus be $\frac{2}{5}$ of PD



42 *If one takes a straight line AB for the thickness of* the crystal its point B being at the bottom and if one divides it at the points C D E according to the proportions of the elevations found making AE $\frac{2}{5}$ of AB AB to AC as 99 324 to 70 283 and AB to AD as 99 324 to 62 163 these points will divide AB as in this figure And it will be found that this agrees perfectly with experiment that is to say by placing the eyes above in the plane which cuts the crystal according to the shorter diameter of the rhombu the regular refraction will lift up the letters to E and one will see the bottom and the letters over which it is placed lifted up to D by the irregu-

he eyes above in the plane which cuts the diameter of the rhombus the regular re- E as before but the irregular refraction will make them at the same time appear lifted up only to C and in such a way that *the interval* CE *will be quadruple the interval* ED which one previously saw

43 I have only to make the remark here that in both the positions of the eyes the images caused by *the irregular refraction do not appear directly* below those which proceed from the regular refraction but they are separated from them by being more distant from the equilateral solid angle of the crystal

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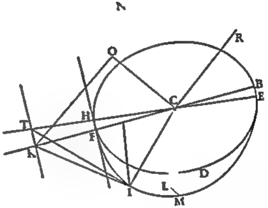
more of that point by the irregular refraction will a appear between the
two heights of D and C passing from one to the other as one turns one s self
and d wn from above And all
e himself
ons which
have con
there be
oid HME
a And let
d

BCK and having in the same plane through RC made CO perpendicular to CR let OK be adjusted across the angle OCK so as to be perpendicular to OC and equal to the line N which I suppose to measure the travel of the light in air during the time that it spreads in the crystal through the spheroid HDEM Then in the plane of the ellipse HDE let KT be drawn through the point K perpendicular to BCK Now if one conceives a plane



drawn through the straight line KT and touching the spheroid HME at I

let this point of contact be at H And having drawn a straight line along CH to meet KT at T let there be imagined a plane passing through the same CH and through CM (which I suppose to be the refraction of the perpendicular

is necessarily the point I which is ought since the plane drawn through TK can touch the spheroid at one point only And this point I is easy to determine

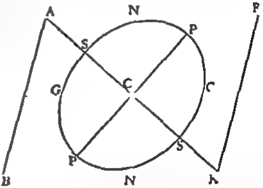
since it is needful only to draw from the point T which is in the plane of this ellip e the tangent TI in the way shown previously For the ellipse HMΓ is given and its conjugate semi diameters are CH and CM because a straight line drawn through M parallel to HE touches the ellipse HME as follows from the fact that a plane taken through M and parallel to the plane HDE touches the spheroid at that point M as is seen from Articles 27 and 23 For the rest the position of this ellipse with respect to the plane through the ray RC and through CK is also given from which it will be easy to find the position of CI the refraction corresponding to the ray RC

Now it must be noted that the same ellipse HME serves to find the refractions of any other ray which may be in the plane through RC and CK Becau e every plane parallel to the straight line HΓ or TK which will touch the spheroid will touch it in this ellipse according to the Lemma quoted a little before

I have investigated thus in minute detail the properties of the irregular *refraction of this crystal in order to see whether each phenomenon that is deduced from our hypothesis accords with that which is observed in fact* And

thereby produced give rise to refractions precisely such as they ought to be and as I had foreseen them according to the preceding theory

In order to explain what these sections are let ABKΓ be the principal section through the axis of the crystal ACK in which there will also be the axis SS of a spheroidal wave of light spreading in the crystal from the centre C and the straight line which cuts SS through the middle and at right angles namely PP, will be one of the *major diameters*



Now as in the natural section of the crystal made by a plane parallel to two opposite faces which plane is here represented by the line GG the refraction of the surfaces which are produced by it will be governed by the ... explained in the preceding ... a plane perpendicular to ... s will be governed by the ... P perpendicularly to the ... ht to be governed by the ... hat if the plane NN was ... angle NCG which is on ... oids NGN would ... N and GG were ... axis SS In con sequence ... -faces which the section through NN produces should effect the same refraction s as the surfaces of the section through GG And not only the surfaces of the section NN but

all other sections produced by planes which might be inclined to the axis at an angle equal to 45 degrees 20 minutes So that there are an infinitude of planes which ought to produce preci ely the same refractions as the natural surfaces of the crystal or as the section parallel to any one of those surfaces which are made by cleavage

I saw al.o that by cutting it by a plane taken through PP and perpendicular
t to be such that the perpen
id that for oblique rays there
ng from the regular and by
be less elevated than by that

other refraction

That similarly by cutting the crystal by any plane through the axis SS such as the plane of the figure is the perpendicular ray ought to suffer no refraction and that for oblique rays there were different measures for the ir regular refraction according to the situation of the plane in which the incident

do bt

That
quare

faces will each make an angle of 45 degrees 20 minutes with the axi

ne pendicular to the axis
triangular pri ms or prisms
ither the sides nor the bases
ey would yet all cause double
refraction for oblique rays The cube is included amongst these prisms the bases of which are sections perpendicular to the axis of the crystal and the sides are sections parallel to the same axis

From all this it further appears that it is not at all in the disposition of the layers of which this crystal seems to be composed and according to which it splits in three different senses that the cause resides of its irregular refraction and that it would be in vain to wish to seek it there

But in order that any one who has some of this stone may be able to find

depoli.hes the surfaces than makes them lucent

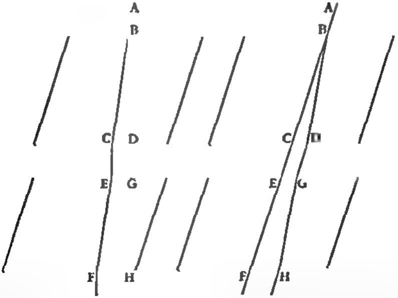
After many trials I have at last found that for this service no plate of metal mu.t be used but a p ece of mirror glass made matt and depolished Upon thi with fine sand and water one smooths the crystal little by little in the same way as spectacle glas.es and pol shes it simply by continuing the work but ever reducing the material I have not however been able to give it per

fect clarity and transparency but the evenness which the surfaces acquire enables one to observe in them the effects of refraction better than in those made by cleaving the stone which always have some inequality

Even when the surface is only moderately smoothed if one rubs it over with a little oil or white of egg it becomes quite transparent so that the refraction is discerned in it quite distinctly And this aid is specially necessary when it is wished to polish the natural surfaces to remove the inequalities because one cannot render them lucent equally with the surfaces of other sections which take a polish so much the better the less nearly they approximate to these natural planes

Before finishing the treatise on this crystal I will add one more marvellous phenomenon which I discovered after having written all the foregoing For though I have not been able till now to find its cause I do not for that reason wish to desist from describing it in order to give opportunity to others to investigate it It seems that it will be necessary to make still further suppositions besides those which I have made but these will not for all that cease to keep their prol

The phenom
them one over
if all the sides of one are parallel to those of the other then a ray of light such as AB is divided into two in the first piece namely into BD and BC following



the two refractions regular and irregular On penetrating thence into the other piece each ray will pass there without further dividing itself in two but that one which underwent the regular refraction as here DC will undergo again onl[illegible] regular refraction at GH and the other CF an irregular refraction at

the same plane without it being necessar[illegible]
be parallel Now it is marvellous why the rays CI and DG incident from the air on the lower crystal do not divide themselves the same as the first ray AB

a regular refraction

But in all the infinite other positions be ides those which I hav just tated the rays DG CE divide themselves anew each one into two by refraction in

depends on the position that one gives to the lower piece whether it divides them both in two or whether it does not divide them and yet how the ray AB above is always divided it seems that one is obliged to conclude that the waves

serve for the two species of refraction and when meeting the second crystal in another position are able to move only one of these kinds of matter But to tell how this occurs I have hitherto found nothing which sati fies me

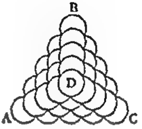
formed with certain *regular angles and figures Thus among flowers there are* many which have their leaves disposed in ordered polygons to the number of 3 4 5 or 6 sides but not more This well deserves to be investigated both as to the polygonal figure and as to why it does not exceed the number 6

Rock cry tal grows ordinarily in hexagonal bars and diamonds are found which occur with a square point and polished surfaces There is a species of small flat stones piled up directly upon one another which are all of pen ta

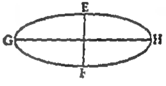
at re there operates But it is not no my intention to treat fully of this matter It seems that in general the regularity which occurs in the e produc

tions comes from the arrangement of the small invisible equal particles of which they are composed And coming to o I- - f there were a m
not

ot o)—I say t en the solid angle of the point D would be equal to the obtuse and equilateral angle of this crystal I say further that if these corpuscles were lightly stuck together on breaking this pyramid it would break along faces parallel to those that make its point and by this means as it is easy to see it would produce prisms similar to those of the same crystal as this other figure represents The reason is that

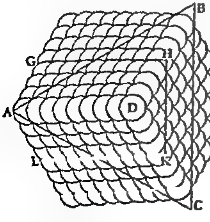


a whole layer
lay



en touches it on its flattened surface and the other two at the edges And the reason that the surfaces separate
of the neighbouring surface would
face which is being separated it wo for it to detach itself from six other spheroids which hold it locked and four of which press it by these flattened surfaces Since then not only the angles of our crystal but also the manner in which it splits agree precisely with what is observed in the assemblage composed of such spheroids there is great reason to believe that the particles are shaped and ranged in the same way

There is even probability enough that the prisms of this crystal are produced by the breaking up of pyramids since Mr Bartholinus relates that he occasionally found some pieces of triangularly pyramidal figure But when a mass is composed interiorly only of these little spheroids thus piled up whatever form it may have exteriorly it is certain by the same reasoning which I have just explained that if broken it would produce similar prisms It remains to be seen whether there are other reasons which confirm our conjecture and whether there are none which are repugnant to it



It may be objected that this crystal being so composed might be capable of cleavage in yet two more fashions one of which would be along planes parallel to the base of the pyramid that is to say to the triangle ABC the other would be parallel to a plane the trace of which is marked by the lines GH HK KL To which I say that both the one and the other though practicable are more difficult than those which were parallel to any one of the three planes of the pyramid and that

therefore when striking on the crystal in order to break it it ought always to split rather along these three planes than along the two others When one h mber of spheroids of the form above described and ranges them in a — — difficult For in

e each spheroid
uches upon their
cts at the edges
ers because each
same layer that

surround it since they only touch it at the edges so adheres readily to the neighbouring layer and the others to it for the same reason and this causes uneven surfaces Also one sees by experiment that when grinding down the crystal on a rather rough stone directly on the equilateral solid angle one verily finds much facility in reducing it in this direction but much difficulty afterwards in polishing the surface which has been flattened in this manner

pal l
heroid is
ere is one

only which touches it on the flattened surface and the other two at the edges only

However that which has made me know that in the crystal there are layers in this last fashion is that in a piece weighing half a pound which I possess one sees that it is split along its length as is the above-mentioned prism by the plane GHKL as appears by colours of the iris extending throughout this

scraping in the opposite sense an incision is easily made This follows mani

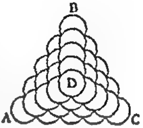
I will not undertake to say anything touching the way in which so many corpuscles all equal and similar are generated nor how they are set in such beautiful order whether they are formed first and then assembled or whether they arrange themselves thus in coming into being and as fast as they are produced which seems to me more probable To develop truths so recondite there would be needed a knowledge of nature much greater than that which we have I will add only that these little spheroids could well contribute to form the spheroids of the waves of light here above supposed these as well as those being similarly situated and with their axes parallel

Calculations which have been supposed in this chapter

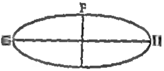
Mr Bartholinus in his treatise of this crystal puts at 101 degrees the obtuse angles of the faces which I have stated to be 101 degrees 52 minutes He states

tions comes from the arrangement of the small invisible equal particles of which they are composed And coming to ... f
there were ...
not ...

of 8)—I ... the solid angle of the point D would be equal to the obtuse and equilateral angle of this crystal I say further that if these corpuscles were lightly stuck together on breaking this pyramid it would break along faces parallel to those that make its point and by this means as it is easy to see it would produce prisms similar to those of the same crystal as this other figure represents The reason is th...



a whole layer ...
layer since e...
from the three ... next layer of which three there is but one which touches it on its flattened surface and the other two at the ...
the ...



f...
six other sph...
tened surface ... of our crystal but also the manner in which it splits agree precisely with what is observed in the assemblage composed of such spheroids there is great reason to believe that the particles are shaped and ranged in the same way

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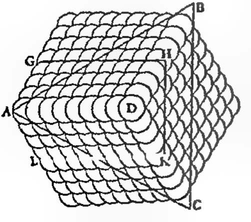

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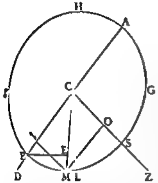
an angle GCS of 45 degrees 20 minutes —it was required to how I say that ... ameter of this ellipse is 105 032

tangent DM at D and Z and from the point of contact ... be drawn as perpendiculars to CP and CS Now because the angles SCP



6 degrees 40 minutes from LCA which is 45 degrees 20 minutes there remains MCP 38 degrees 40 minutes Considering then CM as a radius of 100 000 part MN the sine of 38 degrees 40 minutes will be 62 4 0 And in the right angled triangle MND MN will be to ND as the radius of the tables is to the tangent of 45 degrees 20 minutes (because the angle NMD is equal to DCL, or GCS) that is to say as 100 000 to 101 1 0 whence results ND 63 210 But NC is 8 0 9 of the same part CM being 100 000 because NC is the sine of the complement of the angle MCP which was 38 degrees 40 minutes Then the whole line DC is 141,289 and CP which is a mean proportional between DC and CN since MD touches the ellipse will be 105 032

Similarly because the angle OMZ is equal to CDZ or LCZ which is 44

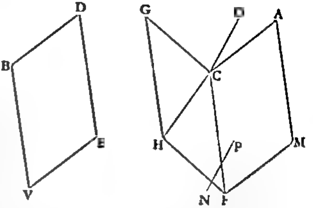
because it is equal to MN the sine of the angle MCP which is 38 degrees 40 minutes Then the whole line CZ is 1 9 655 and CS which is a mean proportional between CZ and CO will be 93 410

is 6 degrees 40 minutes And since the angle LCD is 45 degrees 20 minutes being equal to GCS the side LD is found to be 100 486 whence deducting ML 11 609 there will remain MD 88 877 Now as CD (which was 141 289) is to DM 88 877 so will CP 105 032 be to PE 66 070 But as the rectangle MEH (or rather the difference of the squares on CM and CE) is to the square on MC so is the square on PE to the square on Cg then also as the difference of the squares on DC and CP to the square on CD so also is the square on PE to the square on gC But DP CP and PE are known hence also one knows GC which is 98 779

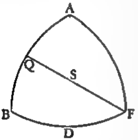
Lemma which has been supposed

If ...
wh
of
m

that he measured these angles directly on the crystal which is difficult to do with ultimate exactitude because the edges such as CA CB in this figure are generally worn and not quite straight For more certainty therefore I preferred to measure actually the obtuse angle by which the faces CBDA CBVF are inclined to one another namely the angle OCN formed by drawing CN perpendicular to FV and CO perpendicular to DA This angle OCN I found to be 105 degrees and its supplement CNP to be 75 degrees as it should be



To find from this the obtuse angle BCA I imagined a sphere having its centre at C and on its surface a spherical triangle formed by the intersection of three planes which enclose the solid angle C In this equilateral triangle which is ABF in this other figure I see that each of the angles should be 105 degrees namely equal to the angle OCN and that each of the sides should be of as many degrees as the angle ACB or ACF or BCF Having then drawn the arc FQ perpendicular to the side AB which it divides equally at Q the triangle FQA has a right angle at Q the angle A 105 degrees and F half as much namely 52 degrees 30 minutes whence the hypotenuse AF is found to be 101 degrees 52 minutes And this arc AF is the measure of the angle ACF in the figure of the crystal



In the same figure if the plane CGHF cuts the crystal so that it divides the obtuse angles ACB MHV in the middle it is stated in Article 10 that the angle CFH is 70 degrees 57 minutes This again is easily shown in the same spherical triangle ABF in which it appears that the arc FQ is as many degrees as the angle GCF in the crystal the supplement of which is the angle CFH Now the arc FQ is found to be 109 degrees 3 minutes Then its supplement 70 degrees 57 minutes is the angle CFH

It was stated in Article 26 that the straight line CS which in the preceding figure is CH being the axis of the crystal that is to say being equally inclined to the three sides CA CB CF the angle GCH is 45 degrees 20 minutes This is also easily calculated by the same spherical triangle For by drawing the other arc AD which cuts BF equally and intersects FQ at S this point will be the centre of the triangle And it is easy to see that the arc SQ is the measure of the angle GCH in the figure which represents the crystal Now in the triangle QAS which is right angled one knows also the angle A which is 52 degrees 30 minutes and the side AQ 50 degrees 56 minutes whence the side SQ is found to be 45 degrees 20 minutes

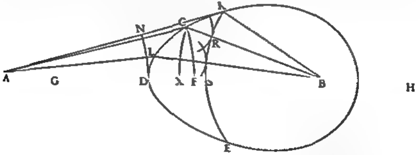
CHAPTER SIX

On the Figures of the Transparent Bodies

Which serve for refraction and for reflexion

AFTER having explained how the properties of reflexion and refraction follow from what we have supposed concerning the nature of light and of opaque bodies and of transparent media I will here set forth a very easy and natural way of deducing from the same principle the true figures which serve either by reflexion or by refraction to collect or disperse the rays of light as may be desired. For though I do not see yet that there are means of making use of these figures, so far as relates to refraction not only because of the difficulty of shaping the glasses of telescopes with the requisite exactitude according to these figures but also because there exists in refraction itself a property which hinders the perfect concurrence of the rays as Mr Newton has very well proved by experiment I will yet not desist from relating the invention since it offers itself so to speak of itself and because it further confirms our theory of refraction by the agreement which here is found between the refracted ray and the reflected ray Besides it may occur that some one in the future will discover in it utilities which at present are not seen

To proceed then to these figures let us suppose first that it is desired to find a surface CDE which shall reassemble at a point B rays coming from an other point A and that the summit of the surface shall be the given point D in the straight line AB I say that whether by reflexion or by refraction it is



only necessary to make this surface such that the path of the light from the point A to all points of the curved line CDE and from these to the point of concurrence (as here the path along the straight lines AC CB along AL LB and along AD DB) shall be everywhere traversed in equal times by which principle the finding of these curves becomes very easy

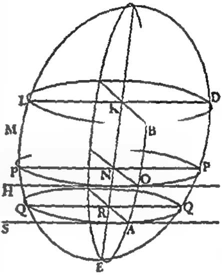
So far as relates to the reflecting surface since the um of the lines AC CB ought

ellipse

media

is the s ... n as the ratio of the sines in the refraction) it is

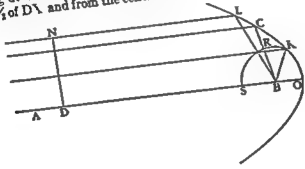
Let LED be the spheroid touched by the line BM ... by the planes parallel ... onstrate that the p ... centre through the ... which in cu ... QAQ which will all be similar and similarly disposed and will have their centres K N R in one and the same diameter of the spheroid which will also be the diameter of the ellipse made by the section of the plane that passes through the centre of the spheroid and which cuts the planes of the three said ellipses at right angles for all this is manifest by Proposition 15 of ... of Arch ... planes ... are drawn through the points O and A will also by cutting the planes which touch the spheroid in these same points generate straight lines as OH and AS which will as is easy to see be parallel to BM and all three BM OH AS will ...

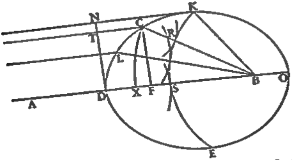


... are drawn the straight lines BK ON AR through the centres of the same ell... centres ... gents B... becau... ameters ... that their conjugate diameters BK ON AR will also be parallel And the centres K N R being as has been stated in one and the same diameter of the spheroid these parallels BK ON AR will necessarily be in one and the same plane which passes through this diameter of the spheroid and in consequence the points R O A are in one and the same ellipse made by the intersection of this plane Which was to be proved And it is manifest that the demonstration would be the same if besides the points O A there had been others in which the spheroid had been touched by planes parallel to the straight line BM

straight lines which are perpendicular to them that is to say which tend to the centre B (for that can be demonstrated in the same way as we have proved above that the pieces of spherical waves are propagated along the straight lines coming from their centre) and these progressions of the pieces of the waves constitute the rays themselves of light It appears then that all these rays tend here towards the point B

One might also determine the point C and all the others in this curve which serves for the refraction by dividing DA at G in such a way that DG is ⅔ of DA, and describing from the centre B any arc CX which cuts BD at X and another from the centre A with its semi-diameter AF equal to ⅔ of GX or rather having described as before the arc CX it is only necessary to make DF equal to ⅔ of DX and from the centre A to strike the arc FC for these two

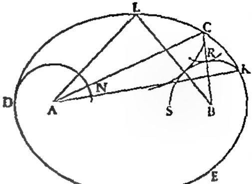




... come back to the first one which was ... he same ... t

Oval.

It is only a part of this oval which serves for the refraction ... DK, ending at K, if AK is the tangent. As to the other part Descartes has remarked that it could serve for reflexions if there were some material of a mirror of such a nature that by its means the force of the rays (or as we should say the velocity of the light which he could not say since he held that the m ement of light was instantaneous) could be augmented in the proportion of 3 to 2 But we have shown that in our way of explaining reflexion such a thing could not arise from the matter of the mirror and it is entirel im possible



only necessary to make DH equal to $\frac{3}{2}$ of DB and having after that described

equal to $\frac{2}{3}$ of FH and the point of in tersection of the two arcs will be one of the points required through which the curve should pass For this point having been found in this fashion it is easy forthwith to demonstrate that the time along AC CB will be equal to the time along AD DB

here more time in proportion as its speed is slo
AH will represent the time along AD DB Similarly the line AC or AF will
present the time along AC and FH being by construction equal to $\frac{3}{2}$ of CB

by revolution

such wise that they tend towards B let there be supposed a point K in the curve farther from D than C is but such that the straight line AK falls from outside upon the curve which serves for the refraction and from the centre B let the arc KS be described cutting BD at S and the straight line CB at R and from the centre A describe the arc DN meeting AK at N

Since the sums of the times along AK KB and along AC CB are equal if ... the time along KB and if from the other one

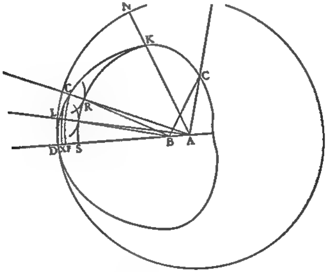
come along

made in the medium from the centre C a partial spherical wave ... semi-diameter equal to CR And this wave will necessarily touch the circumference KS at R since CB cuts this circumference at right angles Similarly having taken any other point L in the curve one can show that in the same time as the light passes along AL it will also have come along AL and in addition will have made a partial wave from the centre L which will touch the same circumference KS And so with all other points of the curve CDF Then ... the light reaches K the arc KRS will be the termination

A which wave may be ...
centre A But all the pieces of the arc KRS are propagated successively along

DA because they represent arcs of circles the centre of which is
f h perpendicular CX with the arc
h to pass
meet
ve at
that
sy as
of
to

are
the distance between
h two cases in which conic sections
by Descartes who first found
the



to DB is giv n of
ume as that which

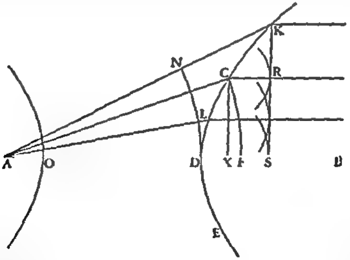
De cartes calls his th
Now the finding and construction of this second oval is the same as that of the first and the demonstration of its effect likewise But it i worthy of remark that n one case th oval becomes a perfect circle namely when the ratio of
h refractions here as 3 to 2 as I ob-
ing only for impossible reflexions

discovered these lines since he has

the surface generated by the revo u

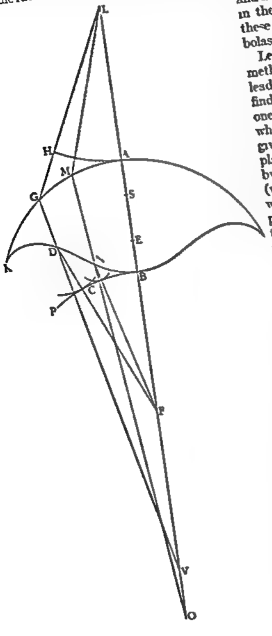
From what has been demonstrated ab ... to find the f... by supposing ... giving ... pse the construction of which differs in no way from that of the oval except that TC which previously was an arc of a circle is here a straight line perpendicular to DB For the wave of light DN being likewise represented by a tr... points of this wave travel ... DB will advance sub equ... the same time As for the ... on it is evident that it will here become a parabola since its focus A may be regarded as infinitely distant from the other B which is here the focus of the parabola towards which all the reflexions of rays parallel to AB tend And the demonstration of these effects is just the same as the preceding

But that this curved line CDE which serves for refraction is an ellipse and is such that its major diameter is to the distance between its foci as 3 to 2 which is the proportion of the refraction can be easily found by the calculus of algebra For DB which is given being called a its undetermined perpendicular DT being called x and TC y FB will be $a-y$ CB will be $\sqrt{xx+aa-2ay+yy}$ But the nature of the curve is such that $2/3$ of TC together with CB is equal to DB as was stated in the la... tion ...
$6/5ay$ ...
the r ... quare on TC Whence it is seen that DC is an ellipse of which the axis DO is to the parameter as 9 to 5 and therefore the square on DO is to the square of the distance between the foci as 9 to 9−5 that is to say 4 and finally the line DO will be to this distance as 3 to 2



Again if one supposes the point B to be infinitely distant in lieu of our first oval we shall find that CDE is a true hyperbola which will make those rays become parallel which come from the point A And in consequence also those which are parallel within the transparent body will be collected outside at the point A Now it must be remarked that CX and KS become straight lines per

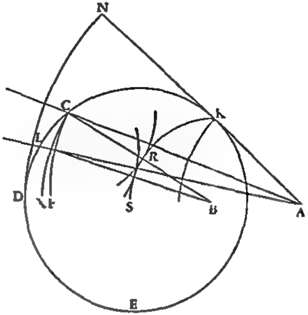
such as K, the straight lines KA, KB the excess by which AK surpasses AD should be to the excess of DB over KB as 3 to 2. For it can similarly be demonstrated by taking another point in the curve such as G that the excess of AG over AD namely NG is to the excess of BD over DG namely DP in this same ratio of 3 to 2. And following this principle M. Descartes constructed these curves in his *Geometry* and he easily recognized that in the case of parallel rays, these curves became hyperbolas and ellipses.



Let us now return to our method and let us see how it leads without difficulty to the finding of the curves which one side of the glass requires when the other side is of a given figure a figure not only plane or spherical or made by one of the conic sections (which is the restriction with which Descartes proposed this problem leaving the solution to those who should come after him) but generally any figure whatever that is to say one made by the revolution of any given curved line to which one must merely know how to draw straight lines as tangents.

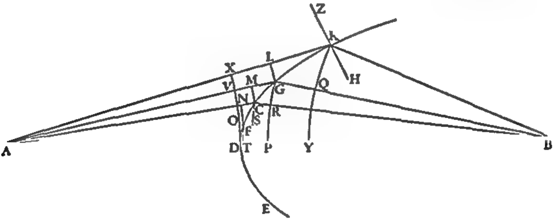
Let the given figure be that made by the revolution of some curve such as AK about the axis AV and that this side of the glass receives rays coming from the point L. Furthermore let the thickness AB of the middle of the glass be given and the point F at which one desires the rays to be all perfectly reunited whatever be the first refraction occurring at the surface AK.

I say that for this the sole requirement is that the outline BDK which constitutes the other surface shall be such that the path of the



the incident rays coming to it from the point A shall deviate them toward the point B Then considering this other curve as already known and that its apex D is in the straight line AB let us divide it up into an infinitude of small pieces by the points G C F and having drawn from each of these points straight lines towards A to represent the incident rays and other straight lines towards B let there also be described with centre A the arcs GL CM FN DO cutting the rays that come from A at L M N O and from the points K G C F let there be described the arcs KQ GR CS FT cutting the rays towards B at Q R S T and let us suppose that the

medium
wn to M
Descartes that the sine of the angle ZKA should be to the sine of the angle



HKB as 3 to 2 supposing that this is the proportion of the refraction of glass

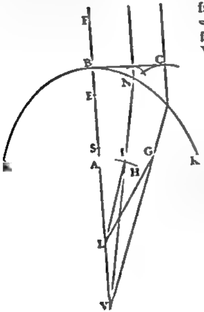
ought to be to DY as 3 to 2 which
of such a nature that having drawn from some point which had been assumed

the axis BA of the line AK, which may be straight or curved Let there be also given in the axis the point L and the thickness BA of the glass and let it be required to find the other surface KDB which receiving rays that are parallel to AB will direct them in such wise that after being again refracted at the given surface AK they will all be reassembled at the point L.

From the point L let there be drawn to some point of the given line AK the straight line LG which, being considered as a ray of light, its refraction GD will then be found. And this line being then prolonged at one side or the other will meet the straight line BL, as here at V Let there then be erected on AB the perpendicular BC which will represent a wave of light coming from the infinitely distant point F since we have supposed the rays to be parallel Then all the parts of this wave BC must arrive at the same time at the point L or rather all the parts of a wave emanating from the point L must arrive at the same time at the straight line BC. And for that it is necessary to find in the line VGD the point D such that having CD

[illegible]

CD plus [illegible] a given length which is a still easier problem than the preceding construction. The point D thus found will be one of those through which the curve ought to pass and the proof will be the same as before And by this it will be proved that the waves which come from the point L, after having passed through the glass KAKB will take the form of straight lines as BC which is the same thing as saying that the rays will become parallel Whence it follows reciprocally that parallel rays falling on the surface KDB will be reassembled at the point L.



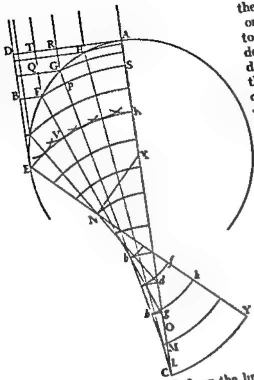
it has been found and about L as centre let there be described GT the arc of a circle cutting the straight line AB at T in case the distance LG is greater than LA for otherwise the arc AH must be described about the same centre

light from the point L to the surface AK and from thence to the surface BDK and from thence to the point F shall be traversed everywhere in equal times and in each case in a time equal to that which the light employs to pass along the straight line LF of which the part AB is within the glass

Let LG be a ray falling on the arc AK Its refraction GV will be given by means of the ... in GV the point D must b... the straight line GL may ... together with $\frac{3}{2}$ of BA and the straight line AI which as is clear make up a given length Or rather by deducting from each the length of LG which is also given it will merely be needful to adjust FD up to the straight line VG in such a way that FD together with $\frac{3}{2}$ of DG is equal to a given straight line which is a quite easy plane problem and the point D will be one of those through which the curve BDK ought to pass And similarly having drawn another ray LM and found its refraction MO the point N will be found in this line and so on as many times as one desires

To demonstrate the effect of the curve let there be described about the centre L the circular arc AH cutting LG at H and about the centre F the arc BP and in AB let AS be taken equal to $\frac{3}{2}$ of HG and SE equal to GD Then con... he point L it is ... the piece A will ... in the transparent body only along AS for I suppose as above the proportion of the refraction to be as 3 to 2 Now we know that the piece of wave which is incident on G advances thence along the line GD since GV is the refraction of the ray LG Then during the time that this piece of wave has taken from G to D the other piece which was at S has reached E since GD SE are equal But while the latter will advance from E to B the piece of wave which was at D will have spread into the air its partial wave the semi diameter of which DC (supposing this wave to cut the line DF at C) will be $\frac{3}{2}$ of EB since the velocity of light outside the medium is to that inside as 3 to 2 Now it is easy to show that this wave will touch the arc BP at this point C For since by construction $FD+\frac{3}{2}DG+GL$ are equal to $FB+\frac{3}{2}BA+AL$ on deducting the equals LH LA there will remain $FD+\frac{3}{2}DG+GH$ equal to $FB+\frac{3}{2}BA$ And again deducting from one side GH and from the other side $\frac{3}{2}$ of AS ... $\frac{3}{2}$ DG equal to FB with $\frac{3}{2}$ o...) is equal to FB with $\frac{3}{2}$ of E... to $\frac{3}{2}$ of EB then deducting these equal lengths from one side and from the other there will remain CF equal to FB And thus it appears that the wave the semi-diameter of which is DC touches the arc BP at the moment when the light coming from the point L has arrived at B along the line LB It can be demonstrated similarly that at this same moment the light that has come along any other ray such as LM MN will have propagated the movement which is terminated at the arc BP Whence it follows as has been often said that the propagation of the wave AH after it has passed through the thickness of the glass will be the spherical wave BP all the pieces of which ought to advance along straight lines which are the rays of light to the centre F Which was to be proved Similarly these curved lines can be found in all the cases which can be proposed as will be sufficiently shown by one or two examples which I will add

Let there be given the surface of the glass AK made by the revolution about



sect one another? It will be seen in the olution of this difficulty that omething very remarkable comes to pass herein and that the waves do not cease to persist though they do not continue entire as when they cross the glasses designed according to the construction we have seen

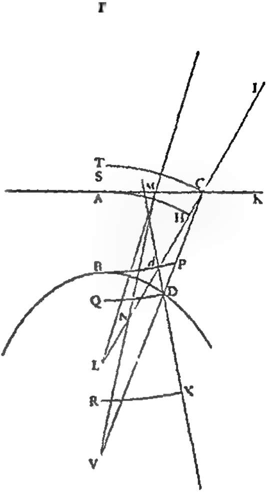
According to what has been shown above the straight line AD which has been drawn at the summit of the phere at right angles to the axis parallel to which the rays come represents the wave of light and in the time taken by its piece D to reach the pherical surface AGE at E its other parts will have met the same surface at F G H etc and will have also formed spherical partial waves of which these points are the centres And the urface EK which all those waves will touch will be the continuation of the wave AD in the phere at the moment when the piece D has reached E Now the line EK i not an arc of a circle but is a ... ENC which touches all the ... med as the evolute of another curve ... h refractions of the parallel rays if we ... d ng describes ... has been thus ... tres F G H

at its end E the said ... described w ... ll how that the said waves ... etc will all t uch t

It is certain that the curve EK and all the others described by the evolution of the curve ENC with different lengths of thread will cut all the rays HL ... h ... les and in such wise that the parts of them inter ... for this follows from what ... ndulorum Now imagining ... other if we consider two of ... RG and if we suppose the ... mm

... the refraction ... o GP as 3 to 2

GM nd similarly ... perpendicular to ... f th ra PG and FP being perpendicular to ... of the refraction as was shown above in explaining th discovery of Descartes. And the same thing occurs in all the small arcs GH HA ... namely that in the quadrilateral which enclose them the side parallel to the axis is to the opposite side as 3 to 2 Then also as 3 to 2 will



cutting the straight line LG at H This arc GT (or AH in the other case) will represent an incident wave of light the rays of which tend towards L Similarly about the centre F let there be described the circular arc DQ which will represent a wave emanating from the point F

Then the wave TG after having passed through the glass must form the wave QD and for this I observe that the time taken by the light along GD in the glass must be equal to that taken along the three TA AB and BQ of which AB alone is within the glass Or rather having taken AS equal to ⅔ of AT I observe that 3/ of GD ought to be ... BQ and ... FD or FC ... to be equ... ... less ½ of SB And this last difference is a given length and all that is required is to draw the straight line FD from the given point F to meet AG so that it may be thus Which is a problem quite similar to that which served for the first of these constructions where FD plus 3/2 of GD had to be equal to a given length

In the demonstration it is to be observed that since the ... BC ... the glass there mu... other side of QD T... wave GT arrives at ... piece ... arrives at Q which is easily deduced from the construction it will be evident as a consequence that the partial wave generated at the point D will touch the arc RX at the moment when the piece Q shall have come to R and that thus this arc will at the same moment be the termination of the movement that comes from the wave TG whence all the rest may be con...

p... surfaces tc... ar... we have several tim... the straight lines which intersect ... angles the waves wh... from ...

For in the ca... face AFE intersec... ... at different point as the figure [P 617] represents what can the waves of light be in this transparent body which are cut at right angles by the converging rays? For they can not be spherical And what will these wave become after the said rays begin to inter

m -diameter BD into two equal
aking
then
beyond
dividing the chord OI at r m ies the
part FO for then F is one of the required points

And as the parallel rays are merely perpendiculars to the waves which fall
ave surface which waves are parallel to AD it will be found that
the surface AB they form on reflexion
hich originate from two opposite evolu
o taking AD as an incident wave when
to say when the piece G shall

on
nue
be
the
h is
that
circle is made to rou ED and whose
centre is D So that it is a kind of cycloid of which now the points can
be found geometrically

Its length is exactly equal to $3/4$ of the diameter of the sphere as can be found and demonstrated by means of these waves nearly in the same way as the mensuration of the preceding curve though it may also be demonstrated in other ways which I omit as outside the subject The area AOBEFY comprised between the arc of the quarter-circle the straight line BE and the curve EFY is equal to the fourth part of the quadrant DAB

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the sum of the one et be to the sum of the other that is to say TF to AS and DE to AK and BE to SK or DV supposing V to be the intersection of the curve EK and the ray FO But making FB perpendicular to DF the ratio of 3 to 2 is also that of BE to the semi diameter of the spherical wave which emanated from the point F while the light outside the transparent body traversed the space BE Then it appears that this wave will intersect the ray FM at the same point V where it is intersected at right angles by the curve EK and consequently that the wave will touch this curve In the same way it can be proved that the same will apply to all the other waves above mentioned originating at the points G H etc to wit that they will touch the curve EK at the moment when the piece D of the wave ED shall have reached E

Now to say what these waves become after the rays have begun to cross one another it is that from thence they fold back and are composed of two contiguous parts one being a curve formed as evolute of the curve ENC in one sense and the other as evolute of the same curve in the opposite sen e Thus the wave KE while advancing toward the meeting place becomes abc whereof the part ab is made by the evol C

b h le l C

b y the same wave becomes

d y and finally CY from whence it subsequently spreads without any fold but always along curved lines which are evolutes of the curve LNC increased by some straight line at the end C

T ich is straight N being the point

wh the sphere falls upon the refrac

tion DE which I no to touch the sphere The folding of

up to the end of the curve C which

the proportion of the refraction as

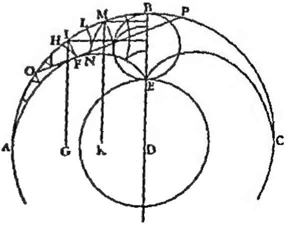
to 2

As many other points as may be desired in the curve NC are found by a Theorem which Mr Barrow has demonstrated in section 12 of his *Lectiones Opticæ* though for another purpose And it is to be noted that a straight line equal in length to this curve can be m F

N F

t

w



o finally the waves that are folded back in reflexion by a concave spherical mirror can be found Let ABC be the section through the axis of a hollow hemisphere the centre of which is D its axis being DB parallel to which I suppose the rays of light to come All the reflexions of those rays which fall upon the quarter circle AB will touch a curved line AFE of which line the end E is at the focus of the hemisphere